sponding self-energy which is spin dependent. In particular, Ω is now replaced by

$$\Omega_\sigma = 0.538 + 0.133\sigma g\mu_B H\Delta/a^2 + 0.048(g\mu_B H\Delta/a^2)^2 . \qquad (13)$$

This predicts a considerable difference on the effective masses of the spin-up and spin-down electrons.

We would like to thank Dr. E. M. Forgan for sending us his experimental results prior to publication and for drawing our attention to the problem. One of us (R.M.W.) also gratefully acknowledges the support of the Alexander von Humboldt Foundation.

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Kosterlitz-Thouless Transition in Proximity-Coupled Superconducting Arrays

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(Received 14 September 1981)

Evidence for the Kosterlitz-Thouless vortex unbinding transition is reported in triangular planar arrays of proximity-coupled Pb-Sn junctions. The temperature dependence of the resistive transition and the nonlinear features of the current-voltage characteristics are consistent with recent theories of topological ordering in two-dimensional superconductors.

PACS numbers: 74.50.+r, 74.40.+k, 74.60.-w

Recent interest in the Kosterlitz-Thouless (KT) transition in two-dimensional superconductors has been motivated primarily by the suggestion of Beasley, Mooij, and Orlando (BMO)[1] that the motion of thermally dissociated vortex pairs should lead to a broadened superconducting transition in films of high normal-state sheet resistance. The experimental results which support this vortex picture are based on measurements of the resistive transition,[2-5] the I-V characteristics,[6] the rf surface impedance,[7] and the voltage noise spectrum.[8]

While these results are generally consistent with the basic ideas of KT and BMO, the interpretation remains in doubt because of concern over film inhomogeneities. As emphasized recently by Bancel and Gray[9] and Lobb *et al.*,[10] thin high-resistivity films are generally inhomogeneous, often consisting of agglomerations of small grains. The BCS transition temperature of small superconducting grains—especially aluminum—is a function of size, impurity content, and other material-dependent parameters. One cannot be certain, therefore, that a broadened transition in a granular film does not result simply from a spread in T_c of the individual grains or, alternatively, from an inhomogeneous distribution of grain-to-grain coupling.

In this Letter we report evidence for the KT transition in ordered arrays of proximity-coupled Pb disks. Our measurements suggest that the KT transition in superconductors is not only intrinsic to homogeneous high-sheet-resistance films but also to proximity-coupled arrays whose effective sheet resistance is very low. Implicit in this conclusion is the idea that on a length scale large compared to the junction size, a proximity array may be viewed as a homogeneous, weak, type-II superconductor with a large transverse penetration depth. While this concept encompasses the properties of granular superconductors in a natural way, there is as yet no integrated theory which extracts the effective superfluid density, transverse penetration depth, and coherence length of superconducting arrays from the coupling strength and lattice spacing of the junctions.

Our samples, prepared by a standard photolithographic technique, consisted of a 1000-Å-thick tin film deposited over a large planar network of about 10^6 lead disks arranged on a triangular lattice. The entire array was supported by an oxidized silicon substrate. As shown in the inset to Fig. 1, the narrow layer of tin between each pair of lead disks formed a proximity-coupled Pb-Sn-Pb junction. The diameter of each lead disk was approximately 13 μm and the disk thickness was about 1500 Å. The distance of closest approach between the disks was about 1.3 μm, comparable to the normal-state coherence length of the tin. The normal-state sheet resistance of the samples, which were typically 5×20 mm^2, was 0.05 to 0.2 Ω/sq.

There are several substantive differences between our samples and granular superconducting films, although each system may be cast theoretically in terms of the well-known X-Y model. Besides the low sheet resistance of our samples, typically five decades below the granular samples studied in Refs. 2–10, the Pb disks in our arrays have sharply defined transition temperatures with negligible variation between disks. The disks themselves have diameters large compared to the superconducting coherence length so

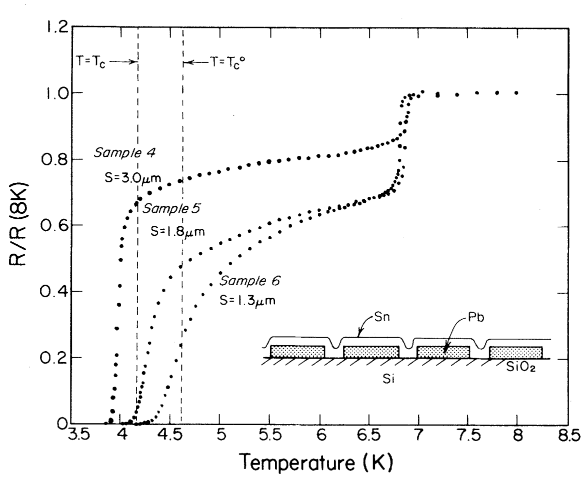

FIG. 1. The temperature dependence of the normalized resistance for samples 4, 5, and 6. The vertical dashed lines mark the values of T_c and T_c^0 for sample 6. A schematic drawing of the sample construction is illustrated in the inset.

that complications arising from zero-dimensional effects (e.g., fluctuations, or capacitive charging[11]) can be ignored. The most important property of our samples, however, is the uniform disk-to-disk coupling which results from the close tolerances maintained by the computer-generated patterns.

