effectively particle-particle in character (proton $Z=50-82$ shell less than half filled) and immediately develop a deformed structure, as evidenced by the sharply increased value of $E(4_1^+)/E(2_1^+)$. Once the $Z=64$ closure is eliminated, this energy ratio becomes almost Z independent and so does its first difference. Finally, the concept of a tendency towards triaxiality for the Nd, Sm nuclei with $N=86, 88$ is further supported by the fact that the quasi-γ-band energy minimizes in exactly this region.

These ideas allow a more detailed understanding of deformation and can be applied to other mass regions as well. Thus with decreasing mass in the heavy rare-earth region (Pt, Os, and W) deformation ensues when the $(h_{11/2})_\pi$ and $(i_{13/2})_\nu$ orbits are filling but is more gradual because the larger separation $[E_{\text{Nilsson}}(K)\sim K^2]$ of the high-K orbits involved leads to a weaker accumulation of a strong n-p interaction. In any heavy-mass region, a certain minimum number of valence particles are a precondition for deformation. Thus, in the Sn nuclei, deformation would be precluded until the neutrons are filling the high-K $g_{7/2}$ orbits. Since the protons would be polarized into the low-K orbits and since the proton gap is larger, deformation does not ensue. In the Sm, Gd region, however, the $(f_{7/2})_\nu$-$(h_{9/2})_\nu$ order of filling allows the former to play the role of a "buffer" orbit.

Finally, the Talmi[3] rule is now seen to be broadened in importance and scope. The point is not so much that singly magic nuclei cannot deform (the Gd and Zr regions belie this) but that the n-p interaction is the essential ingredient. In certain cases of moderately sized gaps (e.g., Sm and Gd region) it can indeed be sufficiently efficacious to create in a sense the very conditions (disruption of a closed shell with ensuing occupation of spatially correlated orbits) to facilitate its own action.

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Elastic Scattering of 500-MeV Polarized Protons from ^{40,48}Ca, ^{90}Zr, and ^{208}Pb, and Breakdown of the Impulse Approximation at Small Momentum Transfer

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New 500-MeV $\vec{p}_{\text{pol}}+{}^{40,48}$Ca, ^{90}Zr, and ^{208}Pb elastic $d\sigma/d\Omega$ and $A_y(\theta)$ data are presented. The small-momentum-transfer data are not reproduced by the second-order Kerman-McManus-Thaler optical potential which uses free nucleon-nucleon amplitudes. The systematics of these results, taken together with previous 800-MeV results, suggest that medium modifications to the NN t matrix are required to obtain an accurate description of the 500–800-MeV proton-nucleus elastic data in general.

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Proton-nucleus elastic differential cross section $[d\sigma/d\Omega]$ and analyzing power $[A_y(\theta)]$ data at 800 MeV have been used to test microscopic descriptions of the scattering in terms of the impulse approximation (IA).[1-4] Encouraging results were obtained, but parameter-free calculations only led to qualitative agreement for $A_y(\theta)$,[4] and to deduced neutron-proton root-mean-square (rms) radius differences (Δr_{np}) about 0.1 fm smaller than theoretically predicted.[1] It was suggested that these difficulties were due to the lack of accurate knowledge of key nucleon-nucleon (NN) amplitudes needed for the calculations and that one should do experiments at energies (for example, 500 MeV) where the IA still is expected to be valid but where the amplitudes are better known.[5]

We present here new data for 500-MeV $\vec{p}_{\rm pol}$ $+^{40,48}$Ca, ^{90}Zr, and ^{208}Pb and the results of initial theoretical work done to investigate the adequacy of existing microscopic approaches for describing medium-energy $\vec{p}$ + nucleus elastic scattering. Based on these 500-MeV results and those obtained at 800 MeV[1-5] we conclude that the IA is inadequate at these energies for momentum transfer $q \lesssim 1$ fm^{-1}; the inadequacies at small q suggest that the calculations should include blocking, binding, and other effects of the nuclear medium. Finally, we note that nuclear structure information deduced through analysis of 500–800-MeV data may be quite sensitive to inclusion of medium effects.

