Relation between the $Z = 64$ Shell Closure and the Onset of Deformation at $N = 88-90$

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The systematics of the onset of deformation in the $N = 88-90$ region is reexamined in light of the recently discovered shell closure at $Z = 64$. It is shown that the microscopic explanation of the origin of deformation already proposed for the $A\sim 100$ region is equally applicable here and leads to a new interpretation of the $A\sim 150$ region.

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The $N=88-90$ transition region from spherical to quadrupole prolate deformation is perhaps the best known and most thoroughly studied nuclear shape/phase transition known. Figure 1 illustrates the transition region in its most familiar aspect, namely the sharply decreasing energy of the first 2^+ state as a function of N for several elements in this region. There have been numerous successful theoretical attempts to understand this sudden onset of deformation, usually in terms of the calculation of equilibrium deformations as a function of neutron number N in some version of a Nilsson potential. The generally accepted interpretation is that since the lowest Nilsson orbits in the $N=82-126$ shell are strongly downsloping as a function of deformation, a deformed minimum in the potential-energy surface eventually becomes energetically favored. Detailed calculations correctly predict the onset of deformation between $N=88$ and 90 and relate it to the occupation of the low-K orbits from the $h_{9/2}$ shell. Indeed, these ideas are substantiated and nicely illustrated by the otherwise surprising fact that the $\frac{3}{2}^-[521]$ neutron orbit from the $h_{9/2}$ shell is the ground state in most $N=91$, 93, and 95 nuclei.[1]

However, the recent discovery[2] of a shell closure at $Z=64$ in ^{146}Gd ($N=82$) presents an enigma. It is well known[3] that the existence of deformation is strongly reliant on the neutron-proton interaction. Indeed, as Talmi has repeatedly emphasized,[3] nuclei which are magic (e.g., Sn) or near magic (e.g., Hg) in one type of nucleon exhibit a remarkable resistance to deformation and a constancy in $E(2_1^+)$ that is a characteristic feature of an unbroken generalized seniority scheme. Yet, with the discovery of a major subshell closure at $Z=64$, the existence of substantial deformation in Gd nuclei with $N\geqslant 90$ would seem to imply the onset of deformation in a singly magic nucleus with only ~ 8 neutrons outside the neutron closed shell.

Unfortunately, the systematics of Fig. 1, while familiar, are not particularly helpful in elucidating this situation: Indeed they obscure precisely the most interesting physical effects. If the same data are instead plotted against *proton* rather than neutron number, the effects of the subshell closure at $Z=64$ should be particularly evident. Figure 2 presents such a plot along with the energy ratio $E(4_1^+)/E(2_1^+)$, which is perhaps an even better structure indicator. While a high (low) 2_1^+ energy is a convenient signature of a spherical (deformed) nucleus, the ratio $E(4_1^+)/E(2_1^+)$ has an opposite and more specific behavior ranging from $\leqslant 2$ (spherical-vibrational nucleus) to 3.33 for a good rotor.

Thus, the 2_1^+ energy can be expected to take on a maximum value, and $E(4_1^+)/E(2_1^+)$ a minimum value, at a shell closure. While it can be seen that these effects clearly manifest themselves for $N\leqslant 88$, it is equally evident that the *exact inverse* of this behavior occurs for $N\geqslant 90$, where the isotones are seen to achieve a *maximum deformation near* $Z=64$. These Z-dependent systematics therefore reveal that not only does a nucleus which is supposedly magic in protons become deformed rather suddenly, but it attains essentially the *maximum* deformation for each neutron number.

To our knowledge, the juxtaposition of these

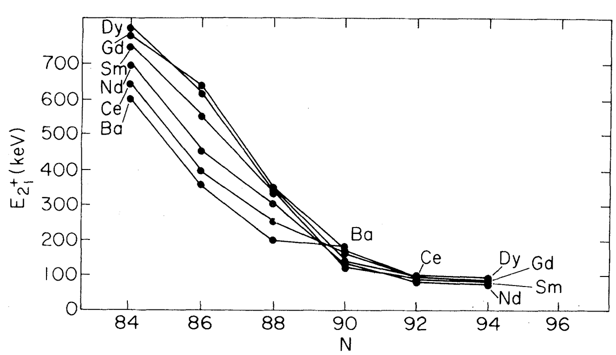

FIG. 1. Systematics of $E(2_1^+)$ in the $A\sim 150$ region.