Data were obtained on eight samples in the temperature range 3.7–8.0 K which spanned the regime between the transition temperature of the Sn [T_c(Sn) $\approx$ 3.75 K] and of the bulk Pb [T_c(Pb) $\approx$ 7.27 K]. The actual transition temperature of the Pb disks was depressed by the Sn overlay by about 0.3 K whereas the transition temperature of the Sn did not depart from the bulk value. Resistivity measurements and I-V characteristics of the samples were obtained with use of a combination of SQUID instrumentation (for sample currents below about 200 μA) and a Keithley 140 nanovoltmeter (for sample currents up to 100 mA). The voltage resolution was about 10^{-12} V for the SQUID system and about 10^{-8} V for the nanovoltmeter. A magnetic field of a few gauss or below could be applied perpendicular to the face of the sample by an external electromagnet.

Figure 1 shows the temperature dependence of the normalized resistive transition for three samples with different disk spacings. The data were obtained at a constant sample current of 15 μA. The sudden drop in resistance at T_c(Pb) $\approx$ 7 K corresponds to the onset of superconductivity in the lead disks and is a measure of the width of the normal-metal region between disks. Although relatively sharp, the Pb transition is slightly broadened by the Sn film overlay (a consequence, most likely, of the superconducting-normal interface resistance[12]). At temperatures below T_c(Pb), the resistance of the arrays decreases gradually, eventually disappearing at a temperature T_c which is substantially higher than the bulk Sn transition at T_c(Sn). The influence of different disk spacings is clearly evident, with the closest spacing ($s = 1.3$ μm) corresponding both to the most pronounced rounding of the transition and to the largest value of T_c.

It is convenient to divide the resistance curves for each sample into three regions, delineated by the dashed lines in Fig. 1 (for sample 6) at temperatures T_c and T_c^0. At temperatures below T_c the specimen is superconducting and exhibits a critical current. In the middle region $T_c < T < T_c^0$ the sample is resistive but shows nonlinear, magnetic-field–dependent I-V characteristics. In the third region, $T > T_c^0$, the sample has field-independent, linear I-V characteristics and shows no superconductive behavior. We believe that T_c may be identified as the Kosterlitz-Thouless vortex unbinding temperature, while $T_c{}^0$ is the effective "BCS-like" transition temperature of the entire array. According to this interpretation, the sample exhibits in the middle region the properties of a homogeneous, weak, type-II superconductor, with a flux flow resistivity caused by the motion of thermally activated free vortices.

In the high-temperature region, $T > T_c{}^0$, the temperature dependence of the resistivity is not influenced by Josephson coupling between array disks. In this region, the proximity effect lowers the average resistance of the sample by effectively enlarging the radius of each disk; however, thermal fluctuations are sufficiently great to inhibit any disk-to-disk coupling. The distance over which superconductivity extends into the Sn is approximately the coherence length ξ_N of the normal Sn,[13]

$$\xi_N(T) = \xi_0[T/T_c(\mathrm{Sn}) - 1]^{-1/2}, \qquad (1)$$

and varies from about 1500 Å at 7 K to about 5000 Å at 4.6 K. Because the disks are essentially isolated from one another in this temperature interval, the effective resistivity of the sample can be treated to a good approximation as a problem in classical electrodynamics. Were the coupling between disks to be entirely suppressed, the resistance of the array would fall to zero at the temperature at which the proximity-enlarged radii of nearest-neighbor disks touched one another. The resistance would exhibit a cusp at this temperature, with a $[1 - 4a(T)^2/L^2]^{1/2}$ variation, where $a(T) = d/2 + \xi_N(T)$ is the proximity-enlarged radius of each disk and L is the center-to-center spacing between disks.[14]

This classical picture fails, however, when the proximity-enlarged radii approach closely enough to permit Josephson currents to flow between disks. This Josephson-tunneling regime, shown as the middle region of Fig. 1, is characterized by the sudden transition of the sample at $T_c{}^0$ from an array of isolated superconducting disks to an array of phase-coherent disks.[15] The existence of this phase-coherent state is clear from the sudden onset of oscillations of the I-V characteristics in a weak external magnetic field. The period of the oscillations indicates that flux quanta from the external field are being trapped in the triangular lattice cells of the phase-coherent array. Similar oscillations have been observed by Fiory, Hebard, and Somekh[16] in superconducting films perforated with a triangular pattern of holes.