The data were taken with the high-resolution spectrometer at the Clinton P. Anderson Meson Physics Facility (LAMPF) and are shown in Figs. 1 and 2. Statistical and other uncertainties as well as experimental details are essentially as given in Refs. 2 and 4. Similar data with poorer angular resolution have been obtained at TRIUMF.[6]

The theoretical calculations now discussed are the same type as those reported in recent analyses of 800-MeV data.[1-5] They employ the multiple-scattering optical-potential formalism of Kerman, McManus, and Thaler (KMT); the second-order, proton-nucleus KMT optical potential is approximated by a local, spin-dependent form, and solution of the Schrödinger equation with relativistic kinematics leads to the proton-nucle-

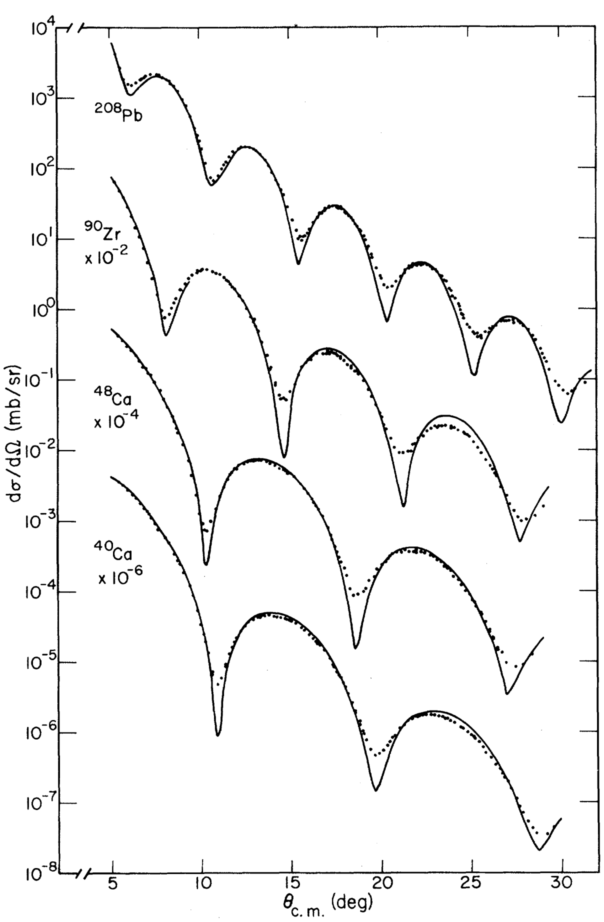

FIG. 1. 500-MeV $\vec{p}_{\rm pol}$ + ^{40,48}Ca, ^{90}Zr, and ^{208}Pb elastic differential cross sections are compared to results (solid curves) of KMT calculations obtained with use of proton-nucleon amplitudes from phase-shift analysis.

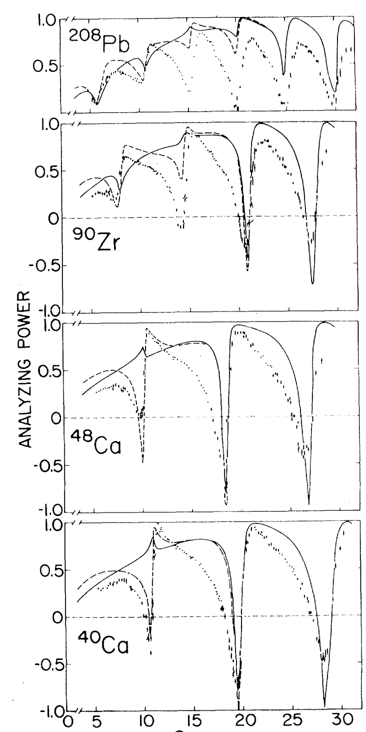

FIG. 2. Same as Fig. 1, but analyzing powers. The dashed curves are the results of KMT calculations with use of an effective t^{so} for $q \leq 0.75$ fm^{-1} (see text and Fig. 3).

us observables.

The solid curves (Figs. 1 and 2) are results obtained with proton-nucleon (pN) amplitudes (solution SL81) obtained recently by Arndt through phase-shift (PS) analysis of NN data.[7] The proton densities were obtained as in Ref. 3 from the empirically known charge densities,[8] while the neutron densities (three-parameter Fermi or Gaussian distributions) were varied to obtain the best fits to the $d\sigma/d\Omega$.

Unfortunately (see Fig. 2), the theoretical $A_y(\theta)$ show poor qualitative agreement with data forward of 20°: Apart from reproducing the correct (positive) overall sign of the envelopes, the curves fail to track the data. Particularly disturbing is the fact that very little structure in $A_y(\theta)$ is predicted forward of 20°, but the data continue to oscillate. Also the minima in the theoretical $d\sigma/d\Omega$ (Fig. 1) are too deep. Noteworthy also is the fact that the Δr_{np} values are about 0.1 fm *smaller* than values obtained at 800 MeV, which themselves are about 0.1 fm *smaller*[1] than predicted by Hartree-Fock calculations. Thus, a consistent analysis leads to an apparent energy dependence for the deduced neutron distributions!