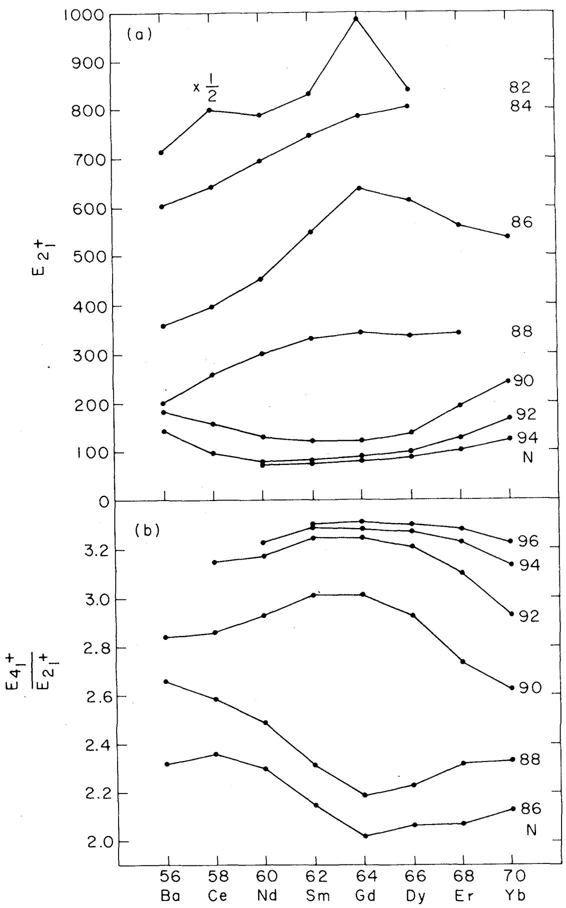

FIG. 2. Systematics of $E(2_1^+)$ and $E(4_1^+)/E(2_1^+)$ vs proton number.

two essential points has not been explicitly treated nor successfully understood. It is the purpose of this Letter to offer an explanation of these Z systematics and, by so doing, to resolve the apparent anomaly of the Gd nuclei. A possible microscopic explanation will be employed which leads to a previously unsuspected prediction of a tendency toward triaxiality in the Nd, Sm, and Gd nuclei. It is shown that this behavior is indeed exhibited in existing data. In turn, the present interpretation permits an understanding of the presence or absence of deformation in other mass regions and leads to a broader understanding of the Talmi rule[3] of the nondeformability of magic or near-magic nuclei.

The essential element in the present interpretation is contained in the pioneering work of Federman and Pittel on the $A\sim100$ region[4] where the most abrupt spherical-deformed transition in heavy nuclei occurs. The nuclei which for $N\leq 58$ are most spherical become the most deformed for $N\geq 60$. Since, in this case, the deformation arises with so few valence nucleons, an explicit shell-model calculation is possible. Federman and Pittel have carried this out and successfully predicted[4] the phase transition. The key factor is the recognition[3,4] that the n-p interaction is largest, and therefore most effective in its deforming tendency, when the particles are in *spin-orbit partner* (SOP) orbits of large spatial overlap. In such orbits the n-p force most effectively counteracts the spherically driving n-n and p-p pairing correlations. Although the largest effects of the n-p force will occur for SOP orbits, other orbits of comparable l and j may also have large spatial overlap with similar but weaker effects. In the $A\sim100$ region, the protons, with $Z=40$, have a filled $p_{1/2}$ shell and the next orbit is the $g_{9/2}$. For a fixed Z, as N increases from 56 to 60 and higher, the neutrons begin filling the $g_{7/2}$ orbit and induce a *polarization of the protons* such that they partially occupy the $g_{9/2}$ orbital. The gain in energy thus obtained by maximizing the n-p interaction in SOP orbits more than compensates the loss in single particle and pairing energies.

