higher than 6 one gets a lower limit for this fraction of about 20%; a pure multipolarity of 4 is not sufficient to reproduce the strong experimental population of the $\frac{13}{2}^+$ state. The more realistic assumption of a distribution around a multipolarity of 6 leads to a fraction of about 40% of the total strength.

This work was supported by the Deutsche Forschungsgemeinschaft and by the Kernforschungszentrum Karlsruhe.

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[2] The spectra in Figs. 1 and 3 are smoothed corresponding to average intervals of about 100 keV.

[3] In the energy interval between about 9.7 and 10.3 MeV this strength gives only an upper limit for the strength decaying into the $\frac{13}{2}^+$ state because of the contribution of the unobserved decay into higher states of ^{207}Pb. Including the information about the decay into these higher states from the α'-γ experiment one gets the dotted curve in Fig. 3(c) as a lower limit.

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^{12}C + ^{12}C Reaction Cross Section between 70 and 290 MeV Obtained from Elastic Scattering

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(Received 21 August 1981)

Measurements of the elastic scattering of ^{12}C on ^{12}C at ten laboratory energies between 120 and 290 MeV were made to determine the reaction cross section σ_R and the total nuclear cross section σ_T. Analysis of these data and existing data at lower energies shows that σ_R reaches a maximum at around 200 MeV in the laboratory whereas σ_T shows little variation with bombarding energy.

PACS numbers: 25.70.Hi

The variation of the nuclear reaction cross section $\sigma_R(E)$ has been investigated recently by DeVries and collaborators.[1,2] They have focused attention on the fact that values of σ_R for light-ion projectiles (up to helium) cannot be parametrized by the usual geometric form

$$\sigma_R(E) = \pi(R+\bar{\lambda})^2\left[1-\frac{Z_1Z_2e^2}{(R+\bar{\lambda})E_{\mathrm{c.m.}}}\right] \qquad (1)$$

over the entire energy range. For example, if R is adjusted to reproduce σ_R at low energies, Eq. (1) overestimates σ_R at high energies. The difference between this geometric limit and the measured values has been parametrized by multiplying the right-hand side of Eq. (1) by a factor $1 - T(E)$ where $T(E)$, the transparency, reflects a finite mean free path for the interacting ions. The authors of Ref. 1 emphasize that values of T increase quite rapidly at energies above 10 MeV/A even for strongly absorbed projectiles such as α particles and that this increase seems to correlate with the known falloff of the nucleon-nucleon cross section[1,3] suggesting the dominance of the latter in determining σ_R.

Calculations of the energy variation of σ_R (Refs. 1 and 2) and σ_T, the total nuclear cross section,[4] based on the Glauber model give quite a good account of the available data and in this sense support the above conclusion. In particular the calculations predict that for ^{12}C + ^{12}C scattering σ_R and σ_T reach a maximum at $E_{\mathrm{c.m.}}$ ~100 MeV.

Although some data for ^{12}C + ^{12}C exist at higher[5,6] and lower[7,8] energies there is a serious lack of experimental data on σ_R for heavy ions in this energy region. Thus measurements of the elastic scattering of $^{12}C+^{12}C$ were undertaken between 120 and 290 MeV. We find that values of σ_R deduced from a parametrized phase-shift analysis of the data reach a maximum at ~200 MeV (lab), in qualitative agreement with the calculations of DiGiacomo *et al.*[2] and with the $^{16}O+^{12}C$ data of Brandan and Menchaca-Rocha.[9] Our values of σ_T, however, indicate that this cross section is approximately constant over the energy range studied.

The experiments were carried out using 4+ and 5+ beams of ^{12}C from the Lawrence Berkeley Laboratory 88-inch cyclotron. Measurements were made in 0.25° steps in the forward angle region (2°–15° lab) which is of critical importance in the determination of σ_R and σ_T. (It can be shown that additional experimental data at angles greater than 25° c.m. have little influence on the deduced values of σ_R and σ_T for the bombarding energies considered here.) Since an accurate knowledge of the scattering angle is essential for the accurate determination of σ_R and σ_T two sets of four lithium-drifted silicon detectors of 3 mm thickness were located on movable arms at equal angles to the left- and right-hand sides of the beam direction. The targets were made of natural carbon of thicknesses between 275 and 1000 $\mu g/cm^2$ determined to ±5% by α energy-loss measurements. In addition, two monitor detectors placed at ±8° above and below the scattering plane monitored the vertical position of the beam.

Data were taken at ten energies between 121.6 and 287.8 MeV. In two cases (121.6 and 161.05 MeV) the angular range of the data was extended beyond 20° lab.

