Subharmonic Route to Chaos Observed in Acoustics

Werner Lauterborn and Eckehart Cramer[a]
Third Physical Institute, University of Göttingen, D-3400 Göttingen, Federal Republic of Germany
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A subharmonic route to chaos including period-doubling bifurcations up to $f/8$ has been observed in experiments on acoustical turbulence (acoustic cavitation noise). The system also shows signs of reverse bifurcation with increasing control parameter (acoustic driving pressure amplitude). In view of the large variety of phenomena observed and yet to be expected the system investigated may well serve as a further experimental paradigm of nonlinear dynamical systems besides Rayleigh-Bénard and circular couette flow.

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There is increasing evidence that period-doubling bifurcations[1] and strange attractors[2] are common phenomena for a large class of nonlinear dynamical systems. Most of this evidence stems from relatively simple mathematical models like the three-variable differential systems of Lorenz[3] and Rössler[4] and one-dimensional iterated maps on the unit interval[5] which show links to dynamical systems via the Poincaré return map. The discovery of universal properties in period-doubling bifurcations of iterated maps by Feigenbaum[1] could be confirmed for the Lorenz model[6] and a five-variable model of the Navier-Stokes equation,[7] and has stimulated the search for additional universal features of nonlinear dynamical systems.[8-11]

Compared with the large body of theoretical work, experiments are rather sparse. Up to now there are only two physical systems where the onset of chaos is studied systematically and which show some analogy to the behavior of the above mathematical models. These are the Rayleigh-Bénard experiment on the flow in a flat convective layer of liquid heated from below[12-14] and experiments on circular couette flow (flow between two cylinders, the inner one rotating).[15,16] Experiments in other fields are just emerging, such as in optics for optically bistable cavities[17] or proposed similar experiments with noisy Josephson junctions,[18] charge-density waves in anisotropic solids and superionic conductors,[19] or pinned dislocation lines.[20]

This paper presents experiments in acoustics which in view of the results to be reported and in analogy to the newly coined terms of "optical turbulence"[17] and "solid-state turbulence"[19,20] may be called experiments on "acoustical turbulence." The experiment consists in irradiating a liquid (water) with sound of high intensity (control parameter is the sound pressure amplitude) and looking for the sound output of the liquid, called acoustic cavitation noise. The physical situation is a somehow fundamental one: The transport of acoustical energy through a liquid is considered. It bears much resemblance with the Rayleigh-Bénard problem where the transport of heat through a liquid is investigated.

To irradiate the liquid a piezoelectric cylinder of 76-mm length, 76-mm inner diameter, and 5-mm wall thickness is used. When driven at its main resonance at 23.56 kHz a high-intensity acoustic field is generated in the interior, and cavitation is easily achieved. The noise is picked up by a broadband microphone[21] and digitized at rates up to 2 MHz after suitable low-pass filtering (to avoid aliasing in the subsequent Fourier analysis) and strong filtering of the driving frequency (to be able to store the noise with just an 8-bit storage). Sound pressure power spectra are calculated via the fast-Fourier-transform algorithm from usually 4K samples out of the 128K storage available. More details of the experimental setup are given elsewhere.[22]

Power spectra of acoustic cavitation noise usually consist of instrumentally sharp lines on a noise background. The lines are related to the driving frequency f_0 and lie at $(n/m)f_0$ ($n, m = 1, 2, 3, \ldots$). Of special interest are the lines at $m \geq 2$, $n < m$, i.e., in the subharmonic region $f < f_0$ of the spectrum. In early experiments the occurrence of lines at $f_0/2$, $f_0/3$, and $f_0/4$ has already been found,[23,24] but no convincing explanation could be given. The explanation that bubbles in water driven at twice their natural resonance are responsible for the $f_0/2$ line[25] had to be abandoned since bubbles of the necessary size could not be found and are unlikely to be present in the experimental situation.[26] Instead, after an extensive numerical investigation of bubble oscillations, it has been argued that special ultraharmonic resonances of bubbles smaller than resonant size (especially the $\frac{3}{2}$ resonance where two oscillations of the driving sound field match three oscillations of the natural oscillation frequency of the bubble) are responsible for the subharmonic line at $f_0/2$ in the power spectrum,[26] but a direct verification of this hypothesis has not yet been possible.

