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Abundance Enhancements in Cosmic Rays Produced by Collisionless Shocks

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It is shown that shocks preferentially accelerate partially ionized heavy elements over protons. Within the framework of a previously published model of injection, the spectra of different ion species are calculated from thermal to ultrarelativistic energies. For typical astrophysical parameters, the predicted enhancement is in qualitative agreement with observations for a 10^6-K preshock plasma; ions with $Z>10$ are enhanced by about an order of magnitude or so. The continuing increase with energy of the heavy-element abundances into the air-shower regime is discussed.

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The observed composition of cosmic rays is an important constraint on theories of their origin. It indicates, given the composition of the thermal plasma, the relative probability that any given type of ion species meets the criterion for becoming a cosmic ray. Heavy elements, $Z>10$, appear to be enhanced by about one order of magnitude relative to protons at ~1 GeV/nucleon, and similar enhancements are seen in solar cosmic rays at about 10 MeV/nucleon.[1] The enhancement of heavy elements in the galactic cosmic rays increases with energy, and recent air-shower studies[2] suggest that at *total* energy of about 10^{15} eV, cosmic rays may be mostly iron. This represents an additional enhancement of one to two orders of magnitude above that at 1 GeV.

Recently, a model for cosmic-ray production was proposed[3] in which energetic particles are drawn directly from a thermal pool. In the model, all particles flowing into the shock are compressed and accelerated to some extent, as required by definition of a shock, but a very few receive a far larger share of energy than most, which makes them cosmic rays. The fact that thermal particles as well as superthermal ones are included in the theoretical description of the acceleration makes it possible, in principle, to calculate the composition of cosmic rays relative to the thermal plasma.

In practice, the calculation is very difficult; however, the enhancements can be calculated with analytic approximation at some sacrifice of accuracy. The purpose of this Letter is to show qualitatively why heavy elements are accelerated more effectively than protons, and to calculate the enhancement with use of an analytic approximation.

The basic equation for shock acceleration is[4]

$$\frac{\partial F}{\partial t} = -\frac{\partial uF}{\partial x} + \frac{1}{3}\frac{du}{dx}\frac{\partial F}{\partial \ln p} + \frac{\partial}{\partial x} D \frac{\partial}{\partial x} F, \qquad (1)$$

where p is momentum per nucleon, F is the number density per unit $\ln p$, D is the particle diffusion coefficient, and u is the fluid velocity, which is assumed to have a shocklike profile.

To show that (1) produces compositional changes at high p, multiply all terms by dx/du, which is negative at $u(-\infty) > u > u(+\infty)$. In steady state, this yields

$$\frac{\partial F_i}{\partial T} = \frac{\partial}{\partial u} u F_i + \frac{\partial}{\partial u} D_i' \frac{\partial}{\partial u} F_i, \qquad (2)$$

where i indexes ion species, $T = 3\ln p$, and $D_i' = -D_i\, du/dx$.

It can be shown that this is equivalent to a time-dependent equation representing the simultaneous diffusion of various species of particles in a background fluid whose velocity towards the origin is proportional to distance from it, viz., a homologously contracting one-dimensional universe. Here the "time" variable corresponds to the logarithm of momentum, and the "space" variable is u. The diffusion coefficients D_i in this analogy have complicated time and space dependence. Independent of these details, a clear qualitative feature of this equation is that at a point far "upstream" of where particles originate at $T=0$, the population is enriched in the species with the largest diffusion coefficients at a given time and place. (This is essentially the principle behind isotope enrichment by diffusion.) Analogously, shock-accelerated particles are enriched in partially ionized heavy elements, which have a larger diffusion coefficient at a given p than light ones. As long as there is velocity change over the diffusion precursor (the region where F changes) the enrichment continues to grow with p and is, of course, an increasing function of D. Physically, the reason for the enrichment at large p is that particles with larger diffusion coefficient see a sharper velocity contrast across the shock, and hence they undergo greater compression, and are accelerated more effectively to large p.

The above discussion can be quantified further by noting that to a good approximation the acceleration of any particle depends only on its diffusion coefficient and on whether its kinematics are relativistic or nonrelativistic. If, for example, $D_i = K_i p^\alpha$, $\alpha > 0$, then (1) can be rewritten as[5]

$$\frac{\partial F}{\partial t} = -\frac{\partial uF}{\partial x} + \frac{\alpha}{3}\frac{du}{dx}\frac{\partial F}{\partial \ln D} + \frac{\partial}{\partial x} D \frac{\partial}{\partial x} F, \qquad (3)$$

where D is now the independent variable. Thus, the abundance ratio at a given value of D is preserved at all higher D. With use of this invariant, it is easy to generate the spectra for other ion species as a function of energy per nucleon given their charge states, the dependence of D on mass to charge ratio, and the proton spectrum.

To obtain the proton spectrum, we first integrate (1) in steady state from $x=-\infty$ to $x=+\infty$.[3,6-9] This gives

$$\frac{d\ln F(+\infty)}{d\ln p} = \frac{3u(+\infty)}{u(+\infty) - \bar{u}(p)}, \qquad (4)$$

where

$$\bar{u}(p) = \frac{\int_{-\infty}^{\infty} u(x)(\partial^2 F/\partial x\, \partial p)dx}{\partial F(+\infty)/\partial p}. \qquad (5)$$

Under the assumption that there is negligible pressure at $x=-\infty$, $u(x)$ is determined by the equation

$$\mathcal{P}(x) + \rho u(x)^2 = J u(-\infty), \qquad (6)$$

where J is mass flux and $\mathcal{P}$ is total pressure.

