Determination of the S-Wave $I=0$ π-π Phase Shifts from Threshold to 0.96 GeV

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(Received 15 June 1981)

The $I=0$ S-wave π-π phase shifts have been determined in the π-π mass range from threshold to 0.96 GeV by use of data from the reaction $\pi^+p \to p\pi^+\pi^0\pi^0$ at 8 GeV/c. The resulting phase shifts confirm the phase shifts determined from previous $\pi^+\pi^- \to \pi^+\pi^-$ data above 0.7 GeV, but are significantly below the accepted phase shifts below 0.6 GeV where our phase shifts are now consistent with current-algebra predictions.

PACS numbers: 13.75.Lb, 13.75.Gx, 14.40.-n

The π-π phase shifts at low mass have been the subject of many analyses[1-11] over the last decade. High-statistics experiments on the reactions $\pi^-p \to n\pi^+\pi^-$ (Refs. 1–4) and $\pi^+p \to \Delta^{++}\pi^+\pi^-$ (Ref. 5) have led to a consensus[12] that the $I=0$ S-wave phase shift rises monotonically below 0.6 GeV, and that the "up-down" ambiguity above 0.7 GeV is resolved in favor of the "down" solution. The presence of P waves in the $\pi^+\pi^-$ final state somewhat masks the S wave and leads to phase-shift ambiguities. Accurate $\pi^0\pi^0$ data are ideal for measuring the S-wave phase shifts since P waves do not contribute to this channel, but most previous $\pi^0\pi^0$ experiments[6-10] have not had sufficient statistical accuracy to obtain on-shell amplitudes. In this work, where the data are extrapolated to the pole, we show that the previously accepted S-wave phase shifts above 0.7 GeV are correct, but the phase shifts below 0.6 GeV are actually significantly smaller than previously thought.

The data for this experiment were obtained at the Argonne National Laboratory with the 1.5-m streamer chamber in combination with a 68-element lead-glass hodoscope. The outgoing charged tracks were detected in the streamer chamber, and the gamma rays from π^0 decay were detected in the lead-glass hodoscope.[13] With an 8.0-GeV/c beam, a sample of 27 000 events consistent with[14] the reaction $\pi^+p \to p\pi^+\pi^0\pi^0$ was obtained. Some results on this data sample have been reported previously.[15] The events were weighted event by event with a Monte Carlo program.[14] Events in the sample have an average detection probability of 0.48 which does not vary strongly with $\pi^0\pi^0$ mass.

In order to study π-π scattering cleanly, events are selected for which the outgoing π^+ and proton are decay products of the $\Delta^{++}(1236)$ and for which the four-momentum transfer from the target proton to the Δ^{++}, $|t|$, is small. Thus we require that the π^+p mass be less than 1.36 GeV, and that $t' = |t - t_{\min}|$ be less than 0.2 GeV.[2] Following these cuts, the data sample consists of 15 000 events.

A π-π amplitude analysis was made with the assumption of dominance of one-pion exchange. This analysis required good acceptance in the $\pi^0\pi^0$ decay angles. In Fig. 1 are shown the weighted and unweighted distributions in $\cos\theta_J$ and Φ_J, defined in the t channel (the Gottfried-Jackson frame) for various $\pi^0\pi^0$ masses. Note that there are no zeros in our acceptance in either variable or, in fact, anywhere in the $\cos\theta_J$ and Φ_J plane (not shown). Also, the weights do not vary strongly with decay angle. To extract the on-shell $\pi^+\pi^- \to \pi^0\pi^0$ amplitude from the data, we begin with the expression

$$|T|^2 = C\,\frac{q}{M_{\pi\pi}^{\;2}}(t+\mu^2)^2\,\frac{d^3N}{dt\,dM_{\pi\pi}\,dM_\Delta}\,\frac{dM_\Delta}{M_\Delta^{\,2}Q\sigma(\pi^+p)}, \tag{1}$$