The present experiments were undertaken to add to our understanding of the subharmonic line problem. They differ from previous experiments in that they are fully computer controlled to realize almost any desired control parameter history.

Figure 1 gives just one example of a pressure power spectrum obtained at a driving voltage of 15 V. The history in this case was to linearly increase the driving voltage to 15 V and then to stay there for some time. These precautions are necessary to arrive at the third period-doubling bifurcation with lines at $nf_0/8$. But even when staying at a constant voltage large fluctuations are observed, and usually only spectra with subharmonics as low as $f_0/4$ can be observed. Figure 1 suggests that our nonlinear acoustical system may follow the period-doubling route to chaos and may belong to the universal class of Feigenbaum systems. Unfortunately this could not yet be proved as the next bifurcation with lines at $nf_0/16$ could not be reached. There may be principal difficulties in achieving this aim because our system can be expected to be a noisy one and this has been shown to limit the bifurcation sequence.[8]

As a result of the total computerization of the experiment we are able to do complex measurements on acoustical turbulence and to gather enormous amounts of data. The question therefore arises of how to present the results. One way that we found very appealing is in the form of grey-scale pictures analogous to "visible speech" where the power spectrum is plotted versus time with the amplitude encoded as grey scale. We have adopted this kind of presentation for our studies of how the system reaches chaos and have plotted the pressure power spectrum as a function of the voltage at the driving transducer. Figure 2 gives an example for the case where the voltage is increased linearly from 0 to 60 V in about 250 ms. During this time 128K samples of the pressure in the liquid are taken at a rate of 500 kHz. The total experiment thus lasts just a quarter of a second. From these data about 1000 overlapping short-time spectra are calculated with 4K data each and a shift of 128 samples from one spectrum to the next. In Fig. 2 three successive spectra are combined to give a total of 370 spectra. The grey level is encoded with the aid of a 3×3 matrix so that a binary plotter can be used.

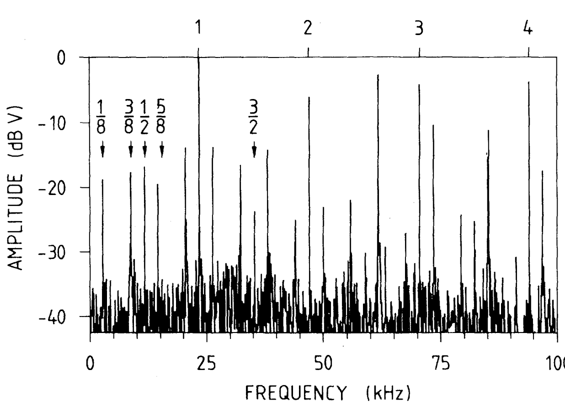

FIG. 1. Example of a pressure power spectrum of acoustic cavitation noise with subharmonic lines as low as $f_0/8$ ($f_0=22.56$ kHz), i.e., three period-doubling bifurcations have taken place. The driving frequency is strongly filtered.

