Quantum Chromodynamic Corrections to the Gluonic Width of the Υ Meson

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The corrections to the gluonic width of Υ-like mesons in quantum chromodynamics (QCD) through first order in α_s are calculated, with the result $\Gamma_g = \Gamma_g{}^0[1+(3.8\pm 0.5)\times\alpha_s(M_\Upsilon)/\pi]$, using the $\overline{\mathrm{MS}}$ scheme definition of α_s, renormalized at M_Υ. With this correction, the measured value of the leptonic branching ratio of the Υ can be used to determine α_s, or, equivalently, the QCD scale parameter $\Lambda_{\overline{\mathrm{MS}}}$. The result is $\Lambda_{\overline{\mathrm{MS}}}=100$ MeV, with experimental errors of $^{+34}_{-25}$ MeV and comparable theoretical uncertainties. Analysis of the ψ and ψ' data is consistent with this value.

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Recent measurements of the leptonic branching ratio ($B_{\mu\mu}$) of the Υ meson provide one of the best determinations of $\alpha_s(M)$, the running coupling constant of quantum chromodynamics (QCD). The ratio of the gluonic and leptonic widths of the Υ is predicted to leading order in α_s to be

$$\frac{\Gamma_g}{\Gamma_{\mu\mu}} = \frac{1-(3+R+\Gamma_{\gamma+\mathrm{glue}}/\Gamma_{\mu\mu})B_{\mu\mu}}{B_{\mu\mu}} = \frac{10(\pi^2-9)}{81\pi e_b{}^2}\frac{\alpha_s{}^3(M)}{\alpha_{\mathrm{em}}{}^2}, \tag{1}$$

where $M\approx M_\Upsilon$, $e_b=-\frac{1}{3}$ is the charge of the b quark,

$$R=\sum_{\mathrm{quark}} e_q{}^2\left[1+\frac{\alpha_s(M)}{\pi}+O\left(\frac{\alpha_s{}^2}{\pi^2}\right)\right]\approx 3.5$$

in QCD with four light quarks, and

$$\frac{\Gamma_{\gamma+\mathrm{glue}}}{\Gamma_{\mu\mu}}=\frac{8(\pi^2-9)}{9\pi}\frac{\alpha_s{}^2(M)}{\alpha_{\mathrm{em}}}\left[1+O\left(\frac{\alpha_s}{\pi}\right)\right]\approx 0.8$$

($\Gamma_{\gamma+\mathrm{glue}}$ is the width into a direct photon plus gluons.) The ratio in Eq. (1) is very sensitive to the value of α_s. Current results for the branching ratio ($B_{\mu\mu}=3.3\pm 0.5\%$)[1] when combined with Eq. (1) give an effective coupling constant

$$\alpha_s(M)=0.158^{+0.012}_{-0.010}. \tag{2}$$

That many processes give α_s in the range 0.1 to 0.3 is in itself a significant success of QCD. However, given the large size of α_s, the order-α_s corrections to QCD predictions are necessary before a test of the theory at the 10%–20% level is possible. The corrections have long been known for the leptonic width.[2] We have calculated the $O(\alpha_s)$ corrections to the gluonic width, thereby completing the analysis of $B_{\mu\mu}$ [Eq. (1)] through next order in α_s.[3]

The leading-order analyses of the gluonic and leptonic decay rates of a nonrelativistic heavy-quark meson like the Υ are essentially identical to analogous calculations for positronium.[4] Each rate is proportional to the square of the nonrelativistic wave function at the origin:

$$\Gamma_g{}^0=\frac{160}{81}(\pi^2-9)\alpha_s{}^3(M_\Upsilon)\frac{|\psi_{\mathrm{NR}}(0)|^2}{M_\Upsilon{}^2},\qquad \Gamma_{\mu\mu}{}^0=16\pi e_q{}^2\alpha_{\mathrm{em}}{}^2\frac{|\psi_{\mathrm{NR}}(0)|^2}{M_\Upsilon{}^2}. \tag{3}$$

To this order, the widths depend on the long-distance structure of the meson only through the overall factor of $|\psi_{\mathrm{NR}}(0)|^2$. This factorization of (nonperturbative) low-energy effects occurs because the annihilation of the heavy quarks is only possible at distances of order $1/M_b$—i.e., at distances much smaller than the meson radius. Corrections to these leading-order rates come from two regions of wave function and loop momenta:

(a) Nonrelativistic p ($\sim\langle p\rangle_\Upsilon$). These include $O(v^2/c^2)$ corrections due to relativistic kinematics, and to perturbative terms in the spin-orbit, spin-spin, and similar interactions. Contributions are also expected from other Fock states in the Υ—e.g., $|b\bar{b}g\rangle$, $|b\bar{b}gg\rangle$, etc. However, both perturbative and nonperturbative couplings between these and the $|b\bar{b}\rangle$ state are almost certainly of order v^2/c^2 or smaller.

