

Resistivity and Relaxation in Spin-Glasses

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The electrical resistivity of spin-glasses is discussed in terms of the elementary excitations of Walker and Walstedt, and excellent quantitative agreement with experimental data is obtained. With the same model an explanation for the local spin relaxation time behavior in spin-glasses is also given.

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Among basic physical properties of spin-glasses which remain to be fully understood is the electrical resistivity, despite a number of theoretical studies.¹⁻⁴ Here I give a simple picture of a spin-glass which can be used to relate directly the electrical resistivity and the excitation density of states. With the excitation density of states determined numerically for a Ruderman-Kittel-Kasuya-Yosida (RKKY) spin-glass by Walker and Walstedt,⁵ I obtain a calculated resistivity curve in excellent agreement with experimental results over the entire temperature range from 1 to 300 K. The local spin relaxation time in the light of the same approach is also discussed.

In a pioneering paper, Yoshida⁶ discussed the resistivity of metals containing dilute magnetic impurities. He showed that if the conduction-electron-impurity interaction is $V - 2J\vec{S}_i \cdot \vec{s}$, then at temperatures well above any ordering the impurity resistivity is

$$\rho_\infty = \lambda c [V^2 + J^2 S(S+1)],$$

where c is the impurity concentration and λ is a constant. At zero temperature in what is now known as the spin-glass phase, the spin disorder is as high as in the paramagnetic state but the spin-flip scattering is frozen out, so that the resistivity becomes

$$\rho_0 = \lambda c (V^2 + J^2 S^2).$$

The problem is how precisely the system passes from one limit to the other.

The overall Hamiltonian for N moments participating in a spin-glass is of the form

$$\mathcal{H} = \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j,$$

with the J_{ij} chosen at random according to some recipe within the range $-|J| < J_{ij} < |J|$. Assuming that the J_{ij} are known, one can imagine a *Gedankenrechnung* where \mathcal{H} is diagonalized directly. For N spin- $\frac{1}{2}$ moments this diagonalization implies resolving a $2^N \times 2^N$ matrix, and so this

is an insuperable problem in practice as soon as $N > 10$; however, we can get quite a reasonable intuitive idea of the shape of the density of states for the energy levels of the whole system. The distribution will contain 2^N states; it will be symmetric (as replacing all J_{ij} by $-J_{ij}$ inverts the distribution) and roughly Gaussian in shape. Calculations for $N < 7$ bear this out. In fact what is important for the low-temperature behavior of the system is the shape of the wing of the distribution, beginning at the lowest energy point, which will be defined as the ground state and the zero of energy.

We will now turn to the very instructive calculation of Walker and Walstedt, who treated numerically a spin-glass Hamiltonian with the assumption of RKKY interactions between the spins. They first found a ground-state configuration, and then derived a density of states $P(\Delta)$ for the N elementary excitations of the system as a function of the energy Δ . Now if the excitation concept is valid, in the low-energy wing of our distribution of energy states of the whole system we will be able to make the identification that each state of the whole system corresponds to a given set of occupation numbers of the elementary excitations. If excitation i is excited n_i times, then (n_1, n_2, \dots) defines one state of the whole system.

Now to calculate the resistivity I make the following assumption: The inelastic electron scattering processes consist of energy-conserving collisions in which one magnetic excitation is created or destroyed; the interaction strength is assumed the same for all excitations. In the low-energy wing this gives us selection rules for passing from one whole-system state to another. On average, the scattering rate for transitions from the states around energy E to the states around energy $E + \Delta$ will depend only on the excitation density of states $P(\Delta)$ together with a temperature-dependent factor expressing the energy conservation. This selection rule is strictly valid

only in the low-energy wing but I will assume that it works over the entire distribution of whole-system states. This is physically plausible as we are dealing with averages, and it allows us to make an explicit calculation.

The energy-conservation requirement for scattering of conduction electrons is well known.¹ The rate for scattering between a state of the system

$$\rho_{in}(T) \propto \sum_E \left(\frac{N(E)/\exp(E/kT)}{\sum_E N(E)/\exp(E/kT)} \sum_{\Delta} \frac{P(\Delta)(\Delta/kT)}{\exp(\Delta/kT) - 1} \right).$$

The sum over the states drops out, leaving the remarkably simple expression

$$\rho_{in}(T) \propto \sum_{\Delta} \frac{P(\Delta)(\Delta/kT)}{\exp(\Delta/kT) - 1}. \quad (1)$$

This $\rho_{in}(T)$ will have positive slope at all temperatures and will tend to saturate as $T \rightarrow \infty$. We now only need to know the excitation distribution $P(\Delta)$ in order to calculate $\rho_{in}(T)$ at all temperatures. Using the $P(\Delta)$ determined by Walker and Wolstedt I have calculated $\rho_{in}(T)$ and compare it with the observed $\rho(T)$ for $AuFe(1\%)$ ⁷ in Fig. 1. The vertical scale is fixed by the experimental room-temperature value of $\rho(T)$ and so the only free parameter is the energy scaling for $P(\Delta)$, which I have chosen appropriately. Specific heat data could also have been used to estimate this scaling; the total energy of the system of excitations (obeying Bose statistics) is

$$E_{ex} = \sum_{\Delta} \frac{\Delta P(\Delta)}{\exp(\Delta/kT) - 1}. \quad (2)$$