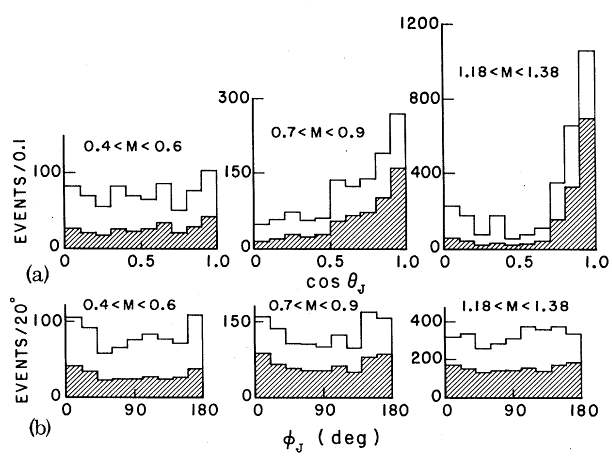

FIG. 1. (a) Unweighted (cross-hatched) and weighted distributions of $\cos\theta_J$ where θ_J is the scattering angle in the t channel for π-π scattering for the π-π mass intervals indicated; (b) Unweighted (cross-hatched) and weighted distributions of the azimuthal angle Φ_J for the mass intervals indicated.

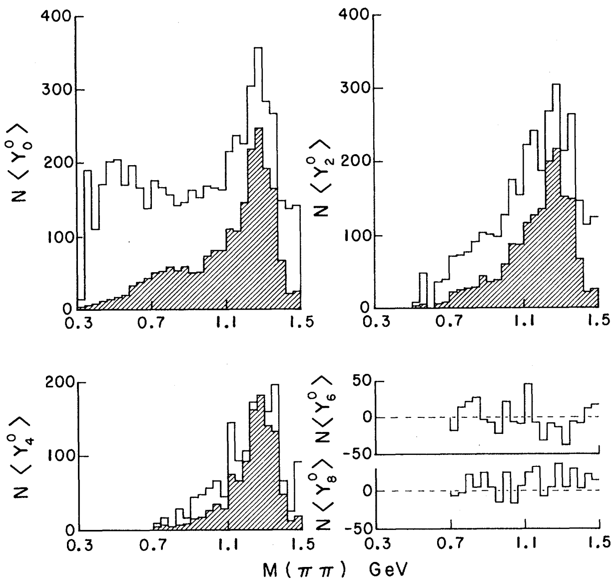

FIG. 2. Distributions of the unnormalized moments $N\langle Y_L{}^0\rangle$ (events/0.04 GeV) as a function of $\pi^0\pi^0$ mass. Cross-hatched distributions are the extrapolated moments.

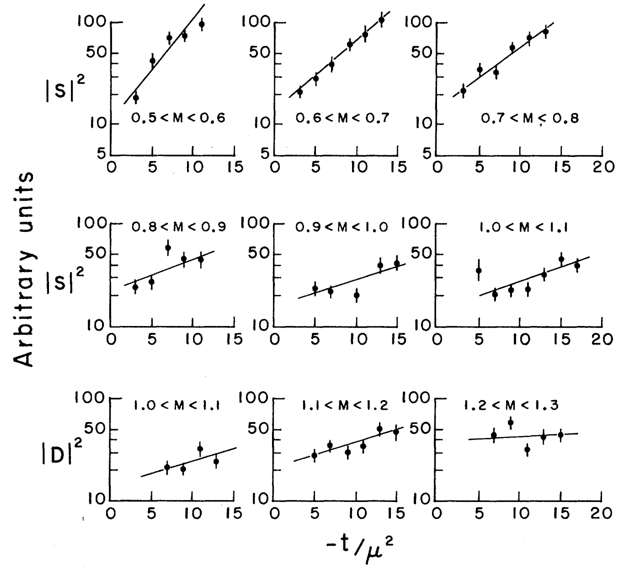

FIG. 3. $|S|^2$ and $|D|^2$ as a function of t for the various $\pi^0\pi^0$ mass intervals indicated. The straight lines are fits to the data used to obtain the extrapolated amplitudes.