From the linear analysis[6-9] it is clear that $F(p)$ decreases with distance ahead of the shock over a diffusion length $\sim D/u$, this region being the "diffusion precursor," and that it is constant behind the shock, so that $F(0) = F(+\infty)$.

Making the same analytic approximations as in Ref. 3, we obtain the proton spectrum in the limit where D increases very rapidly with p, so that the diffusion precursors of the more energetic protons extend much further upstream than those of the less energetic ones. Thus, at the characteristic diffusion length $[D(p)/u]$ upstream of the shock for protons of momentum p, the pressure in protons whose momentum is less than p is negligible, and the pressure in those whose momentum exceeds p is close to its value at $x=0$. We also neglect the pressure of the thermal plasma that has not yet encountered the shock. To within these approximations, the value of $\mathcal{P}(x)$ over the region where the integrand in (5) makes its largest contribution is $\int_p^\infty P(x) \times d\ln p'$, where $P(x)\, d\ln p$ is the differential pressure at x in particles of momentum p. According to (6), $u(x)$ in that region is then $u(-\infty) - J^{-1} \times \int_p^\infty P\, d\ln p$ and, by (5),

$$\frac{d\ln F(+\infty)}{d\ln p} = \frac{3u(+\infty)}{u(+\infty) - u(-\infty) + J^{-1}\int_p^\infty P\, d\ln p'}. \qquad (7)$$

We evaluate the simultaneous solution to (4) and (7) for $F_{\rm H}(p)$, the proton spectrum, assuming (a) that the protons dominate the fluid's mass, (b) that they obey purely nonrelativistic kinematics below $m_p c^2$, $P=(p^2/3m)F_{\rm H}(p)$, and purely relativistic kinematics above $m_p c^2$, $P=(pc/3)F_{\rm H}(p)$ (c) that P is continuous at mc^2, and (d) that the overall compression ratio is 4. The procedure for analytic solution by quadrature and the qualitative features of the solution are discussed in Ref. 3. We also evaluate the spectrum for ions having different mass-to-charge ratios (A/Q) of 2, 5, and 10, assuming that the mean free path scales as $(A/Q)p$ and that the spectral index is the same for all ion species at the same value of D. Within the analytic approximation made here, this is equivalent to letting

$$\frac{dF_i(p)}{d\ln p}=\frac{3u(+\infty)}{u(+\infty)-u(-\infty)+J^{-1}\int_{p_i}^{\infty}P\,d\ln p'}, \tag{8}$$

where the lower limit in the integral, p_i, is the momentum at which protons have the same diffusion coefficient that ions of species i have at p. The stripping time in the interstellar medium is long compared to the acceleration time, and so it is assumed that the preshock charge states of the ions are preserved during the acceleration. The results are plotted in Fig. 1 for a shock velocity of approximately 700 km/s. It is clear that the ratio of any element with $A/Q>1$ is enhanced relative to protons, and that the enhancement increases with both A/Q and energy.

The solution may not be extended below the reference energy E_R, where the analytic approximation yields a spectral index that is an implausibly sensitive function of energy, and it is assumed that viscous heating injects particles at E_R. It is evident that for $A/Q\lesssim 5$, which is satisfied by most ion species in a 10^6-K plasma, the overall enhancement over the relative abundances at E_R is modest—at most an order of magnitude or so. We find that this basic conclusion is not altered by varying the adjustable parameters of the model. It is consistent with observations of cosmic rays in the 1 to 100 GeV/nucleon range.

At very high p, $\bar{u}(p)=u(-\infty)$, and (4) predicts a power law. As all species have the same spectral index, there is clearly no further enrichment with p at a given shock. On the other hand, the observed cosmic-ray spectrum at high energies may be a composite of many individual sources having different spectral indices and high-energy cutoffs. Enrichment of iron above 10^3 GeV in the overall galactic spectrum is possible

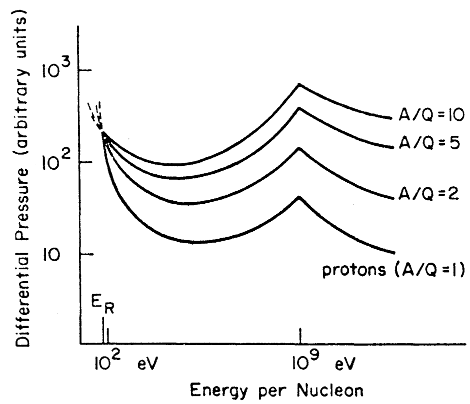

FIG. 1. Ion spectra, as determined by Eqs. (4)–(6), and the approximations in the text, are displayed for the mass-to-charge ratios (A/Q) of 1 (protons), 2, 5, and 10. The overall compression ratio $u(-\infty)/u(+\infty)$ is chosen to be 4. The minimum energy E_R at which the particles are injected by viscous heating is chosen to be 65 eV, corresponding to a shock velocity of roughly 700 km $\rm s^{-1}$. The actual functions plotted are $(p^2/3m)F_i$ below $m_p c^2$ and $(pc/3)F_i$ above $m_p c^2$. The spectra are normalized so that they all intersect at E_R.

if the harder spectra correlate with greater heavy-element enrichment. One reason for such a correlation could be that the spectra are likely to be hardest where the supernova shocks are strongest, i.e., while the supernova remnant is young and close to the stellar collapse site. This may generally be the case within young associations where the heavy-element abundance may be locally enriched by other recent supernovas.

In summary, the calculation involves a number of approximations and needs to be developed further. However, the results at the semiquantitative level are consistent with the observations under the assumption that cosmic rays are produced largely in hot plasma, where the mass-to-charge ratios of heavy ions are less than 10.

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