$\hbar\beta/2$) with the measure

$$d\mu_\beta = \frac{\{\exp -\hbar^{-1}\int_{-\hbar\beta/2}^{\hbar\beta/2} V[q(\tau)]\,d\tau\}\,d\mu_0}{\int_{P(-\hbar\beta/2,\,\hbar\beta/2)} \exp\{-\hbar^{-1}\int_{-\hbar\beta/2}^{\hbar\beta/2} V[q(\tau)]\,d\tau\}\,d\mu_0}. \tag{20}$$

The high-temperature limit (classical limit) of the above expression gives

$$d\mu_\beta \to \rho_{\mathrm{eq}}(q) = Z^{-1} e^{-\beta V(q)}$$

while in the low-temperature limit ($\beta \to \infty$), from the comparison of Eq. (20) with (10) it is obvious that the Feynman process goes over to Nelson's stochastic process for the ground state, provided τ is identified with the physical time of evolution of the process. In other words the Feynman formulation of quantum statistical mechanics, upon lowering of the temperature, produces the switch from the ensemble averages of the classical regime to the time averages of the ground-state process arising in the stochastic quantization. From the general point of view, this makes precise the connection between the zero-temperature limit of quantum statistical mechanics and Nelson's formulation of the quantum theory. With respect to the question originally posed in connection with quantum critical phenomena, the comparison of Eq. (19) with Eq. (3) makes it evident that the framework of stochastic quantization allows us to bypass the current notion of dimensional crossover by introducing the dynamical description of equilibrium, which may open an interesting perspective in view of the application of the Monte Carlo technique to the computation of zero-temperature properties.

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Crossed-Beam Spectroscopy of Hydrogen: A New Value for the Rydberg Constant

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In a crossed laser-atomic beam experiment the wavelengths of the $2s$-$3p$ transitions are measured in H and D to a precision of one part in 10^9. Our value for the Rydberg constant is $R_\infty = 109\,737.315\,21(11)$ cm^{-1}. The fine-structure splittings of the $3p$ states in H and D are 3249.8(8) and 3251.7(7) MHz, respectively; the isotope shifts for the $2s$-$3p_{1/2}$ and $2s$-$3p_{3/2}$ transitions are 124 260.7(7) and 124 262.6(7) MHz, respectively. Our results largely agree with previous, less precise experiments and with theory.

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Because it is the simplest of all atomic systems, hydrogen is still the most important spectroscopically. In recent years, high-precision, optical spectroscopy of atomic hydrogen has taken on renewed interest,[1-10] especially in conjunction with nonlinear techniques[1-6] for reducing linewidths in the wavelength measurements. It is generally agreed,[2-6] however, that it is necessary to use an atomic beam to eliminate pressure and current corrections connected with the gas discharges used by previous workers[1-5] if accuracy is to be improved further. One attempt in this direction has already been reported,[6] and others are underway at Michigan,[7] National Bureau of Standards, Washington, D. C.,[8] Bonn,[9] and Heidelberg.[10]

At Yale University we have succeeded in making precise measurements of the Balmer-α transitions ($2s$-$3p$) in both hydrogen and deuterium. Because crossed beams are employed in the measurements, we determine to high accuracy the wavelength which induces a single-photon, linear transition in the atom. This has allowed us to elicit a value of the Rydberg constant to a precision hitherto unattainable as well as more precise values for the fine-structure splittings in the $3p$ state and the isotope shift between hydrogen and deuterium.

The experiment[5] is an optical analog of the classic Lamb-Retherford radio-frequency determination of the Lamb shift.[11] A hydrogen beam emerges from a tungsten oven heated to 2850 K, is excited by electron bombardment to the metastable $2s$ state, and then collimated to an angle of 1.25 ± 0.15 mrad. Downstream from the collimator, this beam is crossed at right angles by the beam from a low-intensity (~ 0.002 W/cm^2), tunable, cw dye laser. The metastable atoms are detected at the end of their flight path by secondary-electron emission from a Ni plate. The signal from this plate is fed into a lock-in amplifier.

