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Proximity to the Rho-Meson Condensation Threshold in Nuclei

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Critical conditions for instability in the isovector unnatural-parity channel in nuclei are investigated in terms of the random-phase approximation. The particle-hole residual interaction consists of the π and ρ (2π) meson-exchange attractive components in addition to the short-range repulsive Landau term. It is found that the nonresonant part of the iterated two-pion exchange moves the characteristics of the nuclear phase transition closer to those for rho-meson condensation. This mode may dominate at high density.

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Although the possible existence of a pion-condensed mode of hadronic matter has been widely discussed,[1,2] careful studies of nucleon-nucleon correlations have led to the conclusion that the critical density is greater than 1.5 times the normal density of nuclear matter.[3-6] Recent discussions have thus focused on the possibility of observing precritical phenomena in ordinary nuclei.[7-9] The proximity of the pion condensation threshold should amplify the pionic field in finite nuclei and thus increase the cross sections of (e,e'),[8] (p,p'),[9,10] and (π,γ) or (γ,π) reactions[11,12] for isovector unnatural-parity transitions at large momentum transfer.

Quantitative discussions of precritical behavior have centered on the 1^+, $T=1$ state of ^{12}C for which the $M1$ form factor $|F_M|^2$ predicted by the Cohen-Kurath wave functions[13] is substantially smaller than the data from (e,e') reactions.[8,14] Analyses in the context of precritical behavior with π- and ρ-meson exchange interactions have led to values of the Landau-Migdal parameter $g' \lesssim 0.4$.[8] However, by inclusion also of the nonresonant part of the two-pion exchange (TPE) in the isovector channel, good agreement with the data was obtained for $g' \sim 0.55$.[14] Analyses of recent (p,p') data for the same transition have shown little evidence of precritical enhancements and are consistent with a value $g' \gtrsim 0.60$.[15,16]

In examining the previous analyses, we realized that the amount of enhancement that could be produced, and the subsequent estimate of g', were very sensitive to the strength of the $\vec{\sigma}\times\vec{q}$ correlations provided by the ρ-meson exchange (RME) and the iterated one-pion exchange (OPE). The TPE strength is commonly related to the $N\bar{N}\rightarrow 2\pi$

helicity amplitude in the isovector channel.[17] An alternative extraction of the strength of the ρ exchange from $pp \to \pi d$ processes has resulted in a considerably lower value.[18] A large strength may have the implication that nuclear systems are rather close to the instability due to $\vec{\sigma}\times\vec{q}$ correlations among nucleons. This feature would be more observable, for example, in (e,e') reactions rather than (p,p') reactions.[14] We thus explore in a systematic way the critical threshold of nuclear systems due to the $\vec{\sigma}\times\vec{q}$ and $\vec{\sigma}\cdot\vec{q}$ correlations.

The particle-hole interaction introduced for the analysis of the $M1$ form factor in the spin-isospin channel has the form[14]

$$W_{\text{p-h}}(q) = [V_\pi(q)\vec{\sigma}_1\cdot\hat{q}\vec{\sigma}_2\cdot\hat{q} + V_{2\pi}(q)\vec{\sigma}_1\times\hat{q}\cdot\vec{\sigma}_2\times\hat{q} + g'(q)\vec{\sigma}_1\cdot\vec{\sigma}_2 + h'(q)S_{12}(q)]\vec{\tau}_1\cdot\vec{\tau}_2. \quad (1)$$

The one-pion and two-pion exchanges have the q dependence

$$V_\pi(q) = -[f_\pi^2(q^2)/m_\pi^2]\; q^2/(m_\pi^2+q^2) \quad (2)$$

and

$$V_{2\pi}(q) = -\int_{4m_\pi^2}^\infty [\rho(t)q^2/(t+q^2)]dt, \quad (3)$$

where the weighting factor $\rho(t)$ is taken from Ref. 17. In addition to these meson-exchange potentials, all remaining many-body correlations are introduced in terms of the phenomenological Landau-Migdal short-range interaction[1] g' whose q dependence is conveniently expressed as $g'(q) = [f_\pi^2(q^2)/m_\pi^2]g'$. The tensor part h' is generally believed to be quite small compared to the leading tensor pieces from OPE and TPE at finite q and is dropped completely for our numerical study.[19] An isobar-hole interaction is also provided by making appropriate changes of operators $\vec{\sigma}\to\vec{S}$ and $\vec{\tau}\to\vec{T}$ and of coupling constants $f_\pi \to f_\pi{}^*$ and $\rho \to \rho^*$ in Eqs. (1)–(3).[14]

The resonant part of the $V_{2\pi}(q)$ interaction, as determined in Ref. 17, has a momentum dependence very similar to that for exchange of the vector ρ meson,

$$V_\rho(q) = C_\rho(f_\pi^2/m_\pi^2)q^2/(m_\rho^2+q^2), \quad (4)$$

where $m_\rho = 770$ MeV and $C_\rho \simeq 2.0$. The full 2π exchange corresponds to $C_\rho \sim 2.8$ although the momentum dependence is slightly different. This strength might, however, be reduced in the nuclear medium by Pauli blocking. We shall therefore vary the value of C_ρ.