which describes the π-π amplitude at the pole. Here q and Q are the center-of-mass momenta in the $\pi^0\pi^0$ and the π^+p rest systems, respectively, $\sigma(\pi^+p)$ is the elastic π^+p cross section at the energy M_Δ, and C is a constant for a fixed beam energy. In the analysis the right-hand side of Eq. (1) was evaluated event by event, and these values were summed over $\pi\pi$ mass bins for fixed t intervals. With these $|T|^2$ values, the moments of the angular distribution for each mass bin were determined. These moments are shown in Fig. 2. For low mass, it can be seen that $\langle Y_6{}^0\rangle$ and $\langle Y_8{}^0\rangle$ are consistent with zero, and can be neglected. (The moments $\langle Y_L{}^m\rangle$ with $m\neq 0$ are also negligible.) We thus consider only S and D waves for the low-mass region.

The S- and D-wave amplitudes, calculated from these moments, would be the amplitudes for $\pi\pi$ scattering if there were no absorption, background, or off-shell effects. In order to study these effects, the t dependence of $|D|^2=\frac{7}{6}\langle Y_4{}^0\rangle$ and $|S|^2=\langle Y_0{}^0\rangle-|D|^2$ was determined by dividing the data into π-π mass intervals and evaluating $|S|^2$ and $|D|^2$ in various t intervals for each mass interval. Figure 3 shows the dependence of $|S|^2$ and $|D|^2$ on t for various mass intervals. This t dependence could be parametrized[16] by means of the expression

$$|L|^2=|L_0|^2\exp[\alpha_L(t+\mu^2)]\equiv G^2(\alpha_L,t)|L_0|^2.$$

The solid lines in Fig. 3 show the fits to the data. (Note that the empirically determined parameter α_L depends on both $\pi\pi$ mass and L.) The on-shell amplitudes were then calculated with use of the expressions

$$|D|^2=\tfrac{7}{6}\sum Y_4{}^0(\theta_i)G^2(\alpha_{D_i},t_i),$$

$$|S|^2=\sum[Y_0{}^0-\tfrac{7}{6}Y_4{}^0(\theta_i)]G^2(\alpha_{S_i},t_i),$$

and

$$2S^*D=\sum[Y_2{}^0(\theta_i)-(\tfrac{1}{3}\sqrt{5})Y_4{}^0(\theta_i)]G(\alpha_{D_i},t_i)G(\alpha_{S_i},t_i).$$

The effect of this weighting can be seen in the cross-hatched region of Fig. 2 which shows the extrapolated moments.

The on-shell amplitudes were normalized with use of the well-known value $|D|^2=5[(1+\eta_D)/2]^2$ with $\eta_D=0.67$ in the f^0 mass region. The resulting values of $|S|^2$ from threshold to 0.96 GeV are plotted in Fig. 4(a). (An amplitude analysis of the inelastic region above 0.96 GeV is in progress.) The S-wave intensity, $|S|^2=|a_0{}^0-a_0{}^2|^2$, is related to the phase shifts by $a_0{}^I=\exp(i\,\delta_0{}^I)\sin\delta_0{}^I$. For the $I=2$ S-wave phase shift we use $\delta_0{}^2=-q/(1.1+0.88407q^2)$, with q in GeV/c and $\delta_0{}^2$ in radians, which is a

