Double Beta Decay and the Majorana Mass of the Electron Neutrino

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We have calculated the two-neutrino and no-neutrino, lepton-number-nonconserving double beta decay rates for ^{76}Ge and ^{82}Se. Our result for the two-neutrino decay of ^{82}Se is in agreement with a recent cloud-chamber measurement and thus in disagreement with the geochemical determination. We find that a constraint on the Majorana mass of the electron neutrino of $\langle m \rangle_\nu \gtrsim 15$ eV follows from the comparison of our calculations and the experimental limits on the neutrinoless decays of ^{76}Ge and ^{82}Se.

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The prejudice for conservation of lepton number is based in part on the early recognition that a Majorana electron neutrino would lead to neutrinoless double beta decay at rates much faster than found experimentally.[1] Later, with the discovery of parity nonconservation in the weak interaction, it was realized that exact γ_5 invariance of the weak leptonic current would forbid neutrinoless $\beta\beta$ decay, irrespective of the behavior of the neutrino wave function under charge conjugation. Recently, however, evidence is mounting that this γ_5 invariance may be only approximate, broken either by a small neutrino mass or by some explicit mixing of left- and right-handed components in the weak current. If this proves true, then again the presence or absence of neutrinoless $\beta\beta$ decay can place fundamental constraints on possible descriptions of the neutrino.

A recent cloud-chamber search for the two-neutrino $\beta\beta$ decay of ^{82}Se has also renewed interest in this conventional, lepton-number-conserving process. Moe and Lowenthal[2] obtained, from limited statistics, a two-neutrino decay rate which exceeds a preexisting total $\beta\beta$ decay rate, determined geochemically,[3] by a factor of 28. It is imperative that we understand the origin of this discrepancy as other geochemical results, particularly the $\beta\beta$ decay lifetimes of ^{130}Te and ^{128}Te, have been used in limiting possible violations of lepton-number conservation.[1] In this Letter we present what we believe to be the first calculations of sufficient precision to allow a meaningful comparison between theory and these two conflicting measurements. We then calculate the no-neutrino $\beta\beta$ decay rates for ^{82}Se and ^{76}Ge, taking the neutrino mass from a recent measurement of the triton β decay spectrum. The laboratory limits on the neutrinoless decays of these nuclei then indicate that the electron neutrino is not a Majorana mass eigenstate.

The lepton-number-conserving $\beta\beta$ decay of a nucleus occurs as a second-order weak process in which a final state of two electrons and two antineutrinos is produced

$$(A,Z) \to (A,Z+2) + 2e^- + 2\bar{\nu}_e\,.$$

This differs from two ordinary β decays in that the intermediate state $(A,Z+1)+e^-+\bar{\nu}_e$ is virtual. Thus the single β decay to the intermediate nucleus must be forbidden energetically (or at least strongly hindered), a situation not uncommon due to the pairing force which lowers the energy of the initial and final even-even nuclei relative to the odd-odd intermediate nucleus. Consequently, most interesting candidates for $\beta\beta$ decay involve initial and final states with $J^\pi = 0^+$. We specialize our treatment to this case.

The $\beta\beta$ decay amplitude is given by the two-nucleon mechanism of Fig. 1(a). (A single-hadron mechanism where the Δ decays to a neutron and four leptons has been discussed[4]; however, the nuclear operator for this process is a spin vector and thus cannot contribute to $J=0\to 0$ transitions.[5,6]) We evaluate each nucleon β decay in the allowed approximation. The nuclear $\beta\beta$ decay amplitudes in time-dependent perturbation theory then become

$$\sum_M \frac{\langle F|\sum_{i=1}^{A} O(i)|M\rangle\langle M|\sum_{i=1}^{A} O(i)|I\rangle}{E_M{}^N + E^L - E_I{}^N}\,, \tag{1}$$

where the sum extends over a complete set of

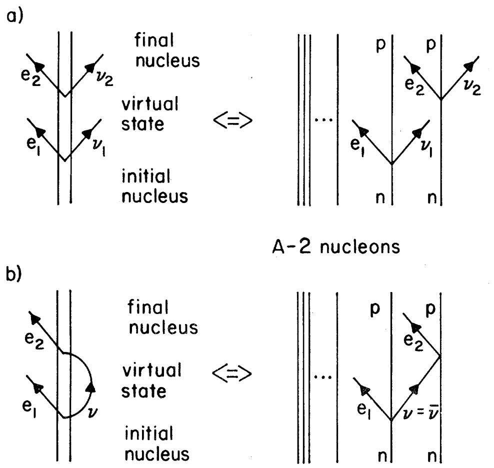

FIG. 1. Two-nucleon mechanisms for (a) two-neutrino and (b) no-neutrino $\beta\beta$ decay.