Investigation of the critical behavior was carried out in the static limit and with the random-phase approximation (RPA). The response function of the nucleus is given by the integral equation for each partial wave J,[20]

$$R_J(q,q';\omega=0) = \Pi_J(q,q';\omega=0) + \int_0^\infty \frac{k^2dk}{(2\pi)^3}\Pi_J(q,k;0)D_J(k)R_J(k,q';0), \quad (5)$$

where $\Pi_J(q,q';\omega)$ is the self-energy or polarization strength of the nuclear medium and $D_J(K)$ is the particle-hole propagator. Explicit expressions for these functions are given elsewhere.[14,20] In finite nuclei they become matrices in the orbital angular momentum $L(L')$. By discretizing of the momentum coordinate, the integral equation can be rearranged into a set of linear equations and solved for $R_J(q,q';0)$ with standard matrix techniques. The critical value g_c' for the phase transition is the largest value of g' for which the (Fredholm) determinant is zero. This condition is equivalent to the RPA instability equation.[20]

Calculations were done for the symmetric doubly closed-shell nuclei ^{16}O, ^{40}Ca, and ^{80}Zr. Excitations of $30\hbar\omega$ for both the nucleon-hole and Δ-isobar–hole terms were included in the self-energy function. This number was needed in order to obtain convergence of the high-momentum components brought in by the 2π exchange. Values of g_c' are plotted in Fig. 1 as a function of the equivalent C_ρ strength for $J^\pi = 1^+$ and 2^- states. We note that $V_{2\pi}$ vanishes for 0^- states.

One sees that g_c' is nearly constant up to $C_\rho \sim 2.2$–2.5 and then rises rapidly after that. To elucidate the behavior, calculations were also done for the 1^+ state of ^{40}Ca with $V_\pi = 0$ and the results are shown as the dashed line in Fig. 1. The behavior of g_c' in the two cases suggests the interesting fact that the $\vec{\sigma}\cdot\vec{q}$ correlation caused by V_π and the $\vec{\sigma}\times\vec{q}$ correlation of $V_{2\pi}$ neither interfere nor cooperate very much in bringing about the nuclear instability, even in nuclei as small as ^{16}O. In infinite nuclear matter they are completely orthogonal.

Since the full two-pion-exchange strength corresponds to $C_\rho \sim 2.8$ (possibly reduced somewhat by Pauli blocking), the above results suggest that the nuclear phase transition is closer to the ρ-mesonic instability than to the pionic instability. A further implication would be that precritical

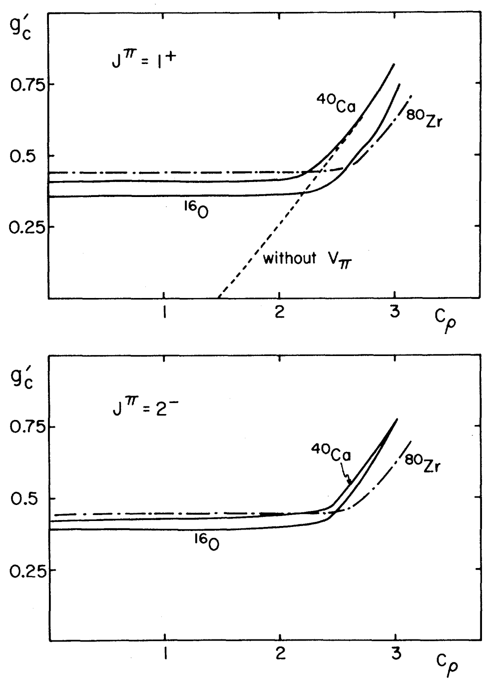

FIG. 1. The critical value g_c' for $J^\pi = 1^+$ and 2^- states in ^{16}O, ^{40}Ca, and ^{80}Zr as a function of the rho-meson coupling strength C_ρ. The dashed curve is obtained with $V_\pi = 0$ for ^{40}Ca.