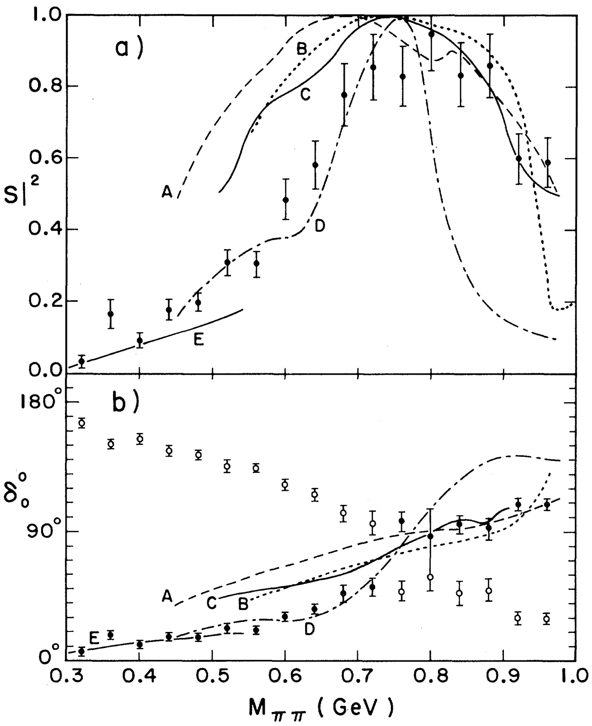

FIG. 4. (a) Extrapolated S-wave intensity as a function of $\pi^0\pi^0$ mass. Shown as smooth curves are predictions based on $\pi^+\pi^- \to \pi^+\pi^-$ scattering (A and D from Ref. 2, B from Ref. 5, and C from Ref. 4) and on current algebra and partial conservation of axial-vector current (curve E). (b) $I = 0$, S-wave phase shifts determined in our experiment. The two ambiguous solutions are shown as open and closed circles. The curves are as in (a).

good parametrization of the available data.[17] The uncertainty in δ_0^2 is small compared to the statistical errors in our data and does not affect our results.

Also shown in Fig. 4(a) as curves A–D are predictions based on analyses of $\pi^+\pi^- \to \pi^+\pi^-$ data. Curves B and C are most representative of the currently accepted[12] S-wave amplitude. Our data are in clear agreement with A, B, and C above 0.84 GeV. They are, however, inconsistent with these predictions below 0.68 GeV. Here our data clearly require a solution of the type represented by curve D. This result is in disagreement with conclusions[2] drawn from previous[7] unextrapolated $\pi^0\pi^0$ data which appeared to favor the currently accepted solution in this entire mass region. It is also in disagreement with the extrapolated $n\pi^0\pi^0$ data[11] at 2.01 GeV. (The extrapolation in that final state is considerably more difficult than in the $\Delta^{++}\pi^0\pi^0$ final state because of the vanishing of the physical amplitude at $t = 0$.)

In Fig. 4(b) are shown the $I = 0$ S-wave phase shifts as determined directly from our data with use of the $I = 2$ phase-shift parametrization described above. There is a discrete ambiguity in that two values of δ_0^0 lead to the same value of $|S|^2$ when an $I = 0$ amplitude is combined with an $I = 2$ amplitude. As shown, the only type of solution for $\pi^+\pi^- \to \pi^0\pi^0$ consistent with the $\pi^+\pi^- \to \pi^+\pi^-$ data is of the type represented by curve D below 0.68 GeV and by the standard solution above 0.84 GeV. This leads to a rather rapid phase variation at approximately 0.75 GeV and a phase shift which goes through 90° at about 0.80 GeV.

It is immediately evident that our data near threshold confirm the predictions[18-19] based on partial conservation of axial-vector current and current algebra. These predictions, which had been previously thought not to be valid,[11] are shown by curve E in Fig. 4.

With regard to the question[20] of a "narrow" $\epsilon(700)$, we find that the phase behavior is not well represented by a Breit-Wigner form: The phase variation is rapid below resonance ("narrow" behavior) and rather slow above resonance ("broad" behavior). Thus, if existence of a narrow $\epsilon(700)$ is assumed, one must postulate the existence of a background which becomes important above resonance.

Finally, we comment on the phase difference $\delta_0^2 - \delta_0^0$ at the mass of the neutral K meson. This difference is related to the CP-nonconservation parameters in K decay[21] and is predicted to be $-40.6° \pm 3°$ with use of current values for these parameters. We obtain $\delta_0^2 - \delta_0^0 = -29.2° \pm 3°$ from our data. The difference between our value and the predicted value is probably within systematic uncertainties associated with the phase-shift measurements and with electromagnetic effects in K decay. Our solution is in significantly better agreement than the result based on the standard solution, $\delta_0^2 - \delta_0^0 = -58° \pm 3°$.

It is a pleasure to acknowledge the important contributions made to the success of our experiment by R. Erichsen, C. Rey, M. Lawson, the Notre Dame scanning staff, and the Argonne streamer chamber technical staff. This work has been supported by the National Science Foundation (Grant No. PHY-8007627) and by the U. S. Department of Energy, Division of Basic Energy Sciences.

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