intermediate nuclear states M. Nuclear energies are denoted E^N, and E^L is the sum of the lepton energies emitted in the first β decay. The $O(i)$ represent either the Fermi or Gamow-Teller operators, $\tau_+(i)$ or $\vec{\sigma}(i)\tau_+(i)$.

The sum over intermediate nuclear states is performed by closure after replacing $E_M{}^N$ by an average value, either $\langle E_F{}^N\rangle$ or $\langle E_{GT}{}^N\rangle$. These are taken from the $(E_M{}^N+E^L)^{-1}$ moments of the Fermi and $T_<$ Gamow-Teller strengths for β^- decay of the initial nucleus, evaluated in a statistical model which incorporates the effects of pairing on the Fermi surface.[5] We find $\langle E_{GT}{}^N\rangle = 8.07$ (7.88) MeV and $\langle E_F{}^N\rangle = 9.17$ (8.79) MeV for ^{82}Se (^{76}Ge). (These are nuclear, not atomic, energies.) The resulting differential $\beta\beta$ decay rate $d\omega/d\epsilon_1 d\epsilon_2 d\nu_1$ then can be expressed in terms of the double Fermi and Gamow-Teller matrix elements[5]

$$M_F = \langle F|\tfrac{1}{2}\sum_{\substack{ij\\ i\neq j}} \tau_+(i)\tau_+(j)|I\rangle$$
$$M_{GT} = \langle F|\tfrac{1}{2}\sum_{\substack{ij\\ i\neq j}} \vec{\sigma}(i)\cdot\vec{\sigma}(j)\tau_+(i)\tau_+(j)|I\rangle \qquad (2)$$

which must be evaluated with realistic wave functions.

We describe the ground states of the initial and final nuclei of interest, ^{82}Se, ^{82}Kr, ^{76}Ge, and ^{76}Se, in weak coupling. First proton (neutron) wave functions are determined separately in full shell model calculations for four, six, and eight valence particles (two, four, six, and eight valence holes) in a model space consisting of the

TABLE I. Calculated and experimental $\beta\beta$ decay half-lives τ, in units of 10^{19} yr. The neutrino mass parameter $\eta = \langle m\rangle_\nu/m_e$. Experimental references are given in the text.

	$\tau_{calc}{}^{2\nu}$	$\tau_{expt}{}^{2\nu}$	$\tau_{calc}{}^{0\nu}$	$\tau_{expt}{}^{0\nu}$
^{82}Se	1.69[a]	1.0 ± 0.4	$1.59\times10^{-7}/\eta^2$	$\geqslant 310$
	2.35	27.6[b]		
^{76}Ge	37.3		$4.42\times10^{-7}/\eta^2$	$\geqslant 500$

[a]Explicit summation over intermediate states below 5 MeV in ^{82}Br makes this the more reliable theoretical value.

[b]This is the total geochemical half-life.

$2p_{1/2}$-$2p_{3/2}$-$1f_{5/2}$-$1g_{9/2}$ shells. This space is nonspurious.[7] The fifty proton states with the lowest energies are combined with a like number of neutron states to form a weak coupling basis for $J^\pi = 0^+$. These basis states are then allowed to mix through the proton-neutron interaction, and the resulting matrix is diagonalized to yield nuclear wave functions and eigenvalues.

