Fractionally Charged Color-Singlet Fermions in a Grand Unified Theory

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Grand unified gauge theories that contain fractionally charged color-singlet fermions are considered. The minimal theory of this type is an SU(7) model which requires a rich structure of new physics in the energy region 100–1000 GeV.

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The observation of electrical charges $\pm\frac{1}{3}e$ has recently been reported.[1] If these observations should be confirmed by further experimental tests, two alternatives may be considered in the context of current particle theory: (1) color SU(3) is broken, and liberated quarks,[2] or diquarks,[3] have been observed, or (2) the observed charges are due to color singlet ($\underline{1}_c$) particles with fractional charge. We have begun a study of the second alternative in the framework of a grand unified theory (GUT) of the electroweak and strong interactions.[4]

Specifically, we have examined minimal extensions of the Georgi-Glashow SU(5) model[5] to include fractionally charged color singlets, subject to requirements described below. We have found SU(6) to be excluded as a candidate GUT, and an essentially unique SU(7) model[6] as the minimal extension of SU(5). The limitations and difficulties encountered in the SU(7) case are instructive, and we wish to report them here.

By now there exists a substantial array of constraints on the candidacy as a GUT of any gauge group G and its fermion representation. We divide these into representation (R) constraints and dynamical (D) constraints.

Representation (R) Constraints.—(1) For *simplicity only,* we limit ourselves to fermion representations which contain only $\underline{1}_c$, $\underline{3}_c$, and $\underline{3}_c{}^*$.[7]

(2) The (reducible) fermion representation $\{F\}$ should be complex, flavor-chiral, and free of Adler–Bell–Jackiw anomalies.[8,9]

(3) The fundamental representation of G must contain at least one $\underline{3}_c$ or $\underline{3}_c{}^*$ [exactly one if G is SU(N), as a consequence of (1)].

(4) The sum of the charges in the fundamental representation should be zero.

(5) In order to allow $\Delta Q = \pm 1$ weak currents, there must exist two $\underline{1}_c$ fields in the fundamental representation which differ in charge by ± 1.

(6) We limit the charges of the fields in a fundamental representation to be 0, $\pm\frac{1}{3}$, $\pm\frac{2}{3}$, ± 1. (This goes beyond simplicity: Higher charges lead to dynamical problems with $\sin^2\theta_W$ and/or the unification mass.)

(7) In order to achieve pure vector couplings of the unbroken $SU_c(3) \otimes U_{em}(1)$ gauge group,[10] the Weyl fields $(\underline{3}_c, q)$ and $(\underline{3}_c{}^*, q)$ (q = electric charge) must occur as pairs in the fermion representation $\{F\}$, as must the Weyl fields $(\underline{1}_c, q)$ and $(\underline{1}_c, -q)$ for $q \neq 0$.

(8) The e and ν type leptons [$(\underline{1}_c, -1)$ and $(\underline{1}_c, 0)$ Weyl fields] must occur as pairs in single irreducible representations, so that the $W^\pm$ gauge fields connect $e\nu_e$, $\mu\nu_\mu$, etc.

(9) Finally, to accomodate the observations of Ref. 1, $\{F\}$ must contain $\underline{1}_c$ fermions with fractional charge. Then also, $\{F\}$ will necessarily contain $\underline{3}_c$ quarks with charges not on the normal sequence $n - \frac{1}{3}$ (n integer).

Dynamical (D) Constraints.—(1) The symmetry must break to $SU_c(3) \otimes U_{em}(1)$ at mass scale $\mu \sim 100$ GeV.

(2) The spectrum of particles and breaking pattern must be such that grand unification can be achieved at $M \gtrsim 10^{14}$ GeV (to inhibit proton decay) with values of $\alpha_s(100\ \text{GeV})$ in the range 0.1 to 0.3 and $\alpha(M) \lesssim 1$.

(3) The value of the weak mixing parameter[11] is $\sin^2\theta_W(\mu) \sim 0.20$.

