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Experimental Limits on Neutrino Oscillations

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A search for neutrino oscillations in a wide-band neutrino beam at Fermilab with use of the 15-ft bubble chamber is reported. No evidence is found for neutrino oscillations and upper limits are set on the mixing angles and neutrino mass differences in the transitions $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$, and $\nu_e \to \nu_{\sim e}$, where $\sim e$ denotes "not *e*."

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Various authors have investigated[1] the possibility of neutrino oscillations, i.e., the time-dependent mixing between different types of neutrinos. These oscillations can only occur if there is a nonzero mass difference between the neutrinos involved and the lepton numbers of the neutrinos are not rigorously conserved. With three or more neutrino types, the situation is quite complex, and depends on many parameters. In this paper, we consider only oscillations between two types of neutrinos at a time. In this case, the observed neutrino types, say ν_α and ν_β, are quantum mechanical mixtures of the neutrino mass eigenstates, ν_1 and ν_2:

$$\nu_\alpha = \cos\theta\,\nu_1 + \sin\theta\,\nu_2,$$

$$\nu_\beta = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2,$$

where θ is the mixing angle between the two types of neutrinos. The probability of the appearance of a neutrino ν_β, when initially a neutrino ν_α was created, is

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\,\sin^2(1.27\Delta m^2 l/E),$$

where $\Delta m^2 = m_1^2 - m_2^2$ is in units of electronvolts squared, E is the neutrino energy in megaelectronvolts, and l is the distance from the source of the neutrinos in meters.

Experimentally, we observe the number of neutrino interactions N_α in which charged leptons of the type l_α (i.e., e^-, μ^-, or τ^-) are produced. To relate these numbers N_α to the oscillation probability $P(\nu_\alpha \to \nu_\beta)$, we have to integrate over the decay space and neutrino energy spectrum. For small oscillation probabilities, we obtain

$$R_{\alpha\to\beta} = \frac{\iint P(\nu_\alpha \to \nu_\beta)\varphi_\alpha \sigma_\beta \, dE\, dl}{\iint \varphi_\alpha \sigma_\alpha \, dE\, dl} = \frac{N_\beta}{N_\alpha},$$

where φ_α is the initial flux of neutrinos ν_α (it is a function of both E and l) and σ_α and σ_β are the total charged-current interaction cross sections of neutrinos ν_α and ν_β, respectively. From this expression we obtain, for small values of Δm^2,

$$\sin(2\theta)\Delta m^2 = [1.27(l/E)_{\text{av}}]^{-1}(N_\beta/N_\alpha)^{1/2},$$

where

$$(l/E)_{\text{av}}{}^2 = \frac{\iint (l/E)^2 \varphi_\alpha \sigma_\beta \, dE\, dl}{\iint \varphi_\alpha \sigma_\alpha \, dE\, dl}.$$

For large values of Δm^2, we see the average of many oscillation wavelengths. Since the average value of $\sin^2(1.27\, l/E \Delta m^2)$ is $\frac{1}{2}$, we obtain in this limit

$$\sin^2(2\theta) = 2\frac{N_\beta}{N_\alpha}\frac{\iint \varphi_\alpha \sigma_\alpha \, dE\, dl}{\iint \varphi_\alpha \sigma_\beta \, dE dl}.$$

In this experiment,[2,3] which used the two-horn-focused wide-band neutrino beam at Fermilab, the dominant component in the beam consists of ν_μ from $(\pi^+, K^+) \to \mu^+ + \nu_\mu$ decays, with a background of about 1% of ν_e from $K \to \pi + e^+ + \nu_e$ decays. We can search for $\nu_\mu \to \nu_e$ oscillations by looking for ν_e in the beam in excess of the expected ν_e flux from K_{e3} decays. We can search for $\nu_\mu \to \nu_\tau$ oscillations by looking for ν_τ's in the initially ν_μ beam via their interactions in the chamber producing a τ^-, followed by the decay of the τ^- into electrons, $\tau^- \to e^- + \nu_\tau + \bar{\nu}_e$. These events will look similar to ν_e interactions in that they have an e^- but no μ^- in the final state, but the ν_τ events will have different kinematics. We can search for the oscillations of ν_e into any other neutrino flavor ($\nu_e \to \nu_{\sim e}$), by using the ν_e component of the beam from K_{e3} decays as a source, and looking for a depletion in the ν_e beam. Among other things, this limit applies to $\nu_e \to \nu_\tau$ oscillations.