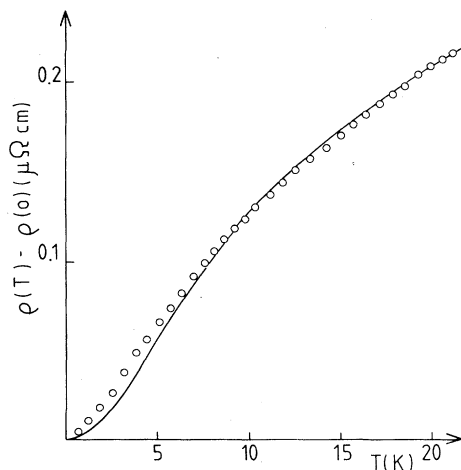


FIG. 1. Resistivity as a function of temperature for $AuFe(1\%)$. Circles, experimental results, Ref. 7. Full curve, calculated using Eq. (1) (see text).

at energy E and a state at energy $E + \Delta$ is proportional to $n(E)\Delta/[\exp(\Delta/kT) - 1]$ times the selection-rule factor. $n(E)$ is the occupation probability of the state at E . Since the resistivity is proportional to T^{-1} times the scattering rate and with the selection-rule assumption made above, the total inelastic resistivity $\rho_{in}(T)$ for the system will be

Comparing this with Eq. (1), we note that in the excitation regime, if the assumptions made so far are valid and with one excitation per magnetic atom, we should find the following exact relationship between the specific heat and $\rho_{in}(T)$:

$$C(T) = R \frac{d[T\rho_{in}(T)/\rho_{\infty}]}{dT}. \quad (3)$$

Figure 2 compares the experimental magnetic specific heat for $AuFe(1\%)$ ⁸ with a curve for $Rd[T\rho(T)/\rho_{\infty}]dT$ deduced directly from resistivity data for the same alloy.⁷ Equation (3) is well obeyed.

In both Figs. 1 and 2 the predictions of the present model are strikingly confirmed. This provides strong evidence in favor of the excitation approach, the Walker and Wolstedt calculation, and the assumptions made here. As the resistivity is a parameter which is considerably easier to measure with precision over a wide temperature range than is the specific heat, a detailed investigation of $\rho(T)$ in different metallic spin-glasses would be well warranted, in order to obtain information on the excitations in different systems and at different concentrations.

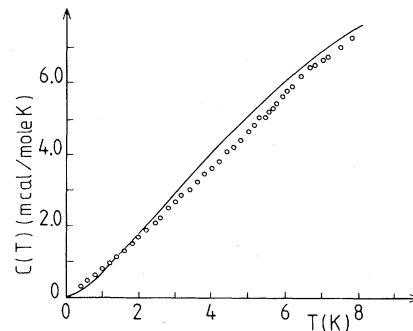


FIG. 2. Magnetic specific heat of $AuFe(1\%)$. Circles, experimental results, Ref. 8. Full curve, calculated from experimental resistivity results of Ref. 7 using Eq. (3).

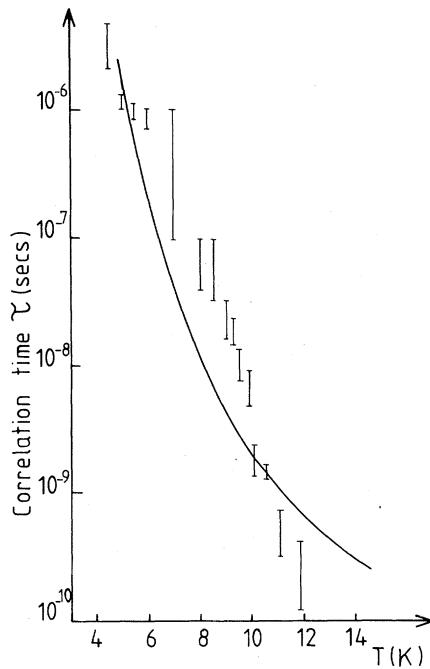


FIG. 3. Local spin correlation time in $AuFe(1\%)$. Vertical bars, experimental results, Ref. 9. Full curve, calculated using Eq. (4) (see text).

The present results can help to understand other spin-glass properties. One example is given here. Muon depolarization measurements have been used to obtain the local spin relaxation time in a number of spin-glasses.⁹ Naively one might imagine that as $\rho_{in}(T)$ is proportional to T^{-1} times the average spin-flip scattering rate, we should find the local spin relaxation rate τ^{-1} proportional to $T\rho_{in}(T)$. This is far from being the case: At low temperature τ^{-1} varies much more rapidly than $T\rho_{in}(T)$.⁹ The physical reason appears to be as follows: The relaxation time to which the muons are sensitive is the relaxation of single local spins; as the elementary spin-flip

processes are mixed together to form the excitations of the system, relaxation of single spins requires excitations taken from over the whole energy spectrum. As a first approximation, we can expect τ^{-1} to be proportional to the relaxation rate corresponding to the average excitation:

$$\tau^{-1} \propto \bar{\Delta} / (e^{\bar{\Delta}/kT} - 1), \quad (4)$$

where $\bar{\Delta} = \sum \Delta P(\Delta) / \sum P(\Delta)$. At low temperatures this will obviously vary much more rapidly with temperature than does $T\rho(T)$, which is dominated by the lowest-energy excitations.

I have calculated τ^{-1} for $AuFe(1\%)$ using the same $P(\Delta)$ and energy scaling as for $\rho_{in}(T)$. The result, Fig. 3, is in good overall agreement with experiment.

Work in progress suggests that the approach in terms of whole-system states and excitations can help to understand quantitatively a wide range of other spin-glass properties, such as the macroscopic relaxation processes and even the existence of a spin-glass ordering temperature.

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