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Extended Hypercolor and the Cabibbo Angle

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We present a semisimple grand unifying $SO(10)_V \otimes SO(10)_H$ gauge theory in which $SO(10)_H$ is an extended hypercolor group. This extended hypercolor group contains both an SO(6) hypercolor subgroup and an SO(4) generation-counting subgroup. The model gives rise to both radiatively induced mass and mass mixing terms for the usual fermions. Under reasonable assumptions, the value $\tan^2\theta_C = m_d/m_s$ for the Cabibbo angle follows provided the strong interaction global vector flavor symmetry is spontaneously broken.

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In an attempt to find a mechanism for breaking the weak interactions dynamically, Susskind[1] and Weinberg[2] introduced a new superstrong TeV-region force known as hypercolor or technicolor. The primary virtue of this new force is that dynamical hyperfermion condensates replace the usual fundamental Higgs fields of the Weinberg-Salam model. These condensates then give masses to the intermediate vector bosons and automatically ensure the Weinberg mixing relation $M_W = M_Z \cos\theta$. The major defect of the scheme is that these same hyperfermion condensates give Goldberger-Treiman masses to the hyperfermions rather than to the usual fermions of weak interactions (the hypercolor singlets of hypercolor theories), so that the usual fermions remain massless. A possible remedy to this problem has been suggested by Dimopoulos and Susskind,[3] namely that there exists a yet stronger and even more speculative force known as extended hypercolor. The gluons of this force could then mediate transitions between hyperfermions and the usual fermions, so that the latter would acquire masses through the radiative corrections of Fig. 1.

What had been lacking in these studies was some general principle which would restrict and specify the structure of the hypercolor and extended hypercolor interactions and allow them to be introduced in any more than an *ad hoc* manner. In a recent paper,[4] we have suggested that such guiding principles could emerge by including hypercolor in grand unifying theories of the strong, electromagnetic, and weak interactions.

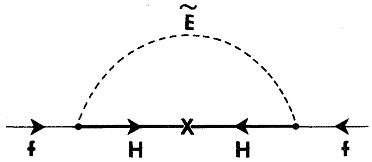

FIG. 1. Radiative self-energy contribution for a usual fermion f due to the exchange of a hyperfermion H and an extended hypergluon $\tilde{E}$.

Indeed, in our study of the candidate $SO(10)_V \otimes SO(10)_H$ vertical-horizontal symmetric grand unifying theory we established a group-theoretical connection between the size of the hypercolor group and the number of generations of usual fermions. In this note we explore our model further and show that it naturally admits extended hypercolor as well. Because of the correlation with the generation problem this will lead us not merely to a mechanism for obtaining light fermion masses but also to a reasonable theory of Cabibbo mixing.

In our model $SO(10)_V$ is a "vertical" symmetry SO(10) group which puts each generation of fermions into the standard $\underline{16}$ dimensional representation.[5] A discrete vertical-horizontal symmetry then fixes the "horizontal" symmetry which is to count generations[6] to also be SO(10). We shall identify $SO(10)_H$ as the complete extended hypercolor group, so that $SO(10)_H$ contains hypercolor just as $SO(10)_V$ contains ordinary color. In Ref. 4 we noted that the only possible classification for the fermions of the theory which was complex, which contained no particles with bizarre quantum numbers, and which possessed the usual $SO(10)_V$ structure was

$$\psi_L = (\underline{16},\underline{10}) + (\underline{10},\underline{16}), \tag{1}$$

with all fermions left handed. Here ψ_L is irreducible because of the discrete vertical-horizontal symmetry, so that all the fermions of our model are contained in one irreducible representation. Decomposing $SO(10)_V \otimes SO(10)_H$ according to $SO(10)_V \otimes SO(6) \otimes SO(4)$ enables us to reexpress ψ_L as

$$\psi_L = (\underline{16},\underline{6},\underline{1}) + (\underline{16},\underline{1},\underline{4}) + (\underline{10},\underline{4},\underline{2}) + (\underline{10},\underline{4}^*,\underline{2}'). \tag{2}$$

The key observation of Ref. 4 was to then point out that if SO(6) [$\approx$ SU(4)] were to be identified as the hypercolor group [the only unitary choice in our model that permits more than two generations is SU(4), see Ref. 4] the model would possess exactly four generations of hypercolor singlet fermions coming from the $(\underline{16},\underline{1},\underline{4})$ piece, with all the other fermions of ψ_L being confined hypercolor nonsinglets. Thus the SO(4) group counts and connects generations, with the size of the hypercolor and the generation-counting groups being simultaneously determined.