We see no evidence for neutrino oscillations in this experiment, and use the data to set upper limits on oscillations in two different ways. The first is to calculate the expected number of ν_e events due to the ν_e contamination in the beam and attribute the excess to oscillations. This approach is limited by systematic uncertainties in the beam-flux calculations. The second is to increase the sensitivity by using low-energy data or kinematic selection procedures in which case no flux subtraction is made and all selected ν_e events are attributed to oscillations.

The data used in this experiment[2,3] come from a 134 000-picture exposure of the Fermilab 15-ft bubble chamber filled with a heavy Ne/H_2 mixture. The neutrinos are produced in a 400-m-long decay region, followed by a 1000-m shield and the 15-ft bubble chamber. Thus l, the distance from the source to the observation of the neutrinos, is 1200 ± 200 m. The energy spectrum in the wide-band beam ranges from a few up to a few hundred gigaelectronvolts and peaks at ~30 GeV. The bubble chamber is 4 m in diameter and is in a 30-kG magnetic field. In the heavy neon (64 at.% neon with 36% hydrogen) mixture, the hadronic interaction length is 125 cm and the radiation length is 40 cm. Hadrons will typically interact, muons will leave the chamber without interacting, and electrons can be reliably identified by their radiation.

All pictures were scanned for events with an $e^\pm$ in the final state. A total of 794 events with an e^- were found where the e^- momentum was over 300 MeV/c. From this sample, e^- events were selected inside a restricted fiducial volume, and to reduce the background of ν_μ-induced events with a Compton electron, the momentum of the e^- was required to be larger than 1 GeV/c, leaving a sample of 595 e^- events. These events were corrected for background (12 ± 6 ν_μ events with Compton electrons over 1 GeV/c) and for efficiencies (electron identification efficiency of 95%, scanning efficiency of 72%, and a 10% loss due to confused events) to yield a corrected number of 942 ± 85 ν_e interactions.

Approximately 3.5% of the pictures, spaced evenly throughout the film, were scanned for ν_μ charged-current interactions, which were defined to be events with at least one negative leaving track. The fiducial-volume cut was imposed, and all the muon candidates were required to have a momentum greater than 1 GeV/c. After correcting for fake μ^- events (the number of neutral-current or neutron events in which a negative hadron left the chamber without interacting was estimated from the number of interacting negative tracks to be 10%) and scanning efficiency of 93%, 68 500 ± 4000 ν_μ charged-current interactions remain.

Both the ν_μ and ν_e interactions were measured. The measured energy was corrected upward by 10% for missing neutral particles, mismeasured tracks, etc. This correction was determined by comparing the measured energy with the predicted energy for a sample of events using a narrow-band neutrino beam in the same heavy-neon bubble chamber.[4]

The number of ν_e interactions from the conventional sources of ν_e's such as K_{e3} and μ decays in the decay pipe relative to the total number of ν_μ interactions has been calculated by a Monte Carlo program to be $(1.5 \pm 0.3)\%$. Many uncertainties such as the overall flux normalizations, etc., cancel out in this ratio, which depends mainly on the overall geometry of the beam, which is well known, and the K/π ratios, which have been measured both at Fermilab and at CERN.[5] We thus expect a total of $(68\,500 \pm 4000) \times (1.5 \pm 0.3) \times 10^{-2} = 1027 \pm 210$ ν_e interactions, which is in agreement with the corrected number of ν_e events, 942 ± 85. We thus have no evidence for an anomalous ν_e flux that can be ascribed to neutrino oscillations. The number of ν_e interactions from anomalous ν_e sources is -85 ± 230, or less than 215 to a 90% confidence limit. Comparing this number with the total number of ν_μ interactions (68 500 ± 4000) and assuming that the ν_μ and ν_e charged-current total cross sections are equal, we obtain the limit

$$R_{\mu \to e} \leq 3 \times 10^{-3} \text{ at } 90\% \text{ C.L.}$$

These numbers can also be used to set a limit on $\nu_\mu \to \nu_\tau$ by considering the process $\nu_\tau + \mathrm{Ne} \to \tau^- + \text{hadrons}$, followed by the decay $\tau^- \to e^- + \nu_\tau + \bar{\nu}_e$, which would look like the e^- events to which the 90% confidence level limit above of 215 events applies. Using the measured $\tau^- \to e^- + \nu_\tau + \bar{\nu}_e$ branching ratio[6] of 17%, we obtain

$$R_{\mu \to \tau} \leq 2 \times 10^{-2} \text{ at } 90\% \text{ C.L.}$$

The above limits rely on a subtraction that depends on a calculation of the ν_e/ν_μ flux ratio. We now describe methods that do not depend on any flux calculation. We take all the observed ν_e interactions and use them to determine an upper limit on neutrino oscillations without making a subtraction for the expected conventional ν_e flux.