Final differential cross sections were obtained by averaging results from left and right detectors thereby eliminating to first order effects of beam movement and misalignment. The angular error of the data was essentially due to the precision with which the angles of the detectors could be set and was estimated to be 0.125°. The angular aperture of the detector collimators in the scattering plane was 0.3° which, when combined with the spot size and angular divergence of the beam, yielded a total "smoothing" angle of approximately 0.5° (lab). An additional measurement at 289 MeV was carried out with use of two position sensitive detectors and a strip target. The angular accuracy in this case was ±0.07°.

The reaction cross section and the total nuclear cross section for identical particle scattering are given in terms of the S matrix elements obtained from a partial-wave expression of the differential cross section as

$$\sigma_R=\frac{2\pi}{k^2}\sum_{\substack{l=0\\ \text{(even)}}}^{l_{\max}}(2l+1)(1-|S_l|^2)\ ,$$

$$\sigma_T=\frac{4\pi}{k^2}\sum_{\substack{l=0\\ \text{(even)}}}^{l_{\max}}(2l+1)(1-\operatorname{Re}S_l).$$

In order to fit the data the following parametrization of S_l was chosen:

$$\eta_l=|S_l|=1-\frac{1}{1+\exp[(l-l_\eta)/\Delta]}$$

and

$$\cos 2\delta_l=\frac{\operatorname{Re}S_l}{|S_l|}=1-\frac{2}{1+\exp[(l-l_\delta)/\Delta]}\ .$$

This parametrization was used in a computer program which optimized the three free parameters (l_η, l_δ, and Δ) using a nonlinear least-squares fitting procedure. The ability of this parametrization to reproduce known values of σ_R and σ_T was determined by fitting theoretical "data" between 5° and 25° (c.m.) generated from an optical-model calculation. Values of σ_R and σ_T so deduced agreed to within 1.5% with the optical-model values.

When the experimental data were fitted all theoretical angular distributions were smoothed over a total angle of 1° in order to simulate the experimental geometry. Several of the resultant differential cross sections are shown in Fig. 1. The results are plotted against momentum transfer rather than c.m. angle to emphasize that the positions of the most forward-angle minima are almost energy independent and that the depths of the minima evolve smoothly over the energy range investigated. Values of σ_R and σ_T obtained from the analysis are displayed in Fig. 2.

The errors in the deduced values of σ_R and σ_T contain various contributions. Tests showed that changes in absolute normalization of ±5% produced only ±0.5% changes in values of σ_R and σ_T. Furthermore the effect of smoothing the theoretical predictions during the fitting procedure produced changes of the same order. However, a relatively large change in σ_R and σ_T could be induced by making a small systematic shift in the experimental angles. At 288 MeV a shift of only ±0.25° (c.m.) produced ∓5% changes in σ_R and σ_T,

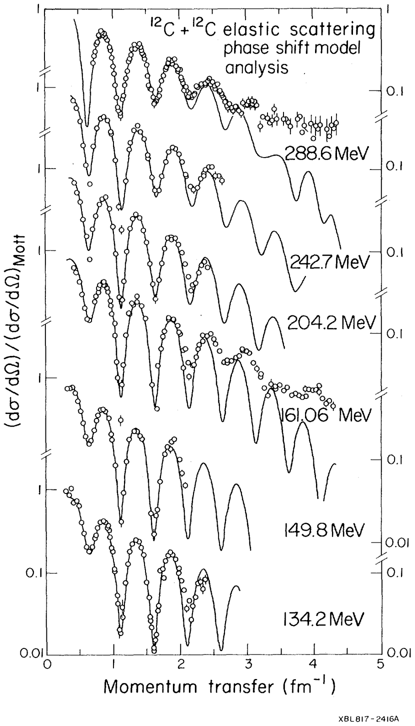

FIG. 1. Elastic scattering cross sections as a function of momentum transfer. The smooth lines are the results of calculations using the parametrized phase-shift model.

and this effect was therefore considered to be the dominant source of experimental error. The total estimated experimental error was obtained by adding in quadrature the errors estimated from all these effects.

The above analysis was also applied to published data[8] between 70 and 127 MeV (lab) and results are included in Fig. 2. These values seem to exhibit some structure. However, a considerable improvement in the quality of the data at forward angles would be required to confirm this suggestion.

To investigate the model dependence of the deduced values of σ_R and σ_T our data were also fitted with use of the standard six-parameter optical model. It should be made clear that in these fits emphasis was placed on obtaining the best description of the forward-angle diffraction structure and not on the determination of a smoothly energy-dependent optical model potential. The deduced values of σ_R, although slightly higher (see Fig. 2), exhibit the same trend with energy as those obtained from the phase-shift model. The values of σ_T are also very similar for the two models except at the lowest energies.