The dye laser is stabilized by locking to an external Fabry-Perot interferometer. The laser, once adjusted to the $2s$-$3p$ transition, is frequency modulated with a 15-Hz square wave of peak-to-peak shift 50 MHz, approximately the linewidth. With the modulating voltage as a reference for the lock-in amplifier, the line shape becomes a derivative trace. By keeping the lock-in amplifier output to zero, we hold the laser to within an rms deviation of 2 MHz from the line center.

The heart of the experiment is the etalon which is used for the wavelength measurements. This is constructed of two multilayered dielectric-coated mirrors with 99.5% reflectivity, has cervit spacers, and is scanned by increasing the N_2 pressure within the cavity. Minimum finesse of the etalon is $F = 40$. The wavelength standard used for the comparison is a ^{3}He-^{22}Ne:^{129}I$_2$ laser stabilized by a third derivative lock[12] (hereafter referred to as the secondary standard). This standard is compared to the portable ^{3}He-^{20}Ne:^{127}I$_2$ primary standard from the National Bureau of Standards.[13] By relating our secondary standard to the primary standard in this manner, our measurements are directly traceable to the absolute value of the meter.[14]

Measurement of the wavelength of the dye laser with respect to the standard is carried out by two methods.[15] First, in order to eliminate effects of phase shifts in the mirror coatings, we use the method of virtual mirrors.[15] Three etalon spacings are used (4.3, 12.5, and 31.4 cm), and the final result is a weighted average of the measurements at each individual spacing, the last of the three being given the most weight. Alternatively, we use the reflection curves furnished by the mirror coaters (Laser Optics, Inc.) to calculate the phase shift at the 31.4 cm spacing, thereby obtaining a completely independent value of each wavelength.[15]

We remove first-order Doppler shifts through 90° beam intersection and retroflection; we minimize light shifts and optical pumping by use of low light intensity. Zeeman shifts are reduced by keeping the magnetic field within the chamber to 0.3 G, although hyperfine splittings pose a particular problem at this low magnetic field. While there is no hyperfine effect in the $2s$ state because the excitation process leads to only the level $F = I + \frac{1}{2}$,[11] the hyperfine structure of the $3p$ state is unresolvable, so that the measured wavelengths must be corrected to account for this.

What we obtain as the result of our measurements is the ratio of the corresponding wavelength of both fine-structure components of the $2s$-$3p$ lines of hydrogen and deuterium to that of the secondary standard. Using the wavelength definition of the Comité Consultatif pour la Définition du Mètre[14] for the meter, $\lambda(^3\text{He-}^{20}\text{Ne}:{}^{127}\text{I}_2, i) = 632\,991.399$ pm, together with our calibration of our standard with respect to that laser, we convert these measurements to absolute wavelengths. After making the reduced mass and nuclear structure corrections calculated by Erickson,[16] we have four independent values of the Rydberg constant using the virtual mirrors measurements and four independent values from the phase-shift calculations. Each of these is weighted equally in calculating a value of the constant within each wavelength determining method. The final value is given as a weighted average of the virtual mirrors data and the phase-shift data with statistical weighting factors of 0.88 for the former and 0.12 for the latter. Our primary result is then

$$R_\infty = 109\,737.315\,21(11)\ \text{cm}^{-1}.$$

We obtain a slightly different value for R if we

TABLE I. Corrections and errors in R (parts in 10^{10}).

Source	Correction[a]	Error[a]
Statistical	–	3
Phase shift	465[b]	3
Refractive index, scanning, and recording	50	6
Magnetic field	−4	2
2s hfs	636	0
Optical pumping, 3p hfs, and light shifts	−1	5
Standard	5	5
Diffraction	−3	1
Second-order Doppler	4	0
Photon recoil	−8	0
rms sum		10

[a] Average value for all four transitions.

[b] For phase-shift data only.

TABLE II. Comparison with earlier measurements.

Reference	$R - 109\,737$ (cm^{-1})
Taylor *et al.*[a]	0.312(5)
Hänsch *et al.*[b]	0.314(1)[e]
Petley *et al.*[c]	0.315 13(85)
Goldsmith *et al.*[d]	0.314 88(32)[e]
Present result	0.315 21(11)

[a] Ref. 18.

[b] Ref. 1.

[c] Ref. 2.

[d] Refs. 3 and 19.