The average energy $\tilde{E}$ $(\equiv \langle 1/E^2 \rangle^{-1/2})$ for the ν_μ events is 18.5 GeV, or an $(l/E)_{\mathrm{av}} = 0.065$ m/MeV. We can obtain more sensitive limits by using only events in the region,[7] $5 \leq E_\nu \leq 10$ GeV, near the low end of our energy spectrum. There are 34 e^- events in this region. Correcting for Compton background (1 ± 1 event), electron identification, and scan efficiency (95% and 72%, respectively) yields 48 ± 9 ν_e interactions.[8] The corresponding number of ν_μ interactions is 4950 ± 1000. These numbers give a 90% confidence level upper limit of

$$R_{\mu \to e} \leq 1.3 \times 10^{-2}.$$

The average energy $(\tilde{E})$ for these events is 7.6 GeV, and $(l/E)_{\mathrm{av}} = 0.16$ m/MeV. This corresponds to a small Δm^2 limit of

$$\sin(2\theta)\Delta m^2 \leq 0.6 \text{ eV}^2$$

and is shown as curve a' in Fig. 1. For the limit on $\sin^2(2\theta)$ for large values of Δm^2, small ener-

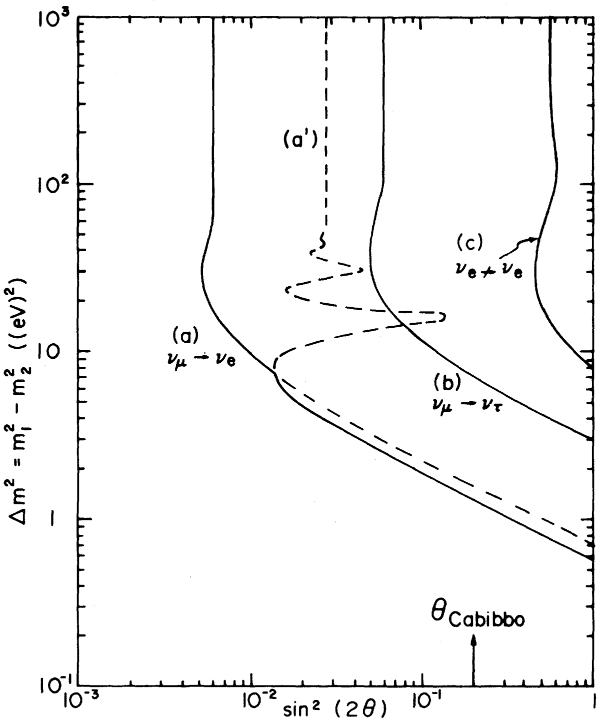

FIG. 1. Limits on the neutrino oscillation parameters $\sin^2(2\theta)$ vs Δm^2. Curves a, b, and c display the 90% confidence level limits for the transitions $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$, and $\nu_e \to \nu_{\sim e}$, respectively, obtained by the flux subtraction method. Curve a' displays the 90% confidence level limit for the transition $\nu_\mu \to \nu_e$, obtained from the low-energy data. The 90% confidence level limit obtained for the $\nu_\mu \to \nu_\tau$ transition by the kinematical method is also given by curve b. For each transition, the region to the right of the solid line is excluded by this experiment. Also shown is the Cabibbo angle.