We therefore see similar problems with IA calculations at *both* 500 and 800 MeV: The $A_y(\theta)$ predictions are poor near $q \simeq 1-1.5$ fm^{-1} for each nucleus studied at both energies, and the deduced neutron rms radii are too small; the discrepancies at 500 MeV are larger, however, than at 800 MeV.

The sensitivity of the 500-MeV theoretical results to density uncertainties, to PS amplitude errors, and to second-order correlation contributions was investigated, and we conclude that such uncertainties *cannot* account for the observed systematic discrepancies. The velocity-dependent, electromagnetic spin-orbit potential which is significant for $A_y(\theta)$ for small q at 800 MeV[4] was included (not included in Fig. 2) but does not account for the discrepancies. The importance of Fermi-motion averaging was also considered. Calculations at 500 MeV were carried out as above, except that several NN amplitudes corresponding to NN laboratory energies varying between 325 and 800 MeV[7] were used; the featureless structure in $A_y(\theta)$ near $q = 1$ fm^{-1} persisted, making it unlikely that the neglect of Fermi-motion averaging is the cause of the discrepancies.

Thus, because of the sensitivity studies, and because there is little reason to suspect the results of phase-shift analysis at 500 MeV, we are *forced* to consider the possibility that a more fundamental (theoretical) inadequacy exists in the conventional application of the KMT formalism. Because systematic and similar problems exist at both 500 *and* 800 MeV, the clear implication is the same inadequacy at both energies. We believe that an important clue to the origin of the difficulties is the fact that theory fails to reproduce $A_y(\theta)$ at both energies over the *same* region of *small* momentum transfer. Since any breakdown of the impulse approximation should be momentum-transfer dependent and most severe at small momentum transfer, we speculate here that momentum-transfer–dependent medium effects are present at 500–800 MeV which lead to a breakdown of the IA at small q. In support of this speculation we simply note the following: (1) Single-particle nucleon binding energies are typically 10–50 MeV while the recoil energy of the struck nucleon (in the proton-nucleus c.m. system) varies between 0 and 47 MeV for $0 \leq q \leq 1.5$ fm^{-1} (target nucleon initially at rest in nucleus); thus, for small q, nucleon-nucleon processes are *necessarily* influenced by the nuclear medium. Inspection of recent 800-MeV (p,p') inclusive data[9,10] indicates that quasielastic processes kinematically follow free NN scattering for $\theta > 5°$ (laboratory), or $q > 0.65$ fm^{-1}, but at smaller angles such processes are severely blocked and/or modified by the low-lying discrete nuclear states. Since quasielastic doorways account for nearly all of the reactive content of the medium-energy optical potential,[10,11] modifications to quasielastic scattering will have nonnegligible effects on the elastic channel. (2) The characteristic time constant for the bound nucleon is[12] $\Delta t \simeq \hbar/(50$ MeV$) \simeq 4$ fm$/c$; if the pN scattering at small q is dominated by the long-range part of the NN potential [one-pion exchange potential (OPEP) with a rms radius of 3.5 fm], we obtain a 500–800-MeV reaction time of $t_r \simeq 4$ fm$/c$; these times are comparable. On the other hand, at large q the NN strong repulsive core dominates and $t_r \simeq 0.6$ fm$/c$; therefore an impulsive scattering description should better represent the effective t matrix (IA) at larger q. Such medium effects have not been investigated theoretically for $E_p > 200$ MeV; for $E_p < 200$ MeV they are known to be significant.[13]