The present Letter suggests that, despite the atypical abruptness of deformation in the $A\sim100$ region, the underlying physical mechanism applies equally well in the Sm and Gd region. This follows, despite the suggestion[4] that the involvement of SOP orbits is not as important in heavy nuclei, from the new recognition that the $Z=64$ shell closure effectively reduces the number of valence particles. The Gd nuclei are doubly magic at $A=146$, where $N=82$ and the protons have filled the $g_{7/2}$ and $d_{5/2}$ orbits above $Z=50$. The addition of neutrons first occurs in the $f_{7/2}$ orbit but, given the recognition that pairing correlations will spread out the occupation amplitudes, will begin to appreciably fill the $h_{9/2}$ orbit near $N=88-90$. In the process, a polarization of the proton shell structure occurs so that protons begin to fill the $h_{11/2}$ orbit even though Z remains constant. The n-p interaction can now operate to the fullest and an immediate transition to a deformed structure occurs, *with the consequent eradication of the $Z=64$ subshell gap.*

Thus, the overall systematics of Fig. 2 can now be easily understood. Starting with a strong proton subshell gap, and therefore a minimum in deformation at $Z=64$, $N=82$, the addition of successive neutrons and the consequent increasing occupation of the $(h_{9/2})_\nu$ and $(h_{11/2})_\pi$ SOP orbits gradually diminishes the severity of the gap. There is, indeed, empirical evidence for this. The most telling is perhaps contained in an extensive series of proton transfer experiments by Burke, Straume, and Løvhoiden. They find,[5] from sums of spectroscopic strengths in the (t, α) reaction, that the number of particles in the $h_{11/2}$ orbit in the Sm isotopes increases from 2.3 to 6.7 from $N = 86$ to $N = 88$ for *constant* Z. By $N = 90$ this process culminates and a deformed structure becomes energetically favored. For $N \geq 90$, the maximum deformation occurs near $Z = 64$. To understand this, it is necessary to consider the role of n-p correlations in a deformed field. A maximum deformation requires not only that valence n-p spherical SOP orbits be available but that the ensuing deformation leads to neutron and proton Fermi surfaces near orbits of comparable K values so that the *spatial overlap is maintained in the deformed field.* Since the ground state for most $N = 91-95$ nuclei is the $\frac{3}{2}^-[521]$ neutron orbit, the maximum spatial correlation in the deformed nucleus will occur when the proton Fermi surface is near the $\frac{3}{2}^-[541]$ orbit from the $h_{11/2}$ shell. This occurs precisely for $Z = 62, 64$ and, indeed, rather more for the former which might explain the slight empirical preference for maximum deformation in Sm rather than Gd. For $Z \geq 68$, the higher-K $(h_{11/2})_\pi$ orbits are filling and the spatial overlap of valence neutrons and protons is reduced, while for nuclei with $Z \leq 60$, it is energetically more difficult to induce protons into the $h_{11/2}$ orbit and hence the deformation is lower. The combination of these ideas suggests that indeed the deformation should be a maximum near $Z = 64$, as observed.

As in the $A \sim 100$ region, although to a lesser extent here, it is the existence of the proton-shell gap that determines the *suddenness* of the onset of deformation. Were there no such gap between the $d_{5/2}$ and $h_{11/2}$ orbits there would be a much larger effective valence proton number, earlier occupation of the $h_{11/2}$ orbit, and therefore a greater cumulative n-p interaction effect and most likely a more gradual shape transition.

The role of the $Z = 64$ shell gap has another, unexpected consequence which was heretofore empirically unrecognized but in fact is rather clearly evident in the data. Figure 3 shows a plot of the ratio $E(4_1^+)/E(2_1^+)$ against neutron number as well as the first differences of this quantity. The usual interpretation is that the ratio changes smoothly from ~2 to ~3.33 as A

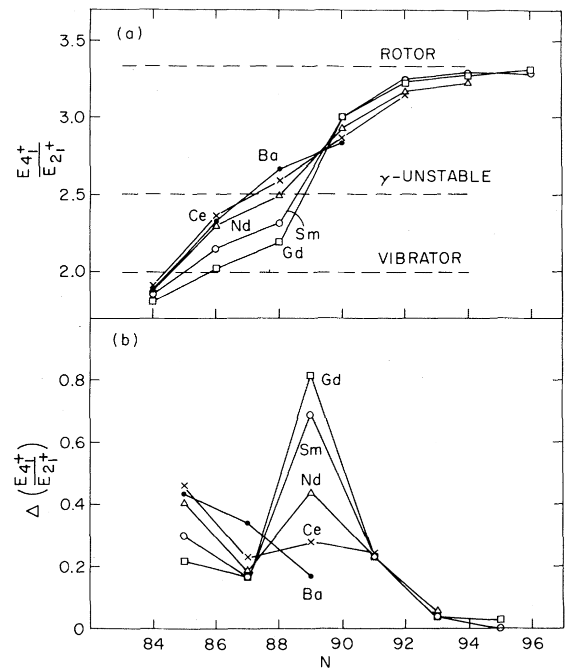

FIG. 3. (a) Systematics of $E(4_1^+)/E(2_1^+)$ in the $A \sim 150$ region. (b) First differences of the data in part (a).