gies are no longer important, and the best limit is the flux-subtracted limit that uses all energies,

$$\sin^2(2\theta) \leqslant 6\times 10^{-3}.$$

The $\nu_\mu \to \nu_\tau$ limit is not improved by going to lower energies because the ν_τ charged-current total cross section falls rapidly below 10 GeV ($\sigma_{\nu_\tau}/\sigma_{\nu_\mu} \leqslant 0.1$). A limit independent of flux calculations can be obtained by using the kinematics of the $\nu_\tau + \text{Ne} \to \tau^- + \text{hadrons}$, $\tau^- \to e^- \nu_\tau \bar{\nu}_e$ events to distinguish them from normal ν_e interactions. In the ν_τ events, the two outgoing neutrinos from the leptonic τ^- decay give rise to a large P_{out}, the momentum imbalance perpendicular to the plane of the electron and the incident neutrino.[9] Selection of events with $P_{\text{out}} \geqslant 1.0$ GeV/c retains[10] 35% of the ν_τ events but reduces the ν_e events by a factor of 15. Only 41 of the 595 e^- events in the reduced fiducial volume have $P_{\text{out}} \geqslant 1.0$ GeV/c and $P_{e^-} \geqslant 1.0$ GeV/c. We use these events to derive an upper limit on the number of ν_τ interactions. Correcting for e^- identification and scan efficiencies, the 35% probability of keeping a ν_τ interaction with $P_{\text{out}} \geqslant 1$ GeV/c, and the $\tau^- \to e^- \nu_\tau \bar{\nu}_e$ branching ratio, we obtain the 90% confidence level upper limit of 1395 on the total number of ν_τ interactions. We compare this with the 68 500 ± 4000 ν_μ interactions, and obtain the limit

$$R_{\mu\to\tau} \leqslant 2\times 10^{-2} \text{ at 90\% C.L.}$$

This limit, which does not depend on a flux subtraction, is the same as the one obtained with the flux subtraction. The corresponding small Δm^2 limit with an $(l/E)_{\text{av}} = 0.04$ m/MeV is

$$\sin(2\theta)\Delta m^2 \leqslant 3 \text{ eV}^2.$$

For large Δm^2, the limit on $\sin^2(2\theta)$ is obtained from the limit on $R_{\mu\to\tau}$ above, taking into account the average $\sigma_{\nu_\tau}/\sigma_{\nu_\mu}$ ratio of ~0.6 for the energies relevant here:

$$\sin^2(2\theta) \leqslant 6\times 10^{-2}.$$

We can set a limit on ν_e disappearance by comparing the observed number of ν_e events (942 ± 85) with the number expected from the ν_e/ν_μ flux ratio calculation (1027 ± 210). The number of events missing is 85 ± 230, or less than 380 to a 90% confidence level. The limit on $P(\nu_e \to \nu_{\sim e})$ is 380/(942 + 380) or

$$R_{e\to\sim e} \leqslant 0.3 \text{ at 90\% C.L.}$$

For the ν_e component in the beam, the average energy $\tilde{E}$ is 22 GeV and the corresponding $(l/E)_{\text{av}}$ is 0.055 m/MeV. This gives a limit of

$$\sin(2\theta)\Delta m^2 \leqslant 8 \text{ eV}^2$$

which is shown as curve c in Fig. 1.

TABLE I. Summary of limits on the neutrino oscillation parameters obtained in this experiment (90% confidence level upper limits).

Oscillation channel $\nu_\alpha \to \nu_\beta$	Limits on Δm^2 for $\sin^2(2\theta) \sim 1$ $(l/E)_{\text{av}}$ (m/MeV)	Δm^2 eV^2	Limits on $\sin^2(2\theta)$ for large Δm^2
$\nu_\mu \to \nu_e$	0.16	$\leqslant 0.6$	$\leqslant 6\times 10^{-3}$
$\nu_\mu \to \nu_\tau$	0.04	$\leqslant 3$	$\leqslant 6\times 10^{-2}$
$\nu_e \to \nu_{\sim e}$	0.055	$\leqslant 8$	$\leqslant 0.6$

All of the limits discussed above are summarized in Table I. It should be pointed out that these limits are calculated with the assumption that only two neutrinos at a time are involved in the oscillation. If all three neutrino types oscillate simultaneously into one another, the analysis becomes more complicated and the limits given here have to be modified somewhat.

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[7] The fake μ^- background in the 5- to 10-GeV region is 18.5%; below 5 GeV this background increases rapidly and makes it hard to estimate the number of μ^- events.

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[9] $\vec{P}_{\mathrm{out}} = (\hat{P}_\nu \times \hat{P}_e) \cdot \vec{\mathrm{P}}_{\mathrm{had}}$, where $\hat{P}_\nu$ and $\hat{P}_e$ are unit vectors in the beam and electron directions, respectively, and $\vec{\mathrm{P}}_{\mathrm{had}}$ is the total hadronic momentum vector.

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