It is therefore interesting to determine if effective amplitudes can be found, through slight modification of the PS amplitudes at small q, which lead to better agreement with experiment. We performed KMT calculations where the effective pN t matrices were given by $t_{\text{eff}}(q) = f(q) t_{pN}(q)$, where $f(q) = (b-a)q/\bar{q} + a$, for $0 \leq q \leq \bar{q}$, and $f(q) = 1.0$ for $q > \bar{q}$; t_{pN} contains the spin-independent and spin-orbit parts of the SL81 PS amplitude: $t_{pN} = t_{pN}{}^0 + t_{pN}{}^{so}\vec{\sigma}\cdot\hat{n}$. The linear form for $f(q)$ was used for simplicity. The value of $\bar{q}$ was set equal to 0.75 fm^{-1} based on the quasielastic (p,p') data and the momentum corresponding to nucleon binding energy $\simeq$ 10 MeV. The phenomenological results are as follows: (1) Modifications to Re(t^0) and Re(t^{so}) do not significantly affect $d\sigma/d\Omega$, $A_y(\theta)$, or Δr_{np}. (2) Increasing Im(t^0) *worsens* the A_y prediction and Δr_{np} gets *smaller* yet (with fits to $d\sigma/d\Omega$ equivalent to those shown in Fig. 1), while a 10% suppression of Im(t^0) at small q ($a = b = 0.9$) improves A_y and *increases* Δr_{np} (with fits to $d\sigma/d\Omega$ equivalent to those shown in Fig. 1). However, variation in Im(t^0) alone cannot resolve the A_y and Δr_{np} discrepancies at both 500 and 800 MeV. (3) On the basis of a series of calculations in which all possible combinations of (a,b), where $a = 0$, 1, 2, and $b = 0$, 1, 2, for Im(t^{so}) were tried, we find that enhancement of Im(t^{so}) for q between 0.4 and 0.75 fm^{-1} (9° to 18° in the NN c.m. system) improves the agreement between experiment and theoretical A_y and suppression in this region worsens the agreement considerably. The calculated $d\sigma/d\Omega$ are not sensitive to this enhancement. Optimized results (with $a = 1.7$, $b = 1.0$) are shown by the dashed curves in Fig. 2. Note that the description of the data forward of 20° is qualitatively improved and that the reasonable IA description beyond this angle is not disturbed. A similar calculation at 800 MeV was shown to provide an excellent fit to the $p + {}^{40}$Ca $A_y(\theta)$ data. This imaginary, effective spin-orbit amplitude for $p + p$ is shown in Fig. 3 (dashed curve) and lies well outside the error corridor of the free amplitude (shaded band) obtained from PS analysis; the $p + n$ comparison is similar. The enhancement of Im(t^{so}) varies from 34% to 0% from 9° to 18° in the NN c.m. system. (4) Increasing Im(t^{so}) at all q results in A_y being too strong with no improvement in the forward angle structure. (5) Finally, we note that setting $a = b = 0.84$ (0.955) for Im(t^0) at 500 (800) MeV and setting $a = 1.7$ and $b = 1.0$ for Im(t^{so}) ($\bar{q} = 0.75$ fm^{-1} in each case) yields the expected $\Delta r_{np} = -0.05$ fm for ^{40}Ca and considerably improved descriptions of the $A_y(\theta)$ data at *both* energies. The fits to the $d\sigma/d\Omega$ data are indistinquishable from the curves shown in Fig. 1.

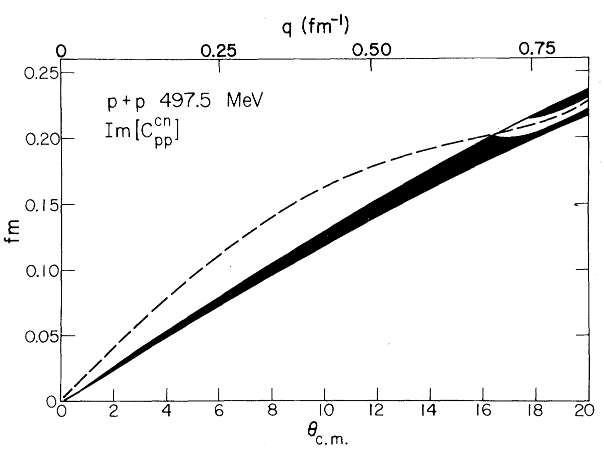

FIG. 3. The shaded band encompasses all phase-shift predictions obtained by Arndt (Ref. 7) for Im$(c_{pp}{}^{CN}) = -(\epsilon_{NN}/2\pi\hbar^2c^2)\,Im(t_{pp}{}^{so})$, where ϵ_{NN} is the NN reduced energy. The dashed curve is the corresponding effective amplitude discussed in the text.

Since little theoretical work has been done concerning effective interactions for energies greater than $\simeq$ 200 MeV, it is not possible to compare the phenomenological findings to theoretical expectations. We point out, however, that the Pauli blocking estimate of Clementel and Villi[14] suggests that Im$[t^0(0)]$ should be *reduced* by about 10% (5%) at 500 (800) MeV, and that an extrapolation of a comparison of IA and medium-modified spin-orbit optical potentials between 100 and 200 MeV hints[13] that the medium-modified Im(t^{so}) amplitude at higher energies is *larger* than the IA Im(t^{so}).

We conclude that a proper explanation of *both* the 500- and 800-MeV data demands an accurate, quantitative accounting of nuclear medium corrections to the IA. Such findings are consistent with results obtained recently in KMT analysis of 400-MeV proton scattering from ^{208}Pb.[6]

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