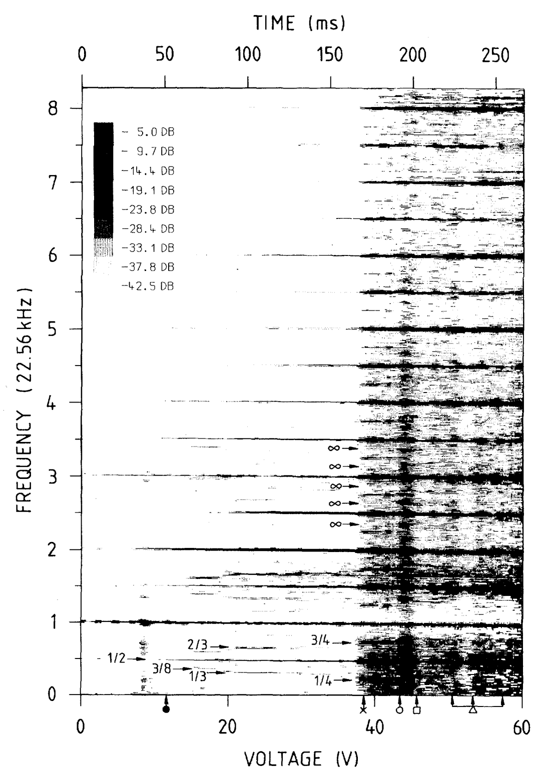

FIG. 2. Sequence of power spectra displayed as a grey-scale picture. The voltage at the driving piezoelectric cylinder is increased linearly from 0 to 60 V in 262 ms.

Many interesting features are immediately visible in Fig. 2:

(1) The first period-doubling bifurcation sets in at about 12 V (see closed circle in Fig. 2).

(2) Further subharmonic lines appear in a process that does not look like period doubling. These lines seem to detach from the $f_0/2$ (and $3f_0/2$) line with stops at $3f_0/8$ ($5f_0/8$) and $f_0/3$ ($2f_0/3$) until suddenly and with large amplitude the $f_0/4$ line and its odd harmonics set in (see 1/2, 3/8, 1/3, 2/3, 1/4, 3/4, and cross in Fig. 2).

(3) The $f_0/4$ bifurcation occurs together with a marked increase in broadband noise and a broadening of the otherwise sharp line spectrum (from cross to open circle in Fig. 2; the white gap around f_0 is due to the band reject filter needed to suppress the otherwise dominating driving frequency).

(4) At about 43 V a further increase in the broadband noise level is observed with the line spectrum still detectable and additional lines at certain odd harmonics of $f_0/8$ (see from open circle to open square and ∞ in Fig. 2). This state of the system ranges from 43 to 46 V only and is the most chaotic state encountered (chaos defined in terms of broadband noise at high level in the power spectrum).

(5) In a process looking like reverse bifurcation,[27] the system returns to a line spectrum with lines only at $f_0/2$ and its harmonics and with much less broadband noise (open square in Fig. 2).

(6) Satellite lines appear around $f_0/2$ and $3f_0/2$ (and also some other lines) which show some periodicity with the control parameter (open triangle in Fig. 2).

A general observation from other experiments is that the state of total chaos seems to be unstable since it cannot be sustained by the driving sound field for a longer period of time. Instead, oscillations are observed between the background noise and the line strength. Also single lines may visibly oscillate. This is best seen in films produced from sequences of up to 2000 power spectra plotted on a graphic display (cathode-ray tube) and filmed with a 16-mm film camera.

The picture of Fig. 2 is a rather condensed form of looking at the properties of a dynamical system. It would be very interesting to see other dynamical systems like those of Lorenz[3] and Rössler[4] as well as the experimental ones of Rayleigh-Bénard and circular couette flow displayed in this way.

Some effort has been spent in modeling the experiments theoretically. As a first step single spherical bubbles have been taken and subjected to a sinusoidal driving pressure of increasing amplitude. The mathematical model is a highly nonlinear ordinary differential equation of second order for the radius of the bubble as a function of time and includes surface tension, viscosity, and compressibility of the liquid (water).[28] Radius-time curves have been calculated numerically for different bubble sizes, sound pressure amplitudes, and frequencies to get response curves for this nonlinear system.[28] To simulate the present experiments bubble wall oscillation power spectra have been calculated and plotted in the manner of Fig. 2 as a function of the driving pressure. Qualitatively similar behavior is observed but strongly depends on the bubble size. Of most importance seems to be the observation that bifurcations are obtained, mostly from f_0 to $f_0/2$ to $f_0/3$ (or also $f_0/4$) to (quickly) $f_0/4$ to chaos and then back to f_0 or $f_0/2$ (directly). Thus both successive bifurcations as well as reverse bifurcations are observed in this simple mathematical model, like those encountered in the experiments.