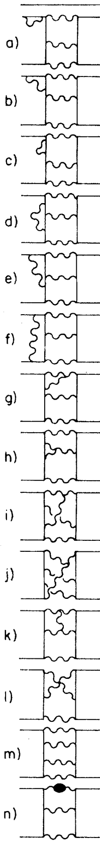
(b) Relativistic p ($\sim M_b$). The only interactions thought to contribute at such short distances are perturbative. The resulting corrections have an expansion in powers of α_s.

We thus assume that the wave function $\psi_{\mathrm{NR}}(0)$ contains the effects of all relevant long-distance nonperturbative interactions, and of the instantaneous Coulomb exchange, which must therefore not be included in the one-loop correction. [This last assumption is unnecessary when computing the ratios of rates as in Eq. (1).] We ignore nonperturbative effects in the short-distance annihilation amplitude, although they must contribute at some level; for example, a linear potential generates relativistic corrections in region (b) of $O(v^3/c^3)$ or less. If more singular nonperturbative interactions existed, they would cause trouble not only here, but also in all other short-distance QCD calculations.

Potential models of quarkonia give $\langle v^2/c^2\rangle_\Upsilon \sim 0.08$.[5] We therefore first ignore corrections from region (a) and concentrate on the $O(\alpha_s)$ relativistic corrections. Clearly, these involve only short distances, as did the leading order, and so we expect the corrected rate to again factorize into $|\psi_{\rm NR}(0)|^2$ multiplying the perturbative on-shell amplitude describing $b\bar{b}\to 3g, 4g, ggq\bar{q}$.

The necessary classes of Feynman diagrams and our results for each class are summarized in Fig. 1. As explained above, the contribution due to Coulomb exchange must be dropped from the result of class f, to avoid double counting. We performed the γ-matrix algebra using the computer program REDUCE.[6] We evaluated the integrals numerically using the adaptive multidimensional integration program VEGAS[7]; the uncertainties listed in Fig. 1 are two-standard-deviation errors as estimated by VEGAS. Since the calculation involves on-shell amplitudes, the result is explicitly gauge invariant; the Feynman gauge was used. Dimensional regularization was employed to regulate the ultraviolet divergences.

The loop integrals were done by closing the p^0 contour at infinity, taking the residues of the enclosed poles, and performing the remaining three-momentum integrals directly. The total decay rate of a pair of massive quarks is known to be infrared finite by the Kinoshita-Lee-Nauenberg theorem. However, the graphs with three gluon vertices have infrared and collinear singularities in the separate three- and four-gluon cuts. These singularities cancel point by point in momentum space when the various cuts of a given forward scattering amplitude are summed, after performing the p^0 integration for the three-gluon cuts as described above. In most classes of graphs, the infrared divergences in the fermion propagators cancel by adding together permutations of the gluons attached to one of the quark lines. The only infrared regularization required in the entire calculation is for classes a and f, which were regulated by giving the gluons a small fictitious mass λ.



Sample Graph	Number of Graphs in Class	Coefficient of α_s/π: Finite Piece	Ultraviolet Divergence	Infrared Divergence
a)	24	-3.591	$-\frac{2}{3}P$	$-4/3 \ln \lambda^2/m_b^2$
b)	24	-0.726 ± .002	$-\frac{1}{6}P$	
c)	24	-0.802 ± .002	$-\frac{4}{3}P$	
d)	12	-0.143 ± .001	$-\frac{1}{12}P$	
e)	24	.594 ± .002		
f)	12	-10.424 ± .022		$4/3 \ln \lambda^2/m_b^2$
g)	24	3.73 ± .26	$\frac{9}{2}P$	
h)	12	9.76 ± .22	$\frac{9}{4}P$	
i)	24	0.0		
j)	12	-9.95 ± .30		
k)	18	10.59 ± .26		
l)	9	-3.02 ± .04		
m)	24	-0.19 ± .04		
n)	72	7.96 ± .02	$\frac{7}{4}P$	
Totals	315	3.79 ± .53	$\frac{25}{4}P$	0

FIG. 1. Classes of diagrams whose imaginary parts contribute to the $O(\alpha_s/\pi)$ correction to the gluonic width of the Υ. The ultraviolet factor P is equal to $\epsilon^{-1}+\ln 4\pi-\gamma_E+\ln\mu^2/M_\Upsilon{}^2$, where $\epsilon=2-n/2$, n being the number of space-time dimensions and $\gamma_E=0.577\ldots$. In the $\overline{\rm MS}$ scheme with the renormalization scale μ set equal to M_Υ, P is absorbed into the effective coupling constant $\alpha_s(M_\Upsilon)$. The numerical uncertainties quoted are two-standard-deviation errors as estimated by VEGAS. The coefficient for class n is for $n_f=4$ light-quark flavors.