The gauge bosons belong to the adjoint representation and can be classified analogously as

$$(\underline{45},\underline{1},\underline{1}) + (\underline{1},\underline{15},\underline{1}) + (\underline{1},\underline{1},\underline{6}) + (\underline{1},\underline{6},\underline{4}). \tag{3}$$

Thus in addition to the $(\underline{1},\underline{15},\underline{1})$ hypergluons and the generation-counting $(\underline{1},\underline{1},\underline{6})$ gauge bosons, our model also contains the $(\underline{1},\underline{6},\underline{4})$ gauge bosons which carry both hypercolor and flavor. These last gauge bosons (to be labeled $E_{\alpha i}$; $\alpha = 1,\ldots,6$; $i = 1,\ldots,4$) can couple a $(\underline{16},\underline{6},\underline{1})$ hyperfermion H_α to a usual $(\underline{16},\underline{1},\underline{4})$ fermion f_i, with an interaction Lagrangian $\mathcal{L}$ of the form

$$\mathcal{L} = g_H \sum_{\alpha i} E_{\alpha i}{}^{\lambda} \bar{f}_i \gamma_\lambda H_\alpha, \tag{4}$$

where we have suppressed the $SO(10)_V$ indices. Here g_H is the horizontal coupling constant which is equal to the vertical g_V at the grand unification mass scale. We thus recognize Eq. (4) as the required interaction of Ref. 3. In order for this interaction to exist we note that since the $(\underline{1},\underline{6},\underline{4})$ gauge bosons transform according to the vector representations of SO(6) and SO(4), they can only couple H_α to f_i if the fermions also transform according to a horizontal vector representation. Moreover, this is a general feature of all extended hypercolor models based on orthogonal groups. Consider, for example, the decomposition of $SO(A+B)$ with respect to $SO(A) \otimes SO(B)$, where $SO(A)$ is identified as the hypercolor group. Then, since the ordinary fermions are hypercolor singlets, and the extended hypergluons belong to the (A,B) representation, the hyperfermions must belong to the representation A, the hypercolor vector representation. This result is particularly significant given the overwhelming preference for spinor representations in the literature. Further, the very existence of the interaction of Eq. (4) is thus seen to be a highly nontrivial property of our model since we had already been obliged to introduce the vector $\underline{10}$ in our classification of ψ_L in Eq. (1) for the completely different set of reasons previously outlined.

When $SO(10)_V \otimes SO(10)_H$ is spontaneously broken the SO(4) group will necessarily have to be broken since there are no known massless gauge bosons associated with it. This will then both give masses to and mix the $E_{\alpha i}$ gauge bosons into new mass eigenstates

$$\tilde{E}_{\alpha i} = \sum_j A_{ij} E_{\alpha j} \qquad (5)$$

to yield a new interaction

$$\mathcal{L} = g_H \sum_{\alpha ij} (A^{-1})_{ij} \tilde{E}_{\alpha j}{}^{\lambda} \bar{f}_i \gamma_\lambda H_\alpha. \qquad (6)$$

From Fig. 1 we thus obtain a light fermion mass matrix

$$m_{ij} \sim g_H{}^2 M_H{}^3 \sum_k (A^{-1})_{ik} A_{kj} / \tilde{M}_k{}^2, \qquad (7)$$

where M_H is the hyperfermion mass and $\tilde{M}_k$ is the mass of $\tilde{E}_{\alpha k}$. Thus the fermions both acquire masses and mix.[3,7]

The eigenstates

$$q_i = \sum_j A_{ij} f_j \qquad (8)$$

diagonalize $[m_{ij}]$ in Eq. (7), with mass eigenvalues

$$\tilde{m}_k = g_H{}^2 M_H{}^3 / \tilde{M}_k{}^2. \qquad (9)$$

Thus the quark mixing angles are simply the same as the gauge boson mixing angles, and the quark masses stand in the same ratios as the inverse squared masses of the gauge bosons.

To proceed further we need a model for the explicit gauge boson mixing mechanism. The simplest way to break the symmetry is in the $\underline{4}$ of SO(4) which can be accomplished by starting with a Higgs multiplet in the $\underline{10}$ of $SO(10)_H$. To illustrate how Cabibbo mixing can be calculated in our model, we shall confine ourselves to the first two generations of quarks; i.e., we take $\tilde{E}_{\alpha 1}$ and $\tilde{E}_{\alpha 2}$ to be heavier than $\tilde{E}_{\alpha 3}$ and $\tilde{E}_{\alpha 4}$ so that we can identify $i=1$ and 2 with the very light (u,d) and (c,s) quark generations. In this case $E_{\alpha 1}$ and $E_{\alpha 2}$ mix through an angle φ so that in the physical d,s sector, for instance, we have

$$\begin{aligned} d &= (f_1 \cos\varphi + f_2 \sin\varphi), \quad m_d = g_H{}^2 M_H{}^3 / \tilde{M}_1{}^2, \\ s &= (-f_1 \sin\varphi + f_2 \cos\varphi), \quad m_s = g_H{}^2 M_H{}^3 / \tilde{M}_2{}^2. \end{aligned} \qquad (10)$$

For the symmetry breaking we introduce two quartet representations, ϕ_a and ϕ_b (the minimum needed), with breaking structure in the requisite 1000 TeV or so region

$$\begin{aligned} \phi_a &= \phi_0(1,0,0,0), \\ \phi_b &= \phi_0(\sin\gamma, \cos\gamma, 0, 0). \end{aligned} \qquad (11)$$

