Upper-Hybrid Wave Collapse

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It is shown that the upper hybrid mode can collapse into thin magnetic-field–aligned filaments as a result of coupling to the electrostatic ion-cyclotron mode.

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Recent experimental work on large-amplitude beam-driven instabilities[1] has shown that both the oblique electron-plasma and the electron-cyclotron modes can collapse into thin magnetic-field–aligned filaments which are confined to the axis of the plasma column. Both types of instability exhibited a quasiperiodic burstlike behavior with multiple modulation time scales characteristic of low-frequency electron, ion-acoustic, and ion-cyclotron waves. Furthermore, in the latter case, correlation measurements were possible which showed that the amplitude modulations on the pump wave were strongly correlated with low-frequency density depletions.

Existing theoretical work on plasma-wave collapse deals largely with the case of an unmagnetized plasma[2,3] and is not directly applicable to the results of Ref. 1. In this Letter, I discuss a special case of wave collapse in a magnetized plasma which explains the filamentary collapse described in Ref. 1. The assumption which is introduced to simplify the calculation is the neglect of the spatial variation of the collapsed structures along the magnetic field. This is the special case of the collapse of the upper hybrid mode due to the coupling to the ion-cyclotron mode.

Apart from its intrinsic interest, this problem is considered to be relevant to the results of Ref. 1 because the collapsed structures were observed to be cigarlike regions having their major axes aligned with the magnetic field. This suggests that the present results would be applicable across transverse sections near the center of the major axis of a collapsed region, inasmuch as one can regard the approximation as equivalent to neglecting end effects.

Following Ref. 4 we split the wave motion into its fast and slow parts. Consider first the fast motion. The fast electrostatic potential, χ_f, is related to the fast electron density perturbation, n_f, by Poisson's equation:

$$\Delta\chi_f = 4\pi e n_f, \tag{1}$$

where $\Delta = \partial_x^2 + \partial_y^2$, x and y being the coordinates perpendicular to the magnetic field $\vec{B}$. The components of the fast electron flow across $\vec{B}$, $\vec{v}_f = (u, v)$, are given by the linearized electron equations of motion:

$$\begin{aligned} \partial_t u &= -(a^2/n_0)\partial_x n_f + (e/m)\partial_x \chi_f - \Omega_e v, \\ \partial_t v &= -(a^2/n_0)\partial_y n_f + (e/m)\partial_y \chi_f + \Omega_e u, \end{aligned} \tag{2}$$

a being the electron thermal velocity, Ω_e the electron gyrofrequency, and n_0 the equilibrium density.

Next, as in previous work,[2-4] the effect of the slow density perturbation n_s is retained in the electron continuity equation for n_f, giving

$$\partial_t n_f + n_0(\partial_x u + \partial_y v) = -\partial_x(n_s u) - \partial_y(n_s v). \tag{3}$$

A stream function ψ and a velocity potential φ are now introduced so that

$$u = \partial_x\varphi + \partial_y\psi \quad \text{and} \quad v = \partial_y\varphi - \partial_x\psi. \tag{4}$$

Then (1), (2), and (4) yield

$$(\partial_t^2 + \omega_{\rm UH}^2 - a^2\Delta)\Delta\varphi = [(\omega_p^2 - a^2\Delta)/n_0](\partial_t n_f + n_0\Delta\varphi), \tag{5}$$

where $\omega_{\rm UH}$ and ω_p denote the upper-hybrid and plasma frequencies, while from (3) and (4) we get

$$\partial_t n_f + n_0\Delta\varphi = -[\operatorname{div}(n_s\nabla\varphi) + (\partial_y\psi\partial_x - \partial_x\psi\partial_y)n_s]. \tag{6}$$

To lowest order on the fast time scale, we can neglect $a^2\Delta$ compared to ω_p^2 on the right-hand side of (5).[2-4] Equations (5) and (6) then yield

$$(\partial_t^2 + \omega_{\rm UH}^2 - a^2\Delta)\Delta\varphi = -(\omega_p^2/n_0)[\operatorname{div}(n_s\nabla\varphi) + (\partial_y\psi\partial_x - \partial_x\psi\partial_y)n_s]. \tag{7}$$

Let us now remove the fast variation at $\omega_{\rm UH}$ from (7) by introducing slowly varying amplitudes defined by

$$(\varphi, \psi) = [(\Phi, \Psi)\exp(-i\omega_{\rm UH}t) + {\rm c.c.}]/2. \tag{8}$$

In addition, ψ is eliminated from the right-hand side of (7) by use of (2) and (4) to get $\partial_t \psi = -\Omega_e \varphi$ from which we obtain

$$i\omega_{\rm UH}\Psi = \Omega_e \Phi. \tag{9}$$

On neglect of the second time derivative of Φ, (7) yields

$$(2i\omega_{\rm UH}\partial_t + a^2\Delta)\Delta\Phi = (\omega_p^2/n_0)[\operatorname{div}(n_s \nabla\Phi) + (i\Omega_e/\omega_{\rm UH})(\partial_x \Phi \partial_y - \partial_y \Phi \partial_x)n_s]. \tag{10}$$