Adopting the $\overline{\rm MS}$ renormalization scheme,[8] the gluonic decay rate of the Υ is

$$\Gamma_g=\Gamma_g{}^0\left(1+\frac{\alpha_s(M)}{\pi}\left\{-19.4(5)+\tfrac{3}{2}\beta_0\left[1.161(2)+\ln\left(\frac{2M}{M_\Upsilon}\right)\right]\right\}+O\left(\frac{v^2}{c^2},\frac{\alpha_s{}^2}{\pi^2}\right)\right)_{\overline{\rm MS}}, \tag{4}$$

where $\beta_0=11-\frac{2}{3}n_f$ and n_f is the number of light quarks.[9] Combining this with the corrected leptonic width,

$$\Gamma_{\mu\mu}=\Gamma_{\mu\mu}{}^0(1-\tfrac{16}{3}\alpha_s(M_\Upsilon)/\pi+\cdots),$$

we obtain our final result:

$$\frac{\Gamma_g}{\Gamma_{\mu\mu}}=\frac{10(\pi^2-9)}{81\pi e_b^{\,2}}\frac{\alpha_s^{\,3}(M)}{\alpha_{\rm em}^{\,2}}\left(1+\frac{\alpha_s(M)}{\pi}\left\{-14.0(5)+\tfrac{3}{2}\beta_0\left[1.161(2)+\ln\left(\frac{2M}{M_\Upsilon}\right)\right]\right\}+\cdots\right)_{\overline{\rm MS}}. \tag{5}$$

All dependence on the wave function cancels in the ratio (to this order), leaving an unambiguous prediction of perturbative QCD.

Equations (4) and (5) are independent of the choice of M up to corrections of $O(\alpha_s^{\,2}/\pi^2)$. A particularly convenient choice is $M=0.48(2)M_\Upsilon$ as then the $O(\alpha_s)$ corrections in Eq. (5) vanish[10] —i.e., Eq. (1) is exact [up to corrections of $O(\alpha_s^{\,2}/\pi^2)$] for $M=0.48(2)M_\Upsilon$, and consequently $\alpha_s[0.48(2)M_\Upsilon]_{\overline{\rm MS}}=0.158^{+0.012}_{-0.010}$ [Eq. (2)]. To this order, the $\overline{\rm MS}$ coupling constant is

$$\alpha_s(M)_{\overline{\rm MS}}=\frac{4\pi}{\beta_0\ln(M^2/\Lambda_{\overline{\rm MS}})}-\frac{4\pi\beta_1\ln\ln(M^2/\Lambda_{\overline{\rm MS}}^{\,2})}{\beta_0^{\,3}\ln(M^2/\Lambda_{\overline{\rm MS}}^{\,2})},$$

where $\beta_0=11-\frac{2}{3}n_f$ $(=\frac{25}{3}$ for the $\Upsilon)$ and $\beta_1=102-\frac{38}{3}n_f$ $(=\frac{154}{3}$ for the $\Upsilon)$. This then implies $\Lambda_{\overline{\rm MS}}=100^{+34}_{-25}$ MeV. We have plotted $\beta_{\mu\mu}(\Upsilon)$ vs $\Lambda_{\overline{\rm MS}}$ in Fig. 2, to illustrate its sensitivity to $\Lambda_{\overline{\rm MS}}$. Choosing a different M in Eq. (5) results in slightly different values of $\Lambda_{\overline{\rm MS}}$ when we fit to the data; for example, choosing $M=M_\Upsilon$ implies $\Lambda_{\overline{\rm MS}}=121^{+44}_{-31}$ MeV.