The R constraints (1)–(2) can be satisfied by choosing anomaly-free reducible representations of SU(N) constructed from irreducible representations corresponding to single-column Young tableaux[9] (i.e., tensors $\psi_\alpha, \psi_{\alpha\beta}, \ldots$ with all indices antisymmetrized). The R constraints (3)–(5), together with the requirement of fractionally charged color singlets, rule out SU(6), while for SU(7), the only representation that satisfies these requirements together with the constraint (7) of vector couplings in $SU_c(3) \otimes U_{em}(1)$ is

$$\{F\} = \underline{7} + \underline{21}^* + \underline{35} \sim \psi_\alpha \oplus \psi^{\alpha\beta} \oplus \psi_{\alpha\beta\gamma}$$

obtained from the 64-dimensional spinor representation of O(14).

If the elements ψ_1, ψ_2, and ψ_3 of the first $\underline{7}$ form a $\underline{3}_c$ with charge Q, then it turns out that only $Q = \frac{2}{3}, -\frac{1}{3}$, 0 do not immediately violate some combination of the R constraints. Moreover, one of the remaining $\underline{1}_c$ must have charge 0 in order to satisfy the vector coupling constraint. Thus we are led to the possible charge assignments

$$Q = \mathrm{diag}(-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 1, 0, \tfrac{1}{3}, -\tfrac{1}{3}), \tag{1}$$

$$Q = \mathrm{diag}(-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 1, 0, \tfrac{2}{3}, -\tfrac{2}{3}), \tag{2}$$

$$Q = \mathrm{diag}(0, 0, 0, 1, 0, -\tfrac{2}{3}, -\tfrac{1}{3}), \tag{3}$$

or

$$Q = \mathrm{diag}(\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{2}{3}, -1, 0, -\tfrac{2}{3}, -\tfrac{1}{3}). \tag{4}$$

The last two assignments contain only one generation of ordinary quarks and leptons. Thus three copies of $\{F\}$ are required to accommodate the known quark and lepton spectrum. Furthermore, any Higgs scalar which contributes to the fermion mass matrix necessarily breaks the weak SU(2) group. Here *all* fermions must be light with respect to M, with the result that their large number (24 Dirac triplets of quarks alone) will violate asymptotic freedom and lead to an unacceptably large $\alpha(M)$.

Assignments (1) and (2) are similar (except for the predicted value of $\sin^2\theta_W$); we give here the $SU_c(3) \otimes U_{em}(1)$ decomposition of assignment (1): Nonexotic Dirac fields,

$$2[(\underline{3}_c, \tfrac{2}{3}) + (\underline{3}_c, -\tfrac{1}{3}) + (\underline{1}_c, -1)];$$

exotic Dirac fields,

$$(\underline{3}_c, 1) + (\underline{3}_c, \tfrac{1}{3}) + (\underline{3}_c, 0) + (\underline{3}_c, -\tfrac{2}{3}) + (\underline{1}_c, \tfrac{4}{3}) + (\underline{1}_c, \tfrac{2}{3}) + 2(\underline{1}_c, \tfrac{1}{3});$$

Weyl fields,

$$3(\underline{1}_c, 0).$$

With two copies of this representation, we can incorporate four generations of (u, d) quark doublets and charged leptons, and six Weyl neutrinos, together with assorted exotic fermions. The six Weyl neutrinos, in contrast to the four of a four-generation SU(5)-like model, allow the possibility of some massive and some massless neutrinos, or a mismatch between the number of singly charged leptons and neutrinos.

The charges observed in Ref. 1 can be either the $(\underline{1}_c, \tfrac{1}{3})$ leptons, or hadronic color-singlet composites of various normal and exotic $\underline{3}_c$ fermions. At least one $\underline{3}_c$ or one $\underline{1}_c$ exotic will be absolutely stable.

With both charge assignments (1) and (2) there is a problem with the symmetry breaking down to $SU_c(3) \otimes U_{em}(1)$. The difficulty is that both the SU(5) charge operator Q_5 (acting on the first five components of the $\underline{7}$), and the exotic charge operator Q_e (corresponding to the σ_3-like generator acting on the 6, 7 components) are conserved separately by any symmetry-breaking Higgs scalar which can be represented as a composite of two fermions (so long as the total charge is conserved).