To obtain an equation for the slow motion, we use the linearized ion equation of motion as in Refs. 2–4. If we ignore ion pressure, the transverse components of the ion flow are given by

$$\partial_t U = -(e/M)\partial_x \chi_s + \Omega_i V, \quad \partial_t V = -(e/M)\partial_y \chi_s - \Omega_i U,$$

where χ_s is the slow electrostatic potential and where Ω_i denotes the ion gyrofrequency. If, in addition, we assume that quasineutrality applies to the slow motion, then, by combining the above equations with the linearized ion equation of continuity, we get

$$(\partial_t^2 + \Omega_i^2)n_s = (en_0/M)\Delta\chi_s.$$

But, after averaging the electron equations over the fast flow, we obtain for the adiabatic electron flow

$$\Delta(e\chi_s/m - a^2 n_s/n_0 - \langle u^2 + v^2\rangle/2) = \operatorname{div}\langle\vec{\omega}_f \times \vec{v}_f\rangle,$$

where $\vec{\omega}_f$ is the fast electron vorticity. Eliminating χ_s gives

$$(\partial_t^2 + \Omega_i^2 - c_s^2\Delta)n_s = (mn_0/M)[\Delta(\langle u^2 + v^2\rangle/2) + \operatorname{div}\langle\vec{\omega}_f \times \vec{v}_f\rangle], \tag{11}$$

where c_s is the ion acoustic speed.

Now the last term on the right-hand side of (11) represents the emission of ion-cyclotron waves by the electron vortex flow. As in previous work,[2-4] we neglect this source term since its role is a secondary one as regards the initiation of collapse. Substituting (4), (8), and (9) in (11) yields

$$(\partial_t^2 + \Omega_i^2 - c_s^2\Delta)n_s = (mn_0/4M)\Delta[(1 + \Omega_e^2/\omega_{\rm UH}^2)|\nabla\Phi|^2 + (2i\Omega_e/\omega_{\rm UH})(\partial_x \Phi \partial_y - \partial_y \Phi \partial_x)\Phi^*]. \tag{12}$$

Equations (10) and (12) have two limitations for applications to the results of Ref. 1. Firstly, they only incorporate one of the three observed modulation time scales (namely, $2\pi/\Omega_i$). Secondly, the relative scales of the collapse regions along and across $\vec{B}$ obviously cannot be calculated. These limitations can only be removed by incorporating the longitudinal variation.[5]

Nevertheless, the present treatment can explain the filamentary collapse at $\omega_{\rm UH}$ described in Ref. 1. To show this, let us change to polar coordinates (r,θ) and assume that $\partial/\partial\theta = 0$. The latter assumption, while not essential, is introduced because the bursts observed to occur on the time scale associated with the ion-cyclotron wave displayed the azimuthal symmetry of the $m=0$ mode.[1] Moreover, it was also shown that there was an approximate balance between the plasma pressure and the ponderomotive force. Therefore, let us also assume static balance.

With these simplifications (12) yields

$$n_s = -(mn_0/4Mc_s^2)(1 + \Omega_e^2/\omega_{\rm UH}^2)|\Phi_r|^2.$$

Hence, from (10), the radial derivative of the velocity potential satisfies

$$\left(2i\omega_{\rm UH}\frac{\partial}{\partial t} + a^2\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\right)\Phi_r + \frac{1}{4\lambda_{\rm D}^2}\left(1 + \frac{\Omega_e^2}{\omega_{\rm UH}^2}\right)|\Phi_r|^2\Phi_r = 0, \tag{13}$$

where $\lambda_{\rm D}$ is the Debye length.

It can now be shown that a singularity will develop after a finite time, by the method of Ref. 4. First note that (13) has the invariant

$$I = \int_0^\infty \left(|\Delta\Phi|^2 - \frac{1 + \Omega_e^2/\omega_{\rm UH}^2}{8a^2\lambda_{\rm D}^2}|\Phi_r|^4\right) r\,dr,$$

where, now, $\Delta = r^{-1}(\partial/\partial r)r\,\partial/\partial r$. Next, consider $A = \int_0^\infty |\Phi_r|^2 r^3 dr$. From (13), we have $d^2A/dt^2 = 2Ia^4/\omega_{\rm UH}^2$. Hence, $A = a^4 It^2/\omega_{\rm UH}^2 + c_1 t + c_2$, where c_1 and c_2 are constants of integration. Since $A > 0$, it follows from the last result that a singularity will develop after a finite time if $I<0$. As is well known,[4] such singularities describe the collapse of the wave field. In the present case, the collapse regions take the form of magnetic-field–aligned filaments.

In view of the high amplitudes encountered in the experiments reported in Ref. 1, the condition $I<0$ would have been satisfied and, consequently, magnetic-field- aligned filamentary collapse was observed.

[1]P. J. Christiansen, V. K. Jain, and L. Stenflo, Phys. Rev. Lett. 46, 1333 (1981).
[2]S. G. Thornhill and D. ter Haar, Phys. Rep. C 43, 43 (1978).
[3]L. I. Rudakov and V. N. Tsytovich, Phys. Rep. C 40, 1 (1978).
[4]V. E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745 (1972) [Sov. Phys. JETP 35, 908 (1972)].
[5]Recently, ter Haar and Statham (to be published) have derived equations describing this coupling.