This entire analysis is applicable to ψ decays as well. Taking $B_{\mu\mu}(\psi)=6.9\pm0.9\%$ gives $\Lambda_{\overline{\rm MS}}=53^{+16}_{-12}$ MeV.[11] We may use the small discrepancy between the ψ and Υ results to estimate the corrections. The corrections expected to fall off most slowly at high energies are v^2/c^2 corrections. The potential model of Ref. 5 gives $\langle v^2/c^2\rangle_\psi\sim0.23$ and $\langle v^2/c^2\rangle_\Upsilon\sim0.077$. If we assume that the dominant corrections to Eq. (5) have the form

$$\frac{\Gamma_g}{\Gamma_{\mu\mu}}=\frac{10(\pi^2-9)}{81\pi e_q^{\,2}}\frac{\alpha_s^{\,3}(M)}{\alpha_{\rm em}^{\,2}}\left(1+C\left\langle\frac{v^2}{c^2}\right\rangle\right),$$

where M is chosen as described above and where C is treated as a free parameter, a fit of C and $\Lambda_{\overline{\rm MS}}$ to the ψ and Υ data yields $C=-2.6$ and $\Lambda_{\overline{\rm MS}}=136$ MeV. The reasonable agreement between the ψ and Υ data thus suggests that v^2/c^2 corrections at the Υ may be no larger than the current experimental errors. Better data on $B_{\mu\mu}(\psi)$ and $B_{\mu\mu}(\Upsilon)$ would be very helpful in pinning down the discrepancy.

The data on $B_{\mu\mu}(\psi')$ give an upper limit of $\Lambda_{\overline{\rm MS}}\leqslant149^{+53}_{-66}$ MeV, which is consistent with the values obtained from ψ and Υ decay.

In Ref. 3 we have reported the one-loop correction for decays into a direct photon plus gluons, and predictions of absolute rates for gluonic, leptonic, and direct photon decays. Reasonable agreement with data was obtained for the rates.

Theoretical uncertainties in this analysis, due to $O(v^2/c^2,\alpha^2/\pi^2)$ corrections, are most likely no larger than the current experimental uncertainty in $B_{\mu\mu}(\sim15\%)$. Greater accuracy requires a much more thorough understanding of the $O(v^2/c^2)$ corrections. With a coupling constant as large as 0.15, the corrections of $O(\alpha^2/\pi^2)$ must also eventually be computed. Indeed, high-precision tests of perturbative QCD will only be possible once these further corrections are known for several processes of interest.

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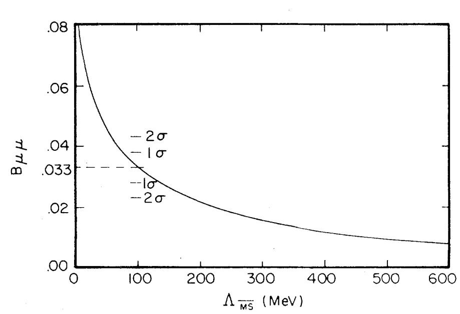

FIG. 2. Our prediction for the leptonic branching ratio $(B_{\mu\mu})$ of the Υ for various values of $\Lambda_{\overline{\rm MS}}$. The current experimental value is $B_{\mu\mu}=3.3\pm0.5\%$.

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[9]Throughout this analysis we have neglected radiative corrections involving b-quark loops. This is appropriate to this order (but not beyond) when expanding in terms of $\alpha_s^{(4)}(M)$, the running coupling constant for a theory with four light quarks. This is the coupling constant used in this paper and in most other QCD analyses involving energies less than 10 GeV. Of course all of the results could be reexpressed in terms of $\alpha_s^{(5)}(M)$, the five-quark coupling constant relevant for energies greater than 10 GeV. In this case, virtual b-quark loops must be included and the $O(\alpha_s^{(5)}/\pi)$ coefficient in Eq. (4) becomes $-18.2(5)+\frac{5}{2}\beta_0[1.161(2)+\ln(2M/M_\Upsilon)]$ with $n_f=5$ for the Υ. In $\overline{\rm MS}$ scheme, the Λ's for $\alpha_s^{(4)}$ and $\alpha_s^{(5)}$ are related by the requirement $\alpha_s^{(4)}(M_b) = \alpha_s^{(5)}(M_b)$, and consequently $\Lambda_{\overline{\rm MS}}^{(5)}=60_{-17}^{+23}$ MeV when $\Lambda_{\overline{\rm MS}}^{(4)}=100_{-25}^{+34}$ MeV.

[10]This method yields predictions essentially identical to the ones produced by the method of G. Grunberg, Phys. Lett. 95B, 70 (1980).

[11]Equation (5) can be used for the ψ (with $n_f=3$) to determine $\Lambda_{\overline{\rm MS}}^{(3)}$, for the three-flavor $\alpha_s^{(3)}$, from $B_{\mu\mu}(\psi)$. Then $\Lambda_{\overline{\rm MS}}^{(4)}=53_{-12}^{+16}$ MeV, for the four-flavor $\alpha_s^{(4)}$, can be derived following the discussion in Ref. 8.