The simplest Higgs vacuum expectation value which violates the separate conservation of normal and exotic charges for assignment (1) is the component $H_{77}^{[1236]}$ from the 756-dimensional representation[12] corresponding to Dynkin weight (201 000). [For assignment (2) an even more elaborate structure is required, so we concentrate on (1).]

These vacuum expectation values will break SU(7) directly to $SU_c(3) \otimes SU(2) \otimes U(1)$—the breaking must occur at a mass scale $\sim M$, since Higgs scalars of such dimensionality will wreak total havoc with the renormalization-group (RG) analysis if they are allowed to appear in the RG β functions below M. In the concluding remarks we will speculate on why complex effective scalars like $\underline{756}$ may be expected to play a role in the symmetry breaking at 10^{14}–10^{15} GeV.

At any rate, with the unbroken symmetry at M determined to be $SU_c(3) \otimes SU(2) \otimes U(1)$, it is straightforward to apply the renormalization group analysis of Georgi, Quinn, and Weinberg[13] to the GUT at hand. For assignment (1), the charge operator is easily identified as

$$Q = T_3 + (19/9)^{1/2}\, T_0,$$

where T_3 and T_0 are the conventionally normalized generators from $SU(2) \otimes U(1)$; hence, at M,

$$\sin^2\theta_W(M) = \tfrac{9}{28}.$$

[Note that, equivalently, $\sin^2\theta_W(M) = 1/(2\,\mathrm{Tr}Q^2)$, where $\mathrm{Tr}Q^2$ is taken in the $\underline{7}$.]

The RG equations can then be combined to give

$$\tfrac{9}{28} - \sin^2\theta_W(\mu) = \frac{\alpha_{em}(\mu)}{2\pi}\,\tfrac{19}{28}(b_1 - b_2)\ln\frac{M}{\mu}, \tag{5}$$

$$\tfrac{9}{28} - \frac{\alpha_{em}(\mu)}{\alpha_s(\mu)} = \frac{\alpha_{em}(\mu)}{2\pi}\left[\tfrac{19}{28}b_1 + \tfrac{9}{28}b_2 - b_3\right]\ln\frac{M}{\mu}, \tag{6}$$

$$\tfrac{9}{28} - \frac{\alpha_{em}(\mu)}{\alpha(M)} = \frac{\alpha_{em}(\mu)}{2\pi}\left[\tfrac{19}{28}b_1 + \tfrac{9}{28}b_2\right]\ln\frac{M}{\mu}, \tag{7}$$

where the b_i are defined by the one-loop β-function equation

$$16\pi^2\, dg_i/d(\ln\mu) = b_i\, g_i^{\,3}. \tag{8}$$

As discussed above, all fermion masses must be taken light ($\lesssim m_W$), and hence in the usual way[13] do not contribute to the right-hand side of Eqs. (5) and (6). Ignoring for the moment the contributions of scalar fields, we find for $M/\mu \approx 10^{12}$, $\alpha_{\rm em}^{-1}(\mu) = 128$ (Ref. 14), that

$$\sin^2\theta_W(\mu) \cong 0.15\,,\quad \alpha_s(\mu) \cong 0.32\,,\quad \alpha(M) \cong 0.22\,. \tag{9}$$

This value of $\sin^2\theta_W$ is unacceptably low. It can be improved by including contributions of scalar fields. For example, if the $\mathrm{SU}_c(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$ symmetry is broken by a $\underline{7}$ (H_α) and $\underline{140}$ ($H_{[\alpha\beta]}{}^{\gamma}$) of scalars, and if the color singlets in these representations are light while the nonsinglets are superheavy, then the contribution of the light scalars to the b_i is given by

$$(b_1)_s = \tfrac{159}{114}\,,\quad (b_2)_s = \tfrac{5}{2},\quad (b_3)_s = 0\,. \tag{10}$$

This leads to a greatly improved value,

$$\sin^2\theta_W(\mu) \sim 0.19\,, \tag{11}$$

for $M/\mu \sim 10^{12}$ as above.

