effect was observed in the FeNi alloys, while in pure Ni the magnetic field effect was not appreciable.[11]

Finally, note that in Fig. 2 we varied M/M_0 by varying $\overline{V}$ for $T=0$. In reinterpreting the result of Fig. 2 by appropriately translating the role of $\overline{V}$ to that of T we should note that the phonon effect on magnetization is enhanced by the thermal excitation of phonons. With such enhancement effect,[6] which is proportional to $\sim T/\Theta$ and starts from $T\sim\Theta/5$, Θ being the Debye temperature, the size of phonon effect can be larger for higher temperatures (smaller M) than for lower temperatures (larger M) even in the situation of Fig. 2(b).

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Observation of Zero-Point Fluctuations in a Resistively Shunted Josephson Tunnel Junction

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The spectral density of the voltage noise has been measured in current-biased resistively shunted Josephson junctions in which quantum corrections to the noise are expected to be important. The experimental data are in excellent agreement with theoretical pretions, demonstrating clearly the contribution of zero-point fluctuations that are generated in the shunt at frequencies near the Josephson frequency and mixed down to the measurement frequency.

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In a recent Letter, we[1] considered the effects of quantum corrections on the voltage noise in a current-biased resistively shunted Josephson junction. For measurement frequencies much less than the Josephson frequency and for a heavily overdamped junction we predicted a spectral density for the voltage noise $S_v(0)$:

$$\frac{S_v(0)}{R_D{}^2}=\frac{4k_BT}{R}+\frac{2eV}{R}\left(\frac{I_0}{I}\right)^2\coth\left(\frac{eV}{k_BT}\right). \tag{1}$$

Here, I_0 and R are the critical current and shunt resistance of the junction, I and V are the current and voltage, and R_D is the dynamic resistance. Equation (1) is based on the assumption that the noise arises from equilibrium noise currents in the shunt resistor with a spectral density

$$S_I(\nu)=(2h\nu/R)\coth(h\nu/2k_BT)$$
$$=(4h\nu/R)\{[\exp(h\nu/k_BT)-1]^{-1}+\tfrac{1}{2}\} \tag{2}$$

at frequency ν. The first term on the right-hand side of Eq. (1) represents noise from the resistor at the measurement frequency, while the second term arises from noise mixed down from frequencies near the Josephson frequency. In the limit $eV\gg k_BT$, the latter term represents zero-point fluctuations; provided, in addition, $2eV(I_0/I)^2 > 4k_BT$, the measured noise will be dominated by the mixed-down zero-point fluctuations. These limits require a junction with[1] $\kappa \equiv eI_0R/k_BT \gg 1$. Previous[2] accurate measurements of the noise were in the limit $\kappa \ll 1$, so that quantum corrections were insignificant. In this Letter we report experimental results that are in excellent agreement with Eq. (1). In particular, the contribution of the zero-point fluctuations that are generated at frequencies near the Josephson frequency and mixed down to the measurement frequency is clearly demonstrated. The inferred spectral density of the current noise in R is also in excellent agreement with Eq. (2).

We fabricated Pb(20 wt.% In)-In_2O_3-Pb tunnel junctions with a diameter of 2.5 μm defined by a window in an SiO insulating layer. Each junction was shunted with a CuAl film with a resistance of about 0.5 Ω. The capacitance and critical current were typically 0.5 pF and 0.5 mA, so that[3] $\beta_c = 2\pi R^2 CI_0/\Phi_0$ was about 0.2. The junction bias current, supplied by batteries, passed through cooled low-pass filters. The cryostat, batteries, and preamplifier were enclosed in a shielded room. The shunt resistance, R, of the junction was measured with the critical current reduced to zero by trapped flux or by an applied magnetic field. The voltage noise produced by the junction was amplified with cooled 70-, 106-, and 180-kHz LC resonant circuits coupled in turn to a room-temperature low-noise preamplifier. The noise at each frequency was mixed down to frequencies below 500 Hz, and the spectral density of this low-frequency noise was measured with a computer. The junction produced a noise across the tank circuit with a spectral density $Q^2S_v(0)$, where $Q = \omega L_t/R_D$, $\omega/2\pi$ is the resonant frequency, and L_t is the tank circuit inductance. Thus, the quantity $S_v(0)/R_D{}^2$ was independent of Q, and could be compared directly with Eq. (1) using measured values of I_0, R, I, V, and T. The gain of the amplifier-mixer-computer chain was calibrated frequently during each experimental run by connecting a known resistor at the input of the preamplifier, and measuring the Nyquist noise.

We took into account the following extraneous noise contributions:

(i) The voltage and current noises of the preamplifier, at most 30% of the total spectral density, were subtracted from each measurement. The maximum error introduced was no more than $\pm 3\%$.