More sophisticated models must take into account that many bubbles are present in the liquid which all couple via their sound radiation. A fuller discussion of these questions as well as of the physical processes responsible for the observed phenomena is given in Ref. 22.

We arrive at the conclusion that in our acoustic system there is a subharmonic route to turbulence, but one which does not solely proceed via successive period-doubling bifurcations although this route is strongly involved. We therefore propose to make a distinction between a subharmonic and a period-doubling route to chaos. Moreover, our system shows signs of reverse bifurcations which may be worthwhile to study in greater detail.

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[a]Present address: Physikalisch Technische Bundesanstalt-IB, Abbestr. 2-12, D-1000 Berlin 10, West Germany.

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Determination of Transition in Couette Flow in Finite Geometries

Kwangjai Park, Gerald L. Crawford, and Russell J. Donnelly
Department of Physics, University of Oregon, Eugene, Oregon 97403
(Received 10 August 1981)

The transition from laminar to Taylor vortex flow in a couette apparatus with fixed ends has been studied. The reversibility of the transition has been examined with use of a reflectance technique with Kalliroscope tracer with a variety of geometries and fluids. Very little hysteresis is found, providing the speed is ramped with a dimensionless acceleration less than a certain value. Contrary to earlier expectations, it is not possible to neglect the effect of the ends of the apparatus simply by making the apparatus longer.

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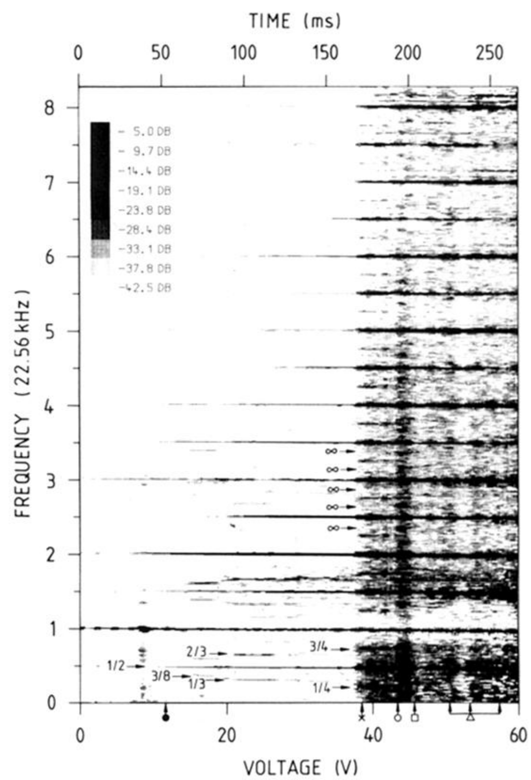
Consider a couette-flow apparatus with cylinders of radii R_1 and R_2, gap $d=R_2-R_1$, and length L between fixed ends. The radius ratio is defined as $\eta=R_1/R_2$ and the aspect ratio $\Gamma=L/d$. Most theoretical work on the couette problem assumes that there are no ends, and experimentally many results with large aspect ratio have been in reasonable agreement with theory. The experimenter, however, needs to know how rapidly the Reynolds number $N_{\rm Re}=\Omega R_1 d/\nu$ can be changed and still maintain quasistatic conditions (here Ω is the angular velocity of the inner cylinder and ν is the kinematic viscosity of the fluid). This paper advances such a criterion based on a series of careful measurements of the transition. The results, however, demonstrate that an attempt to neglect the effects of the ends by simply increasing L is unrealizable in an actual laboratory experiment. These results have important consequences for time-dependent experiments in couette flow and possibly other experiments such as Bénard convection.

We have used glycerol-water solutions with various amounts of Kalliroscope tracer (polymeric flakes) and have measured the light reflectance at a local minimum near the center of the apparatus (a minimum in the reflectance corresponds to a maximum in the radial speed of the fluid). The fluid temperature was measured with a HP 2804A quartz thermometer. The output of the photocell



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