increases. While this is qualitatively correct, more careful inspection of Fig. 3(a) shows a more detailed structure that is physically meaningful, namely, a definite change in curvature at $N = 88-90$ for Gd, which becomes gradually less distinct as Z decreases, finally vanishing for Ba. This kink in the systematics is instantly obvious from the difference plot. Furthermore, inspection of the ratio plot for Nd and Sm shows that, before the upward turn at $N = 90$, the curves are asymptotically approaching a value close to 2.5 which is exactly the limiting value for a γ-unstable nucleus and close to the value for triaxial nuclei with large γ. This is contrary to all prior conceptions of these nuclei but is entirely reasonable and, indeed, *a priori* expected in light of the present analysis, wherein the $Z = 64$ shell closure is effective for $N \leq 88$ but vanishes for $N \geq 90$. Thus, for $N \leq 88$, the Gd nuclei show $E(4_1^+)/E(2_1^+)$ ratios near 2.0, and nuclei such as Nd and Sm, with proton *holes* (proton $Z = 50-64$ shell more than half filled) and neutron particles, can be expected to show a tendency toward triaxiality. The Ba and Ce nuclei are rather more particle-particle in character and approach a deformed character nearly monotonically. For $N \geq 90$, the effects of the $Z = 64$ closed shell vanish, and the Nd, Sm, and Gd nuclei suddenly become effectively particle-particle in character (proton $Z=50-82$ shell less than half filled) and immediately develop a deformed structure, as evidenced by the sharply increased value of $E(4_1^+)/E(2_1^+)$. Once the $Z=64$ closure is eliminated, this energy ratio becomes almost Z independent and so does its first difference. Finally, the concept of a tendency towards triaxiality for the Nd, Sm nuclei with $N=86, 88$ is further supported by the fact that the quasi-γ-band energy minimizes in exactly this region.

These ideas allow a more detailed understanding of deformation and can be applied to other mass regions as well. Thus with decreasing mass in the heavy rare-earth region (Pt, Os, and W) deformation ensues when the $(h_{11/2})_\pi$ and $(i_{13/2})_\nu$ orbits are filling but is more gradual because the larger separation $[E_{\text{Nilsson}}(K)\sim K^2]$ of the high-K orbits involved leads to a weaker accumulation of a strong n-p interaction. In any heavy-mass region, a certain minimum number of valence particles are a precondition for deformation. Thus, in the Sn nuclei, deformation would be precluded until the neutrons are filling the high-K $g_{7/2}$ orbits. Since the protons would be polarized into the low-K orbits and since the proton gap is larger, deformation does not ensue. In the Sm, Gd region, however, the $(f_{7/2})_\nu$-$(h_{9/2})_\nu$ order of filling allows the former to play the role of a "buffer" orbit.

Finally, the Talmi[3] rule is now seen to be broadened in importance and scope. The point is not so much that singly magic nuclei cannot deform (the Gd and Zr regions belie this) but that the n-p interaction is the essential ingredient. In certain cases of moderately sized gaps (e.g., Sm and Gd region) it can indeed be sufficiently efficacious to create in a sense the very conditions (disruption of a closed shell with ensuing occupation of spatially correlated orbits) to facilitate its own action.

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Elastic Scattering of 500-MeV Polarized Protons from ^{40,48}Ca, ^{90}Zr, and ^{208}Pb, and Breakdown of the Impulse Approximation at Small Momentum Transfer

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New 500-MeV $p_{\text{pol}}+{}^{40,48}$Ca, ^{90}Zr, and ^{208}Pb elastic $d\sigma/d\Omega$ and $A_y(\theta)$ data are presented. The small-momentum-transfer data are not reproduced by the second-order Kerman-McManus-Thaler optical potential which uses free nucleon-nucleon amplitudes. The systematics of these results, taken together with previous 800-MeV results, suggest that medium modifications to the NN t matrix are required to obtain an accurate description of the 500–800-MeV proton-nucleus elastic data in general.

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Proton-nucleus elastic differential cross section $[d\sigma/d\Omega]$ and analyzing power $[A_y(\theta)]$ data at 800 MeV have been used to test microscopic descriptions of the scattering in terms of the impulse approximation (IA).[1-4] Encouraging results were obtained, but parameter-free calculations