In such calculations both the shell model effective interaction and the single particle energies must be specified. Potential matrix elements were taken from Kuo,[8] modified as discussed below, with the Coulomb interaction between protons added. The [illegible] matrix elements are designed for the mass region near ^{56}Ni; it proved necessary to weaken this interaction somewhat to obtain a systematic fit to level spacings of nuclei near the middle of the fpg shell, reflecting the larger mean nucleon separations in these heavier nuclei. This reduction factor ($\simeq 0.8$) and the proton single particle energies were determined by fitting thirteen levels in the closed-shell nuclei ^{84}Se, ^{85}Br, ^{86}Kr, ^{87}Rb, and ^{88}Sr. A similar fit to fourteen levels in the closed-shell nuclei ^{62}Ni, ^{63}Ni, and ^{65}Ni yielded the neutron energies. The mean square deviation from the experimental levels included in these fits is approximately 270 keV. Further details are given in Ref. 5.

The two-body density matrices determined from these weak coupling wave functions immediately yield $|M_{GT}| = 0.94$ (1.28) for ^{82}Se (^{76}Ge). The Fermi operator can be rewritten in terms of T_+T_+, where T_+ is the isospin raising operator, and thus cannot change the total isospin. Its matrix element differs from zero only to the extent that charge-dependent terms in the shell model potential mix the analog of the initial nuclear state (with $T = T_i = T_f + 2$, $M_T = -T_f$) into the ground state of the final nucleus. As the weak coupling approximation itself yields states without exact isospin, the Coulomb mixing of these states was calculated in perturbation theory. We find $|M_F| < 0.02 \ll |M_{GT}|$.

The results are shown in Table I. The calculated two-neutrino half-life for ^{82}Se, $\tau_{1/2}^{2\nu} = 2.35 \times 10^{19}$ yr, is in reasonable agreement with the result of Moe and Lowenthal,[2] $(1.0 \pm 0.4) \times 10^{19}$ yr, and more than an order of magnitude shorter than the total half-life determined from geochemical measurements of Srinivasan, Alexander, and Manuel,[3] 2.76×10^{20} yr. Calculations have also been performed[5] for the $\beta\beta$ decay transitions ^{128}Te$\to$^{128}Xe and ^{130}Te$\to$^{130}Xe, and rates much faster than the geochemical values are again found. Although we feel that the difficulty of the Te and Xe structure calculations render those results somewhat less reliable than the ^{82}Se results reported here, the suggestion of a systematic discrepancy between theory and the geochemical measurements is clear.

In view of this, we emphasize that the calculated ^{82}Se $\beta\beta$ decay amplitude is largely determined by a few dominant two-body density matrix elements which add coherently. Thus minor wave function adjustments are unlikely to alter our present result. Also, although no Gamow-Teller transition involving the intermediate nucleus ^{82}Br is known, a recent measurement of the $\frac{3}{2}^- \to \frac{1}{2}^-$ (0.19 MeV) transition from ^{81}Br$_{\text{g.s.}}$ to ^{81}Kr does provide an opportunity for checking our weak coupling techniques. Use of proton bases identical to those employed here yields[9] $\log(ft) = 4.99$, in good agreement with the experimental value[10] $\log(ft) = 4.88 \pm 0.11$.

However, the most troublesome theoretical point may be in the derivation, and not the evaluation, of the matrix elements. The terms in Eq. (1) are not positive definite, so in principle the non-energy-weighted sum evaluated by closure may differ significantly from the exact result, particularly if terms with small E_M are important. To check the sum rule we generated the weak coupling 1^+ states lying within 5 MeV of the ground state of ^{82}Br and performed the summation of Eq. (1) explicitly for those states. Treating the higher excitations as before, this yields $\tau_{1/2}^{^{82}\text{Se}} = 1.69 \times 10^{19}$ yr. Thus this more reliable estimate of $\tau_{1/2}^{^{82}\text{Se}}$ is in good agreement with our previous value.

With these checks, in addition to the agreement with Moe and Lowenthal, encouraging some confidence in the present calculations, we turn to the neutrinoless $\beta\beta$ decay of ^{82}Se and ^{76}Ge. Primakoff and Rosen[1] have stressed that the nuclear matrix elements mediating the neutrinoless decay are expected to track those for two-neutrino decay; thus we expect, and find by explicit calculation, strong matrix elements for these cases as well.

