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New Formula for the Cabibbo Angle and Composite Quarks and Leptons

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A composite model for quarks and leptons is proposed. An SU(3)-generation symmetry, which is badly broken not only by the generator F_8 but also by the generator F_3, is introduced. The model leads to a new formula for the Cabibbo angle θ_C represented in terms of lepton masses, which predicts $\tan\theta_C = 0.225$.

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It has, for many years, been a longed-for goal of investigations for the unified description of the weak interactions to give both a successful calculation of nonleptonic weak interactions and a reasonable estimate of the Cabibbo angle. Recently, for the purpose of a unified description of leptons and quarks, many authors[1] have discussed subquark models where quarks and leptons are made of more fundamental constituents (subquarks), but few authors have directed their attention to those two problems[2,3] on the weak interactions.

The present author[4] has very recently proposed a composite quark model which provides a further enhancement of the SU(4) $\underline{20}$ piece by a factor of $2\cos^2\theta_W$ and a further suppression of the SU(4) $\underline{84}$ piece by a factor of $2\sin^2\theta_W$ in the effective nonleptonic weak Hamiltonian as compared with the conventional Glashow-Iliopoulos-Maiani (GIM) model.[5] In that model, the quarks are made of a color-triplet fermion (ν_a, l_a), whose left-handed components form doublets of $SU(2)_w$, and a boson θ^f $(f=1,2,3)$ as follows:

$$\begin{aligned} u_a' &= \nu_a\theta^1, \quad c_a' = \nu_a\theta^2, \quad t_a = \nu_a\theta^3, \\ d_a' &= l_a\theta^1, \quad s_a' = l_a\theta^2, \quad b_a = l_a\theta^3, \end{aligned} \tag{1}$$

where $u' = u\cos\alpha - c\sin\alpha$, $c' = u\sin\alpha + c\cos\alpha$, $d' = d\cos\beta - s\sin\beta$, $s' = d\sin\beta + s\cos\beta$, and the Cabibbo angle θ_C is given by $\theta_C = \alpha - \beta$. The additional enhancement and suppression are caused by a rearrangement of the fermions (ν_a, l_a) at the nonleptonic weak vertex.

In this paper, the rest of the problem (on the origin of the Cabibbo angle) is discussed on the basis of the model proposed previously,[4] but by pushing further the speculation and regarding leptons and the bosons θ^f as composite systems made of some fundamental constituents. We will obtain a new formula for the Cabibbo angle θ_C represented in terms of lepton masses:

$$\tan\theta_C = \frac{\sqrt{3}(m_\mu^{1/2} - m_e^{1/2})}{2m_\tau^{1/2} - m_\mu^{1/2} - m_e^{1/2}}, \tag{2}$$

which predicts $\tan\theta_C = 0.225$.

In this model, there exist 21 subquarks of spin $\frac{1}{2}$:

$$\begin{pmatrix} \nu_a \\ l_a \end{pmatrix}, \quad \begin{pmatrix} \nu_\alpha \\ l_\alpha \end{pmatrix}, \quad h_\alpha{}^i; \tag{3}$$

$$a = 1,2,3, \quad \alpha = 1,2,3, \quad i = 1,2,3,$$

where a, α, and i are SU(3)-color, SU(3)-subcolor, and SU(3)-generation indices, respectively. The left-handed components of (ν_a, l_a) form doublets of the gauge group $SU(2)_w$. We regard $SU(2)_w \otimes U(1)$, $SU(3)_c$, and $SU(3)_{sc}$ as local gauge groups, while we consider $SU(3)_G$ as merely a phenomenological symmetry which is badly broken. The charges of the subquarks (ν_a, l_a), (ν_α, l_α), and $h_\alpha{}^i$ are $(\frac{2}{3}, -\frac{1}{3})$, $(\frac{1}{3}, -\frac{2}{3})$, and $-\frac{1}{6}$, respectively, in units of e.

In this model, a lepton is a subcolor-singlet state which is composed of the fermion (ν_α, l_α) and two-fermion hh. Since the state hh is antisymmetric in subcolor, the state must be symmetric in SU(6) which is composed of $SU(3)_G \otimes SU(2)_{spin}$, so that the ground state (spin-0) of $\{hh\}$ belongs to $\underline{3}^*$ of $SU(3)_G$. Therefore leptons of spin-$\frac{1}{2}$ have three generations:

$$\begin{aligned}&\nu_e=\{\nu h^2h^3\}, \quad \nu_\mu=\{\nu h^3h^1\}, \quad \nu_\tau=\{\nu h^1h^2\},\\ &e^-=\{lh^2h^3\}, \quad \mu^-=\{lh^3h^1\}, \quad \tau^-=\{lh^1h^2\},\end{aligned} \tag{4}$$

where the curly brackets denote subcolor-singlet state and the subcolor index is omitted. We cannot, of course rule out the possibility that there exist leptons of spin-$\frac{3}{2}$ with six generations.