The nontrivial assumption of Eq. (11) is that ϕ_a and ϕ_b share a common scale ϕ_0 and differ only in their orientation in the SO(4) generation space. We then find that the gauge boson mixing angle satisfies

$$\tan^2\varphi = (1 - \sin\gamma)/(1 + \sin\gamma) \qquad (12)$$

while the mass eigenvalues are

$$\begin{aligned} \tilde{M}_1{}^2 &= g_H{}^2 \phi_0{}^2 (1 + \sin\gamma), \\ \tilde{M}_2{}^2 &= g_H{}^2 \phi_0{}^2 (1 - \sin\gamma). \end{aligned} \qquad (13)$$

Thus the d-s quark mixing angle satisfies

$$\tan^2\varphi_{ds} = \tilde{M}_2{}^2 / \tilde{M}_1{}^2 = m_d / m_s. \qquad (14)$$

While Eq. (14) is an encouraging result we note that since the hyperfermions are generation singlets an analogous relation also exists in the u-c sector where the mixing angle satisfies

$$\tan^2\varphi_{uc} = \tilde{M}_2{}^2 / \tilde{M}_1{}^2 = m_u / m_c. \qquad (15)$$

Equations (14) and (15) imply that $m_d/m_s = m_u/m_c$ (Ref. 8) and $\varphi_{ds} - \varphi_{uc} = 0$. Therefore, an identification of $\varphi_{ds} - \varphi_{uc}$ with the observed Cabibbo angle θ_C would lead us to conclude that there would be no Cabibbo mixing in our model. In fact this result can immediately be seen to be a general property of all extended hypercolor models in which the hyperfermions are generation singlets. (In passing we also remark that were the hyperfermions to carry a generation index the theory would then suffer from an alternative disease in that the hyperfermion condensates would break flavor spontaneously to produce potentially large flavor changing rare decay processes.)

However, we would now like to point out that $\varphi_{ds} - \varphi_{us}$ as determined from Eqs. (14) and (15) is not necessarily the observed Cabibbo angle. Indeed such an identification is only justified provided the strong interactions (quantum chromodynamics) preserve the vector flavor symmetry exactly. The point is as follows.

In all discussions of the Cabibbo angle in the literature it is tacitly assumed that the global chiral symmetry $SU(n)_L \otimes SU(n)_R$ (n is the number of flavors) of quantum chromodynamics is only broken down by the color dynamics to $SU(n)_{L+R}$, so that the Cabibbo angle arises from the "current" masses (m_u, m_d, m_s, m_c, etc.) only. Suppose, however, that $SU(n)_L \otimes SU(n)_R$ is broken in some of the 3, 8, 15, etc., directions so as to split the "constituent" dynamical Goldberger-Treiman masses of the quarks as $m_u{}^0$, $m_d{}^0$, $m_s{}^0$, $m_c{}^0$, etc.[9] This then contributes an extra diagonal piece to the fermion mass matrix. Explicit calculation in the two-generation case under consideration shows that the mixing angles now satisfy

$$\tan 2\varphi_{ds} = \frac{\sin 2\varphi}{K_{ds} + \cos 2\varphi},$$
$$\tan 2\varphi_{uc} = \frac{\sin 2\varphi}{K_{uc} + \cos 2\varphi}, \qquad (16)$$

where

$$K_{ds} = \frac{m_s^0 - m_d^0}{m_s - m_d}, \quad K_{uc} = \frac{m_c^0 - m_u^0}{m_c - m_u}. \qquad (17)$$

Now while the strong interaction is thought to possess an approximate $SU(3)_{L+R}$ symmetry it apparently does not possess any $SU(4)_{L+R}$ symmetry.[10] Hence we can effectively take m_s^0 equal to m_d^0, and m_c^0 very large. In this limit we find $\varphi_{ds} = \varphi$, $\varphi_{uc} = 0$, so that the observable mixing angle, now the actual Cabibbo angle, satisfies

$$\tan^2\theta_C = m_d/m_s, \qquad (18)$$

a relation which is believed to be well satisfied.

While our remarks about the influence of strong interaction breaking on the Cabibbo angle are straightforward we find it somewhat mystifying that the above analysis appears to be absent from the literature. Moreover our analysis is even amenable to a direct experimental test. Specifically, if $SU(4)_L \otimes SU(4)_R$ is spontaneously broken beyond $SU(4)_{L+R}$ there would have to exist some observable scalar Goldstone bosons in addition to the fifteen familiar pseudoscalar Goldstone bosons such as the pion. With m_c^0 large some of these scalar Goldstone bosons would be coupled strongly to the charmed sector and would obey appropriate low-energy theorems. Moreover, we would also advocate a vigorous search for such particles even independently of our work to see just how much $SU(4)_L \otimes SU(4)_R$ is in fact spontaneously broken. [The search for a scalar κ meson[11] coupled to strangeness has not been conclusive, indicating that $SU(3)_{L+R}$ is probably a good symmetry.]

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