The neutrinoless $\beta\beta$ decay amplitude is taken from the two-nucleon process shown in Fig. 1(b). (The comment made earlier on the absence of Δ-to-nucleon $\beta\beta$ decay transitions applies equally here.[5,6]) Details of the general evaluation of the decay rate (i.e., allowing both massive neutrinos and left- and right-handed admixtures in the current) are given in Ref. 5. In the present work we assume a Majorana electron neutrino and break the γ_5 invariance of the weak current by the neutrino mass. As in the two-neutrino case, the transition amplitude is taken from time-dependent perturbation theory; the integral over the intermediate neutrino momentum can be evaluated exactly for small m_ν, and the sum over nuclear states is performed by closure. The resulting differential decay rate $d\omega/d\epsilon_1$ is given in terms of nuclear matrix elements of the form[5]

$$M = \langle F | \tfrac{1}{2} \sum_{\substack{ij \\ i \neq j}} f(r_{ij}) [O(i) \times O(j)]_{00} | I \rangle$$

with $r_{ij} = |\vec{r}_i - \vec{r}_j|$, with $f(r_{ij})$ not too different from $1/r_{ij}$, and with $O(i) = \tau_+(i)$ or $\vec{\sigma}(i)\tau_+(i)$. [The third rotational invariant, $O = \vec{\sigma}(i) \cdot \hat{r}_{ij} \tau_+(i)$, arises only for admixed left- and right-handed currents.]

Our weak coupling wave functions then yield

$$\tau_{1/2}^{0\nu} = 1.59 \times 10^{12} \text{ yr}/\eta^2, \ ^{82}\text{Se},$$

$$\tau_{1/2}^{0\nu} = 4.42 \times 10^{12} \text{ yr}/\eta^2, \ ^{76}\text{Ge},$$

where $\eta = \langle m \rangle_\nu / m_e$ for this lepton-number- (and $B-L$)-nonconserving process. (Here $\langle m \rangle_\nu = \sum_i C_i^2 m_i$, with C_i^2 the probability of finding the electron neutrino in the eigenstate with Majorana mass m_i.) The laboratory limits for ^{82}Se and ^{76}Ge, $\tau_{1/2}^{0\nu} \geq 3.1 \times 10^{21}$ yr (Ref. 11) and $\tau_{1/2}^{0\nu} \geq 5 \times 10^{21}$ yr (Ref. 12), then yield $\langle m \rangle_\nu \leq 12$ eV and $\langle m \rangle_\nu \leq 15$ eV, respectively, at the 68% C.L. Clearly these constraints also apply separately to each $C_i^2 m_i$. Assuming a definite mass $m_1 = \langle m_\nu \rangle$, a recent measurement of the triton endpoint β spectrum gives $14 \leq m_1 \leq 46$ eV at the 99% C.L.[13] Thus if these experiments and the present calculations are correct, the electron neutrino is not a Majorana mass eigenstate.[14] The open question of the more general implications of our

$\langle m\rangle_\nu$ limit should encourage less restrictive analyses of the triton β spectrum.

It is apparent from this discussion that, given $\langle m\rangle_\nu \geq 14$ eV, existing limits on 0ν $\beta\beta$ decay are at the threshold of testing lepton-number conservation unambiguously. We thus believe it imperative to proceed with the new generation of experiments currently under discussion, including very sensitive searches for the neutrinoless decays of ^{76}Ge,[15] ^{82}Se,[2] and ^{136}Xe.[16] We also strongly urge a definitive laboratory measurement of the 2ν $\beta\beta$ decay of ^{82}Se,[2,17] as this rate provides a crucial test of the nuclear theory and of the assumptions implicit in geochemical determinations.

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Relativistic Theory of the Effective Interaction in Nuclei

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We show that the relativistic many-body theory that has recently been used to explain the saturation properties of nuclear matter also explains the remarkable density dependence (found in phenomenological studies) of the Landau-Migdal parameter, F_0. This result provides further evidence that the nucleus is a relativistic system.

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There has been an ongoing effort to understand the effective interaction in nuclei which uses reaction matrices obtained from interactions that fit nucleon-nucleon scattering data.[1-4] It has become conventional to specify the effective interaction in nuclei in terms of the Landau parametrization[5] and then to compare the phenomenological parameters with those calculated from a Brueckner reaction matrix.[1-4] Usually one calculates the Landau parameters in infinite nuclear matter since it is assumed that corrections to these parameters due to finite nuclear size are small.[6] (This assumption is considered to