However, a new problem is created: $\alpha_{\rm em}(\mu)/\alpha_s(\mu)$ and $\alpha_{\rm em}(\mu)/\alpha(M)$ are now poorly determined by Eqs. (6) and (7), since the right-hand sides are now $\sim\frac{9}{28}$. This does not signal a failure of perturbation theory; it is simply that higher-order (two-loop) terms become decisive in determining the small quantities $\alpha_{\rm em}/\alpha_s \approx \alpha_{\rm em}/\alpha(M)$. We have not yet calculated these terms.

We conclude with a summary and some remarks.

(1) We have explored a minimal extension of the "standard" minimal GUT SU(5) in order to allow for the possible existence of color-singlet fractional charges. The candidate group is SU(7), with an anomaly-free fermion representation $2(\underline{7}+\underline{21}^*+\underline{35})$ which contains four ordinary generations of quarks and leptons, along with two extra neutral Weyl fields, and a wide variety of exotic quarks and leptons.

(2) The structure of the fermion representation requires a Higgs scalar multiplet of dimension 756 (or larger) to break SU(7) directly to $\mathrm{SU}_c(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$ at $M \gtrsim 10^{14}$ GeV. The reader may be uneasy about the need for such a representation, and we wish to add a speculative comment about this possibility.

It has been traditional to minimize one's misgivings about the Higgs sector of gauge theories by seeking the smallest possible representations which will accomplish the required symmetry breaking. This makes sense in the low-energy domain, because the general model of dynamical symmetry breaking[15] has hyperquarks condensing into low-lying scalar representations under the action of Yang-Mills forces.

However, the symmetry-breaking condensates at M are composed of hyperquarks which may have masses large enough to bring gravitational forces into the picture. In this case, there is a real conflict between Yang-Mills forces, which tend to favor low-lying representations in the condensate, and the gravitational force, which is universally attractive, and favors massive conglomerates, i.e., multihyperquark condensates. The balance between the Yang-Mills and gravitational sectors might indeed favor medium-sized representations (such as $\underline{756}$) to break the symmetry at M.

(3) The renormalization group analysis (including a low-lying $\underline{7}\oplus\underline{140}$ of scalars) gives $\sin^2\theta_W \approx 0.19$ for $M/\mu \approx 10^2$. Evaluation of $\alpha(M)$ and $\alpha_s(\mu)$ will require higher-order calculations including the effects of the scalars. The $\underline{7}\oplus\underline{140}$ are required to break the $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ to $\mathrm{U}_{\rm em}(1)$, and to give fermions masses, with the exotic fermions heavier than the normal fermions (we should note that the hierarchy of fermion masses is not required *a priori*, but can be generated only by adjusting parameters in the Lagrangian).

(4) Besides allowing the presence of fractionally charged color singlets to accomodate the observations of Ref. 1, the model presented here (and others like it) predicts an inundation of new physics in the subteraelectronvolt range. As a striking manifestation of this, the parameter R which is a measure of the e^+-e^- annihilation cross section is predicted in our model to reach a value of 16 (from quarks alone) or $20\frac{8}{9}$ (including exotic leptons) for energies above the fermion thresholds.

(5) Our model should certainly be extended; it is clearly of interest to look for a GUT which will also provide flavor unification for at least three generations of ordinary fermions. The vector coupling conditions for $\mathrm{SU}_c(3)\otimes\mathrm{U}_{\rm em}(1)$ are most easily satisfied for representations of $\mathrm{SU}(2n+1)$ derived from the spinor representations of $\mathrm{O}(4n+2)$, and hence the next natural candidate is SU(9). Work is in progress on this and other possibilities.

(6) It is known that heavy charged leptons drop out of equilibrium in the early universe, at a high abundance (10^{-2}–10^{-3}) relative to nucleons.[16] If the stable fractionally charged objects in Ref.

1 are leptons, then such an abundance is much too large. In a forthcoming publication, one of us (H. G.) will show that annihilation during presupernova states of first generation star formation may reduce this abundance to values consistent with the results of Ref. 1.

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Note added.—After this work was submitted for publication, we received a preprint by L.-F. Li and F. Wilczek which also examines an SU(7) model to accomodate fractional charges. These authors seem not to have included the effects of the Higgs scalars in the continuation of $\sin^2\theta_W$ from high to low energies.

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