[e] Refs. b and d are corrected to $\lambda(^3\mathrm{He}\text{-}^{20}\mathrm{Ne};\ ^{127}\mathrm{I}_2,\ i) = 632\,991.399$ pm (Ref. 14).

assume the tentative definition of the meter[17] in terms of the velocity of light: $c \equiv 299\,792\,458$ m/s, namely $R = 109\,737.315\,37(11)$ cm^{-1}.

In Table I we present a list of all sources of error which contribute to the Rydberg determination. In Fig. 1 and Table II we make comparisons of our Rydberg value with results from previous investigators. We disagree slightly outside the estimated errors with the results of Hänsch *et al.*[1] and are in fair agreement with the results of Goldsmith, Weber, and Hänsch,[3] which is based on a single transition. We are in good agreement with Petley, Morris, and Shawyer.[2] The number given here for Goldsmith, Weber, and Hänsch[3] differs slightly from that quoted in the original article and reflects a corrected wavelength standard and a new analysis of the data by van Dierendonck.[19]

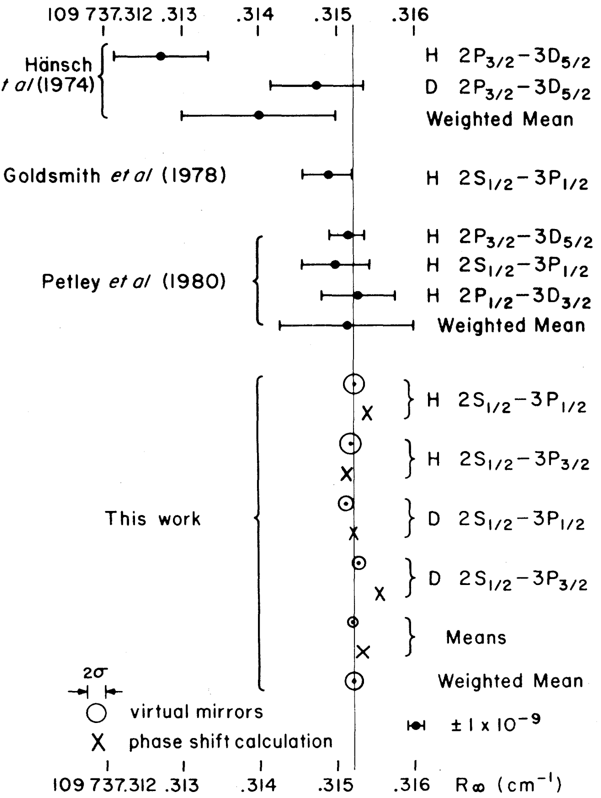

FIG. 1. Comparison of recent measurements of the Rydberg constant. Errors for individual measurements are statistical. Errors for weighted means include systematic errors. See Table II.

Once we have the wavelengths of both the fine-structure transitions, it is a straightforward matter to determine the fine-structure splittings within each isotope and the isotope shift between each line. Direct subtraction of the two wave numbers followed by multiplication by $c = 299\,792\,458$ m/s (Ref. 17) yields $\nu(\frac{3}{2}) - \nu(\frac{1}{2}) = 3249.8(8)$ MHz for hydrogen and $\nu(\frac{3}{2}) - \nu(\frac{1}{2}) = 3251.7(7)$ MHz for deuterium. These errors are obtained from the statistical and systematic errors for each of the above numbers with the phase-shift calculations and the virtual mirrors determinations weighted with the same factors as for the Rydberg calculation. In Table III we give a comparison of these values with those from the previous most accurate experiments[20-23] and with the theoretical values.[16]

Following the same procedure with regard to determining errors and assigning weights to the different methods, we obtain for the isotope shift between each of the two lines: $\mathrm{D}(p_{1/2}) - \mathrm{H}(p_{1/2}) = 124\,260.7(7)$ MHz; $\mathrm{D}(p_{3/2}) - \mathrm{H}(p_{3/2}) = 124\,262.6(7)$ MHz. These numbers represent a precision of 6 parts in 10^6 and compare well with the theoreti-

TABLE III. Fine-structure splittings and isotope shifts (MHz).