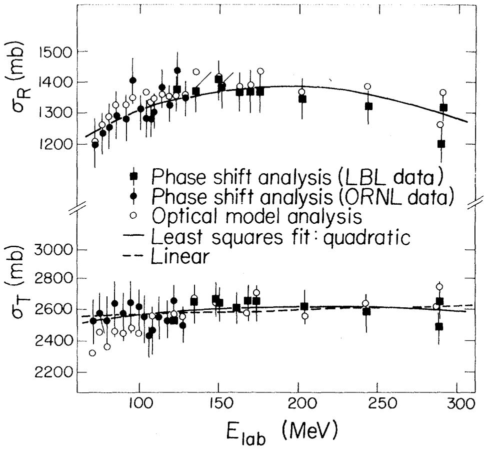

FIG. 2. Deduced values of σ_R and σ_T from parametrized phase-shift analysis shown with least-squares quadratic fit for $\sigma_R(E)$ and least-squares linear and quadratic fits for $\sigma_T(E)$. Results of optical-model fits are also given.

In Fig. 3 our results for σ_R and σ_T are compared with the calculations of DiGiacomo *et al.*[2] and Peng, de Vries, and DiGiacomo,[4] respectively.

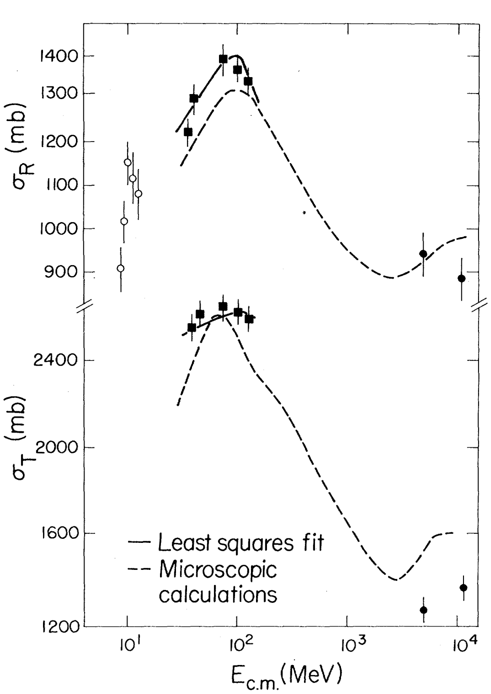

FIG. 3. Least-squares quadratic fits from Fig. 2 shown with microscopic calculations for σ_R (Ref. 2) and σ_T (Ref. 4). Representative data points from the present work (squares) as well as low-energy data taken from Ref. 7 (open circles) and high-energy data from Ref. 6 (filled circles) are given in the figure.

The calculation for σ_T disagrees somewhat with our data which do not exhibit the sharp rise and fall of the predicted values and which, in fact, are consistent with a constant value ($\sigma_T = 2570$ mb) over the energy range studied. It should be pointed out, however, that the calculation[4] of σ_T did not include the effects[3] of Fermi motion, Pauli blocking, and the real nuclear potential whereas these effects were included in the calculation[2] of σ_R. Indeed, the agreement with experiment is more satisfactory for σ_R. In particular it does appear that both the theoretical and experimental reaction cross sections reach a maximum at $E_{\text{c.m.}} \approx 100$ MeV.

Some caution is required, however, in drawing conclusions from these comparisons. Brink and Satchler[10] have noted that a maximum in the reaction cross section may arise in part from the attraction of the real nuclear potential. Over a limited energy range this attraction produces trajectories with a significant curvature thus pulling flux into the region of strong absorbtion and lengthening the path over which the two nuclei are in close contact. Although Brink and Satchler concluded from calculations using energy-independent optical potentials that this effect could not produce all of the observed falloff in σ_R, relatively small changes in the real potential could produce large effects at low and medium energies ($E_{\text{c.m.}} \approx 100$ MeV). This may be seen from the simple semiclassical formula

$$\sigma_R = \pi R^2 \left(1 - \frac{V_C(R) + V_N(R)}{E_{\text{c.m.}}}\right), \tag{2}$$

where the radius R is deduced from the high energy σ_R values (Ref. 6 and Fig. 3). Equation (2) is similar to Eq. (1) but does not neglect the nuclear attraction in the vicinity of the Coulomb barrier. Unfortunately the strength of the nuclear attraction in the case of $^{12}\text{C} + {}^{12}\text{C}$ cannot be deduced with high precision since present optical models are unable to give a satisfactory account of the elastic scattering data at all angles (Ref. 8).

In conclusion our experiments indicate a maximum in $\sigma_R(E)$ at $E_{\text{c.m.}} \approx 100$ MeV as predicted in Refs. 1 and 2. However, the role of the energy dependence of the nucleon-nucleon force in producing such a maximum remains uncertain. Accurate measurements of σ_R and σ_T over a wider range of energies will undoubtedly be of use in understanding the respective roles of "mean field" and "nucleon-nucleon" aspects of the problem. Further theoretical work in the low- and medium-energy region near the maximum of $\sigma_R(E)$ would also be valuable.

This work was supported by the Director, U. S. Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics, and by Nuclear Sciences of the Basic Energy Sciences Program of the U. S. Department of Energy under Contract No. W-7405-ENG-48.

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