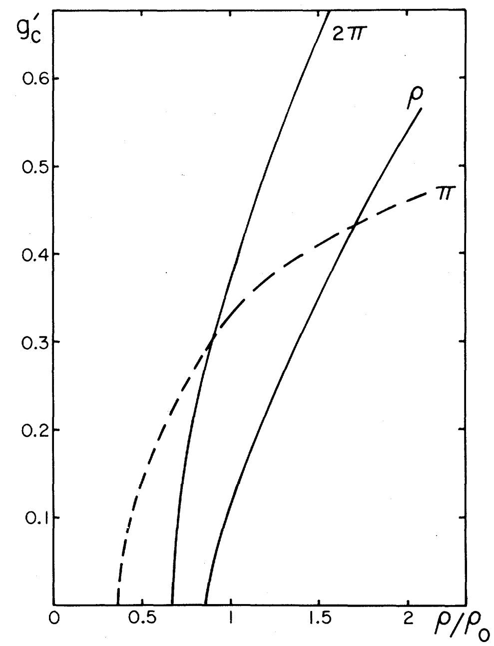

FIG. 2. The critical value g_c' as a function of the density in infinite nuclear matter. The dotted line gives g_c' for pion condensation. The solid lines are for ρ-meson condensation, one with the full TPE and the other with RME. For this calculation, $g'(q)$ is constant.

phenomena would be more pronounced in processes dominated by the $\vec{\sigma}\times\vec{q}$ interaction, such as (e,e') reactions, than in processes mediated by the $\vec{\sigma}\cdot\vec{q}$ interaction, such as (p,p') or (γ,π) reactions.

If the preceding suggestions are correct, it is interesting to consider the situation at higher density where the higher-q components of nuclear matter are brought into the ground state. Since $V_{2\pi}$ is greater than V_π at large q, rho-meson condensation can be expected to be the dominant form of the phase transition at high density. To elucidate, we consider the case of infinite nuclear matter. The full response function of the medium with respect to a ρ meson with momentum q can be written in terms of the lowest-order response (ρ-meson self energy) and the iterated pieces mediated by the $V_{2\pi}$ and g' terms. This condition leads to the expression

$$R_\rho(q,\omega) = -q\frac{U(q)}{1+[V_{2\pi}(q)+g'(q)]U(q)}q, \qquad (6)$$

where $U(q)$ denotes the sum of the Lindhard functions for the nucleon-hole and Δ-isobar–hole excitations. The Lindhard functions are exactly the same as those considered for pion condensation and are given explicitly in Refs. 1 and 2. A similar expression is written for $R_\pi(q,\omega)$.[10]

The critical condition for instability is given by the vanishing of the denominator,

$$\epsilon_\rho(q_c) = 1+[V_{2\pi}(q_c)+g'(q_c)]U(q_c)=0.$$

Calculated results for g_c' are shown in Fig. 2 as a function of the nuclear matter density. As the density increases, the critical value g_c' changes more rapidly for ρ-meson condensation than for pion condensation. The reason is that the increase in the density introduces higher-momentum components and thus emphasizes the high-momentum parts of the interactions. The TPE increases more rapidly than the OPE at $q \gtrsim 2m_\pi$ and surpasses it at $q\sim 3m_\pi$. Because of the blocking effect, the actual values of g_c' should be between the two curves with the TPE and RME. On the basis of this estimate, rho condensation may be realized before pion condensation at high densities.

The above discussion has been confined to the spin-isospin channel in the particle-hole space. Other types of correlations would enter the discussion only through exchange graphs. We have assumed here that they are incorporated in the phenomenological parameter g_c'. In particular, rho exchange also brings in spin-independent correlations. Since they are repulsive, they cannot affect the phase transition that would occur first in the spin-isospin channel, except through the exchange contributions in g'. However, once the nuclear system is in the rho-meson condensed phase, they should work against the increase of the ρ-meson amplitude and keep the condensate moderate at high density. The problem is an interesting one, but is beyond the scope of the present study.

In summary, we find that the widely used amplitudes for RME and iterated OPE[17] place the response of the nuclear medium close to the threshold for rho-meson condensation and that this mode becomes more important as the density increases. Precritical effects should be more evident in reaction probes that are sensitive to $\vec{\sigma} \times \vec{q}$ correlations instead of $\vec{\sigma} \cdot \vec{q}$ correlations. Present analyses of $pp \to \pi d$ processes are not consistent in the interpretations of the ρNN and $\rho N\Delta$ coupling constants.[17,18,21] Additional studies of this and other nuclear processes should lead to better estimates of the strength of the ρ-meson exchange interaction in the nuclear medium.

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