	Fine structure	
	H	D
This work	3249.8(8)	3251.7(7)
Last value	3250.0(11)[a]	3250.7(20)[b]
Theory[c]	3250.086(1)	3250.973(1)
	Isotope shift	
	1/2 component	3/2 component
This work	124 260.7(7)	124 262.6(7)
Theory[c]	124 261.10(2)	124 261.97(2)
	Weighted average	
This work	124 261.9(6)	
Theory[c]	124 261.68(1)	
Last value[d]	124 269(4)	

[a] Combinations of measurements of Refs. 20–22.
[b] Ref. 23.
[c] Ref. 16.
[d] Ref. 1.

cal calculations of Erickson,[16] which give 124 261.10(2) and 124 261.97(2) MHz. Thus this measurement removes the discrepancy between the theoretical value and that obtained by Hänsch *et al.*[1] (see Table III and Fig. 1).

The measurement of the Rydberg constant described in this paper is more precise, by a factor of 3, than any previous measurement to date. It emphasizes the value of making these measurements in a crossed atomic-beam–laser-beam system. While the present accuracy has been limited by the bandwidth of the laser and certain hyperfine effects, minor improvements of the apparatus should make it possible to extend this technique to a measurement of this fundamental constant into the realm of precision of comparison of standards of length[13] (2 parts in 10^{10}). The same improvements would also furnish more accurate determinations of the fine-structure splitting and the isotope shift of these levels. Such determinations are important for critical tests of quantum electrodynamics and for probes of nuclear structure. Lastly, they serve as a check on the electron-proton mass ratio.

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Duality Transformations for Two-Dimensional Directed Percolation and Resistance Problems

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It is shown that the percolation problem with blocked, one-way, or two-way bonds is self-dual on a square lattice. The usual directed percolation (blocked or one-way bonds) is dual to a simpler process with one- or two-way bonds. The results of a Monte Carlo simulation of the latter are reported and an improved bound on the critical probability is derived. The corresponding resistance problem in which circuit elements have different forward and backward resistances is also shown to be self-dual.

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The effect of introducing directionality into bonds in percolation problems is currently the subject of much interest. Besides its relevance to many physical situations,[1,2] the problem is interesting because directionality brings in qualitatively new effects.[1] In this Letter we present a duality transformation for a generalized directed-bond percolation process on planar lattices. In particular we show that the dual of the usual directed percolation[3-8] is a new process with one- or two-way bonds which, in some respects, is a simpler problem.

Consider a square lattice in which each bond is blocked, one way (right or up), or two way, with probabilities p_0, p_1, and p_2, respectively ($p_0+p_1+p_2=1$), independently of other bonds. In electrical terminology, we have a random network of insulators (I), diodes (D), and resistors (R). Obukhov studied this percolation problem in $5-\epsilon$ dimensions for a small concentration of diodes. It includes as special cases the usual (undirected) resistor-insulator percolation[9] (RIP), the diode-insulator percolation[3-8] (DIP), and the diode-resistor percolation (DRP).

We show that this diode-resistor-insulator percolation problem is self-dual. Unlike RIP, where there is a single transition between two phases, our general problem has three phases and two phase transitions. Duality relates these. We show that the corresponding resistance problem is also self-dual, and discuss the bulk resistivity of a random I-D-R network.

DIP ($p_0=q$, $p_1=p$, $p_2=0$) has been shown to be equivalent to Reggeon field theory.[5] It lies in a universality class different from RIP and quite accurate estimates of its critical parameters are available.[4-8] For $p>p_c^{\text{DIP}}$ there is a nonzero probability $P_\infty^{\text{DIP}}(p)$ that a point source wets an infinite region. Far away from the source, this region is confined to a wedge of angle $\theta^{\text{DIP}}(p)$. This wedge angle increases from 0 to $\pi/2$ as p increases from p_c^{DIP} to 1 and varies as $(p-p_c^{\text{DIP}})^b$ for p close to p_c^{DIP} (b is a critical exponent).

The DRP process is defined as the case $p_0=0$, $p_1=q$, $p_2=p$. Suppose there is a source of fluid at the origin. If a site (x,y) of the lattice is wetted by the source, it wets all sites (x_1,y_1) with $x_1\geqslant x$, $y_1\geqslant y$. Conversely if (x,y) is dry, so are all sites (x_2,y_2) with $x_2\leqslant x$, $y_2\leqslant y$. Thus the wet sites are separated from the dry ones by a