(ii) From the noise measurements at the three frequencies and at each bias voltage and temperature we determined that some junctions contained a small excess noise with a spectral density close to $1/f$. In the junction reported on in this Letter, the noise at 180 kHz was at most 10% of the total spectral density of the noise generated in the junction. The error due to the subtraction of this noise was no greater than $\pm 2\%$.

(iii) The temperature rise of the shunt resistor due to Joule heating was determined at each temperature as a function of power. We reduced the critical current of the junction almost to zero by trapping flux in it or by applying an external magnetic field, and estimated the temperature from the measured Nyquist noise. At low bias voltages, where the self-heating was negligible, the measured noise agreed with the Nyquist formula to within $\pm 3\%$. The temperature rise varied from 0.25 K/μW at 4.2 K to 1.6 K/μW at 1.6 K, so that the correction for self-heating was important only at relatively high voltages where $eV/k_BT > 1$. Thus, the mixed-down noise was not significantly affected, and we corrected the data by subtracting $4k_B\Delta T/R$ from the measured spectral density, where ΔT is the temperature increase. The maximum uncertainty in the heating corrections was $\pm 10\%$, introducing a maximum uncertainty in the data of $\pm 3\%$.

(iv) The quasiparticle current of the junction, I_{qp}, contributes a noise with a spectral density[4] $2eI_{qp}\coth(eV/2k_BT)$. Therefore, the ratio of the quasiparticle noise to the predicted mixed-down noise is of order $I_{qp}/(V/R)$, which we estimate to be $\lesssim 10^{-2}$ at 4.2 K. Thus, the contribution of the quasiparticle noise should be negligible; this result was confirmed by the fact that the measured noise at low voltages with nearly zero critical current was very close to the Nyquist value.

In this Letter, we focus on the results of one of the three junctions we have studied in detail. In Fig. 1 we plot measured values of $S_v(0)/R_D{}^2$ vs voltage (open circles) after the preamplifier noise has been subtracted. The solid circles are the noise after the $1/f$ noise subtraction and the heating correction have been made. The upper solid line through the solid circles is the prediction of Eq. (1) with the appropriate measured parameters. The upper dashed line is the predicted noise in the absence of zero-point fluctuations, that is,

$$\frac{S_v'(0)}{R_D{}^2} = \frac{4k_BT}{R} + \frac{4eV}{R}\left(\frac{I_0}{I}\right)^2 \frac{1}{\exp(2eV/k_BT) - 1}. \quad (3)$$

The triangles represent the measured mixed-

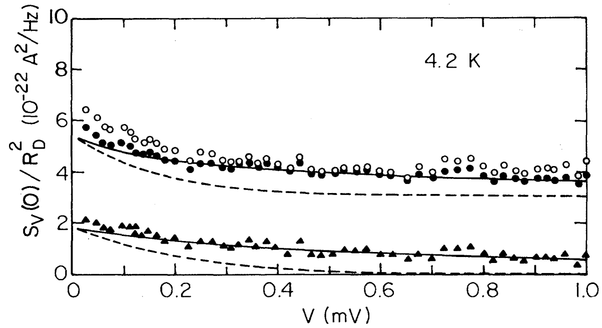

FIG. 1. $S_v(0)/R_D^2$ vs voltage for junction at 4.2 K with $I_0 = 0.514$ mA, $R = 0.70\ \Omega$, and $\kappa = 0.99$. The open circles show the total measured noise across the junction; solid circles below show the noise remaining after correction for $1/f$ noise and heating. Upper solid and dashed lines are predictions of Eqs. (1) and (3). Solid triangles are measured mixed-down noise, lower solid and dashed lines are mixed-down noise predicted by Eqs. (1) and (3).

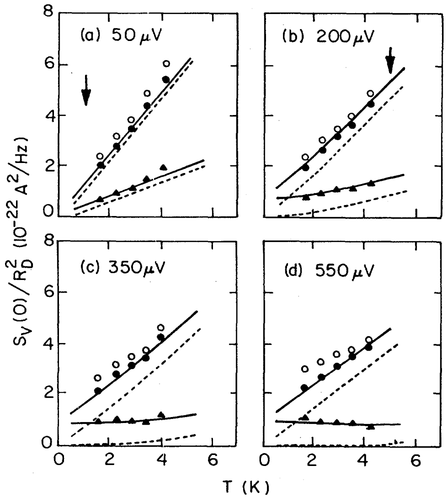

FIG. 2. $S_v(0)/R_D^2$ vs temperature for the junction of Fig. 1 at four bias voltages. Notation is as for Fig. 1. Arrows indicate $2eV = k_BT$.

down noise, which was computed by subtracting $4k_BT/R$ from the solid circles. The lower solid line is the mixed-down noise predicted by Eq. (1), $(2eV/R)(I_0/I)^2 \coth(eV/k_BT)$, while the lower dashed line is the mixed-down noise predicted by Eq. (3), $(4eV/R)(I_0/I)^2[\exp(2eV/k_BT) - 1]^{-1}$. It is evident that both the total measured noise and the measured mixed-down noise are in excellent agreement with the theory that includes a contribution from the mixed-down zero-point fluctuations, and are substantially higher than a theory that does not include this contribution.