A quark is made of the fermion (ν_a, l_a) and a subcolor-singlet boson $(h\bar{h})$. We assume, for the present, that the physical quarks are restricted to those composed of the diagonal elements of $(h\bar{h})$, that is, $(h^1\bar{h}^1)$, $(h^2\bar{h}^2)$, and $(h^3\bar{h}^3)$. The bosons θ^1, θ^2, and θ^3 in Eq. (1) are regarded as the three $(h\bar{h})$ bosons $(\Phi^{(8)})_{10}$, $(\Phi^{(8)})_{00}$, and $(\Phi^{(1)})_{00}$, respectively, where

$$\begin{aligned}&(\Phi^{(8)})_{10}=(h^1\bar{h}^1-h^2\bar{h}^2)/\sqrt{2},\\ &(\Phi^{(8)})_{00}=(h^1\bar{h}^1+h^2\bar{h}-2h^3\bar{h}^3)/\sqrt{6},\\ &(\Phi^{(1)})_{00}=(h^1\bar{h}^1+h^2\bar{h}^2+h^3\bar{h}^3)/\sqrt{3}.\end{aligned} \tag{5}$$

Mixings of t and b with u, c and d, s, respectively, are neglected, since we consider that the $SU(3)_G$-singlet quarks t and b are very massive relative to the $SU(3)_G$-octet quarks.

In order to understand the generation mixing, we assume that the generation symmetry $SU(3)_G$ is badly broken not only by the generator F_8 but also by the generator F_3, and introduce the spurion

$$S=c_0+c_8F_8+c_3F_3, \tag{6}$$

where $F_i=\frac{1}{2}\lambda_i$, and by which the mass matrix of the fermion h is badly broken. We consider that the c_0 term is dependent on the representation of $SU(3)_G$ because the c_0 term contains the operator $F^2=\sum F_iF_i$, while the coefficients c_8 and c_3 are constant through all the representations of $SU(3)_G$. We define "i spin" and the operators $i_3=F_3$ and $y=(2/\sqrt{3})F_8$ analogously to those of the usual hadronic SU(3) group.

We assume that the mass $(M^{(8)})_{ij}$ of the $SU(3)_G$-octet meson $(\Phi^{(8)})_i{}^j=h^i\bar{h}^j-\frac{1}{3}\delta_{ij}\sum h^k\bar{h}^k$ is given by

$$\begin{aligned}(H^{(8)})_{\rm mass}&=\sum_{i,j}[(M^{(8)})_{ij}]^2(\Phi^{(8)})_i{}^j(\Phi^{(8)})_i{}^j\\ &=M_0{}^2\,{\rm Tr}(\Phi S\Phi S),\end{aligned} \tag{7}$$

that is, the mass $(M^{(8)})_{ii}$ of the "diagonal" meson $(\Phi^{(8)})_i{}^i$ is given by

$$\begin{aligned}[(M^{(8)}{}_{ii}]^2&=M_0{}^2S_i{}^iS_i{}^i\\ &=M_0{}^2\sum_j S_i{}^jS_j{}^i=M_0{}^2(S^2)_i{}^i,\end{aligned} \tag{8}$$

where

$$S^2=\left(c_0{}^2+\frac{c_8{}^2+c_3{}^2}{6}\right)+\left(2c_0c_8-\frac{c_8{}^2-c_3{}^2}{2\sqrt{3}}\right)+F_8 + \left(2c_0c_3+\frac{2c_8c_3}{2\sqrt{3}}\right)F_3, \tag{9}$$

and use has been made of the property that S is a diagonal matrix. The mass matrix (8) leads to the mixing between the $i=1$ meson $(\Phi^{(8)})_{10}$ and the $i=0$ meson $(\Phi^{(8)})_{00}$, and we get the relation for the the mixing angle θ,

$$\tan 2\theta=\frac{2c_8c_3}{c_8{}^2-c_3{}^2}, \quad \text{or } \tan\theta=\frac{c_3}{c_8}, \tag{10}$$

where it has been assumed that $c_0\simeq 0$ for the octet meson $(h\bar{h})$. Here we have neglected the mixing with the $SU(3)_G$-singlet meson $(\Phi^{(1)})_{00}$ since we suppose that the state $(\Phi^{(1)})_{00}$ is very massive.

We suppose that the mass matrix of the $SU(3)_G$-$\underline{3}^*$ meson $(\Phi^{(3^*)})_{ij}=\frac{1}{2}\epsilon_{ijk}\varphi^k$ is given by

$$\begin{aligned}&{\rm Tr}(\bar{\Phi}^{(3^*)}S\Phi^{(3^*)}S)\\ &=\tfrac{1}{2}\sum(S_j{}^kS_i{}^l-S_j{}^lS_i{}^k)(\bar{\Phi}^{(3^*)})^{ij}(\Phi^{(3^*)})_{kl}\\ &=\tfrac{1}{2}\sum_{i,j,k}\epsilon_{ijk}\bar{\varphi}_i\varphi^iS_j{}^jS_k{}^k,\end{aligned} \tag{11}$$