Surprisingly, the resistance of the sample is increased by the onset of Josephson tunneling, and it is this increase which we attribute specifically to thermally activated free vortex motion within the array lattice. Our evidence for this conclusion is the following:

(1) The resistive transition exhibits the temperature dependence predicted by Halperin and Nelson (HN)[17] and Doniach and Huberman (DH)[18] above the vortex dissociation temperature. This agreement is illustrated for samples 5 and 6 in Fig. 2, which is a logarithmic plot of R/R_n (in the low-current limit) as a function of the reduced temperature $t = T_c/(T - T_c)$.

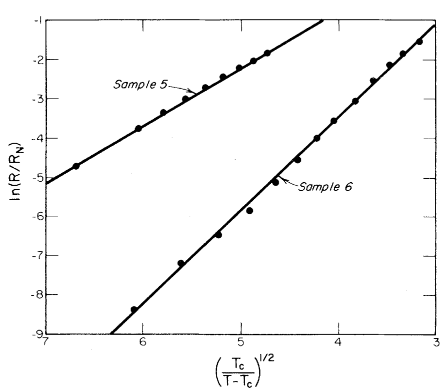

FIG. 2. A logarithmic plot of the resistance of samples 5 and 6 in the temperature range $T_c < T < T_c{}^0$. The data are plotted as a function of the reduced temperature variable $t^{1/2} = [T_c/(T - T_c)]^{1/2}$. For sample 5, $T_c = 4.07$ K while for sample 6, $T_c = 4.25$ K.

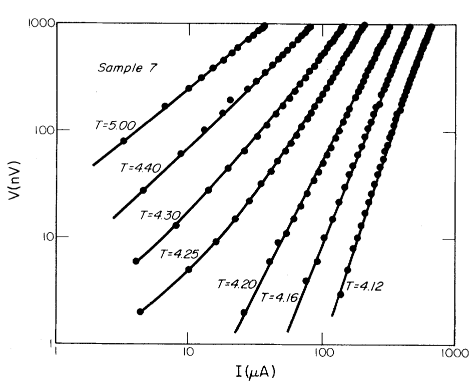

FIG. 3. A logarithmic plot of the current-voltage characteristics of sample 7 at temperatures spanning the critical region $T_c < T < T_c{}^0$. For this sample, $T_c = 4.16$ K and $T_c{}^0 = 4.40$ K. At $T_c{}^0$ the I-V curve is approximately linear, while at T_c, V increases as I^3.

(2) The resistance versus temperature data of Fig. 1 fall along the "universal" curve of Minnhagen,[19] believed applicable to all KT systems in this temperature regime. This agreement will be illustrated in a subsequent paper.

(3) The low-current I-V characteristics of all samples, shown in Fig. 3 for sample 7, are consistent with a KT interpretation. Above T_c^0, the I-V characteristics are strictly linear. As the temperature falls below T_c^0, the curves increasingly acquire a nonlinear component which is superimposed upon the linear term. At T_c, the linear term has decreased to zero and the I-V curves follow a simple power law of the form $V \sim I^3$, as predicted by HN. Below T_c, the current exponent increases, also in accord with HN, although the exponent is enhanced above the HN value by the incipient superconductivity of the Sn.

In interpreting our data as evidence for a KT transition, we note that the two characteristic temperatures, T_c and T_c^0, may be inferred several ways, each of which gives the same result (to within 2%). The choice of T_c which provides the best fit to the HN and DH resistance theory also corresponds to the value at which the I-V curves have an exponent of 3. As expected, this temperature is always slightly below (typically 0.03 K) the temperature at which the voltage difference across the specimen becomes immeasurably low. Similarly, the value of T_c^0 which provides the best agreement with the Minnhagen theory also corresponds to the temperature at which the I-V curves become linear and the field oscillations vanish.

In summary, therefore, we have found that the resistive transition in low-resistance proximity-coupled arrays reproduces the main features of the two-dimensional topological ordering theories. Our evidence suggests that the key ingredient for observing vortex unbinding in a type-II film is not the sheet resistance but the weakness of the superconductivity, as exemplified by a large transverse penetration depth. As a model system in which to study KT effects, our Pb-Sn-Pb planar arrays offer both advantages and disadvantages over high-sheet-resistance granular films. The advantages are that the samples are well characterized and are free of inhomogeneities and zero-dimensional complications. Furthermore, the scale is set by the normal-metal coherence length and is of the order of microns (rather than angstroms), which is an experimentally convenient size. The primary disadvantage is that the coupling strength between junctions is strongly temperature dependent at low temperatures because of the singularity in the coherence length of the Sn.

The authors are grateful to Mr. M. Lacher for assistance with sample preparation and to Professor T. R. Lemberger for a critical reading of the manuscript. This research was supported in part by National Science Foundation Grants No. DMR-8106121 and No. DMR-7921989.

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