In Fig. 2 we show the temperature dependence of the noise for four bias voltages. The temperature $T = 2eV/k_B$ is indicated for the lowest two voltages; for the highest two voltages all the data lie below this temperature. Thus, as we expect, the mixed-down noise at 350 and 550 μV is independent of temperature,[5] and in excellent agreement with the predicted value $S_v(0)/R_D^2 = (2eV/R)(I_0/I)^2$. As the voltage is lowered, the mixed-down noise becomes increasingly temperature dependent. At 50 μV the mixed-down noise is always in the classical limit, and the spectral density of the mixed-down noise is proportional to T, as we expect.

We can extract from our data the measured spectral density of the current noise $S_I(\nu)$ generated by the resistance R. We multiply each value of the mixed-down noise by $2(I/I_0)^2$ and set $2eV = h\nu$. [This procedure converts the mixed-down term in Eq. (1) into Eq. (2).] The results are plotted in Fig. 3 for 4.2 K (solid circles) and 1.6 K (open circles). The solid lines are the corresponding predictions of Eq. (2) with measured values of $\nu = 2eV/h$, R, and T. The agreement between the predictions and the data is rather good, if one bears in mind that no fitting parameters are used. The larger scatter in the data at the highest voltages shown occurs near a self-resonance on the I-V characteristic; the additional nonlinearity introduced by the resonance mixes down noise near higher harmonics of the Josephson frequency. The dashed lines represent the theoretical prediction in the absence of the zero-

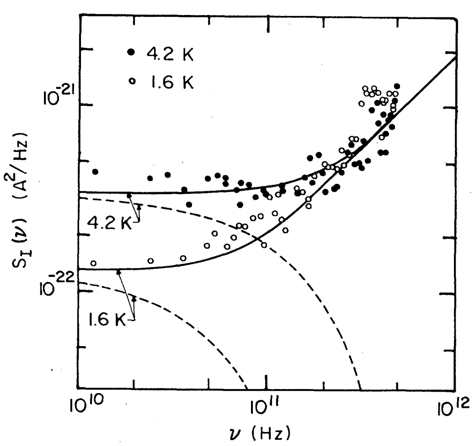

FIG. 3. Measured spectral density of current noise in shunt resistor vs the Josephson frequency $\nu = 2eV/h$ at 4.2 K (solid circles) and 1.6 K (open circles). Solid lines are predictions of Eq. (2), while dashed lines are $(4h\nu/R)[\exp(h\nu/k_BT) - 1]^{-1}$.

point term, $(4h\nu/R)[\exp(h\nu/k_BT)-1]^{-1}$, and fall far below the data at the higher frequencies. The evidence for zero-point fluctuations in the measured spectral density of the current noise is rather compelling.

The noise in the two other junctions that we have investigated in detail is in good agreement with our model. For one junction, we progressively reduced I_0, and hence κ, by trapping flux; the noise at each value of κ followed the prediction of Eq. (1) rather well. The other junction showed pronounced structure on the I-V characteristic associated with the resonance of the shunt inductance and junction capacitance. The additional nonlinearities mixed down noise from frequencies near higher harmonics of the Josephson frequencies, substantially increasing the measured voltage noise. A computer simulation that took these effects into account was in good agreement with the data.

In conclusion, we emphasize that in comparing theory and experiment we have used only measured parameters; there is not fitting of the data. We believe our results are a convincing demonstration, first, of the existence of a zero-point term in the spectral density of the current noise of a resistor in thermal equilibrium, and, second, that these fluctuations give rise to the limiting voltage noise in a current-biased resistively shunted Josephson junction in the quantum limit for $I>I_0$. Furthermore, the good agreement between our results and the predictions of our model justifies our use of a Langevin treatment[1] to predict quantum noise effects in a current-biased Josephson junction in the overdamped limit when it is in the free-running mode, $I>I_0$. We were not able to examine the validity of the theory in the noise-rounded case, $I<I_0$, since quantum effects are negligible in the ^{4}He temperature range for the parameters of our junctions.

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