analogously to that of the $(h\bar{h})$ boson given by Eq. (7), using the fact that the matrix S is diagonal. Therefore, it is assumed that the mass m_i of the $\underline{3}^*$ lepton ψ_i which is made of φ_i is given by

$$\begin{aligned}m_i&=m_0\sum_{j,k}\tfrac{1}{2}\epsilon_{ijk}S_j{}^jS_k{}^k+\text{const.}\\ &=m_0\left[\left(c_0{}^2-\frac{c_8{}^2+c_3{}^2}{12}\right)-\left(c_0c_8+\frac{c_8{}^2-c_3{}^2}{2\sqrt{3}}\right)F_8 - \left(c_0c_3-\frac{2c_8c_3}{2\sqrt{3}}\right)F_3\right]_i{}^i+\text{const.}\end{aligned} \tag{12}$$

Note that the mass matrix of $\underline{3}^*$ is diagonal, so that there is no mixing among the lepton generations. In order to simplify Eq. (12), we define

$$m_i{}^{1/2}=m_0{}^{1/2}(d_0-d_8F_8-d_3F_3)_i{}^i. \tag{13}$$

Then, by comparing the square of Eq. (13) with Eq. (12), we get

$$\begin{aligned}&d_9=c_8, \quad d_3=c_3, \quad d_0=\tfrac{1}{2}c_0,\\ &\text{const}=\tfrac{1}{4}(c_8{}^2+c_3{}^2-3c_0{}^2).\end{aligned} \tag{14}$$

Therefore, we obtain the relation

$$\tan\theta=\frac{c_3}{c_8}=\frac{d_3}{d_8}=\frac{\sqrt{3}(m_2{}^{1/2}-m_1{}^{1/2})}{2m_3{}^{1/2}-m_2{}^{1/2}-m_1{}^{1/2}}, \tag{15}$$

from Eqs. (10), (13), and (14).

We suppose that such a symmetry-breaking effect appears only in the bound state with (ν, l) and therefore the magnitudes of c_8 and c_3 are dependent of the weak-isospin component I_{w3}. We assume that $c_3=0$ in the "up" quarks and "up" leptons, that is, $m(\nu_e)=m(\nu_\mu)$ ($=0$ or $\simeq 0$). Therefore there is no mixing between u and c quarks. (The u-c mass difference is induced by $c_8 \neq 0$, but the numerical value cannot be determined from the lepton masses because of arbitrariness of the additional parameters.) On the other hand, in the "down" quarks and "down" leptons, the symmetry-breaking c_3/c_8 is nonvanishing, that is, $m_e \neq m_\mu$, and therefore we obtain the relation[6] (2) by putting $\theta=\theta_C$, $m_1=m_e$, $m_2=m_\mu$, and $m_3=m_\tau$ into Eq. (15).

In conclusion, this model can successfully provide a new SU(4)-$\underline{20}$ enhancement and SU(4)-$\underline{84}$ suppression mechanism[7] in the nonleptonic weak Hamiltonian and a new formula for the Cabibbo angle represented in terms of lepton masses. However, some problems to be investigated still remain in the model: What force combines (ν_a, l_a) with $(h\bar{h})$? Why are other $SU(3)_{sc}$-singlet states [bosonic leptons $(\nu_\alpha \bar{h}_i{}^\alpha)$, ..., nondiagonal $SU(3)_G$-octet quarks $\nu_a(h^1\bar{h}^2)$, ..., and so on] absent? In order to reply to these questions, we must presumably introduce further a new binding force.[8]

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$$\tan^2\theta_C = \frac{c_3{}^2}{c_8{}^2} = \frac{\sqrt{3}(m_\mu - m_e)}{2m_\tau - m_\mu - m_e},$$

which predicts $\tan\theta_C = 0.229$. It is an open question whether or not this coincidence is accidental.

[7]Introduction of the subcolor to the model is not essential in the derivation of the formula for the Cabibbo angle. We may substitute the usual color index a for subcolor index α in (ν_α, l_α) and $h_\alpha{}^i$, and identify (ν, l) inside a lepton with (ν_a, l_a) inside a quark. The purpose of the introduction of the subcolor is as follows: We consider the rearrangement of (ν_a, l_a) at the nonleptonic weak vertex, while we neglect such a rearrangement of (ν_α, l_α) at the semileptonic and pure leptonic weak vertices, by assuming that the subcolor-singlet state is a very tightly bound state, so that the rearrangement of (ν_a, l_a) from $(h\bar{h})$ is only allowed.

[8]For example, we suppose that each of h^i has independently a sort of charge and (ν, l) has all the three charges but with opposite signs so that h^1-$\bar{h}^1$, h^2-$\bar{h}^2$, h^3-$\bar{h}^3$, ν-h^i, and l-h^i are attractive, while the other combinations are repulsive or not interactive. We assume that all the subquarks in the physical states must be combined by this attractive force: For example, as h^2-ν-h^3 in a lepton, ν-h^1-$\bar{h}^1$ in a quark and so on.