Inclusive Proton Decay in a Nucleus

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The conversion of a proton inside a nucleus into a positron, energy-momentum being shed into the final state, is consdered. For ^{16}O it is found that the lifetime of a proton due to this mechanism is of the same order (probably ~3 times smaller) as that due to the conventional two-quark fusion. The experimental signature is different, a positron track against a star of nuclear fragments, with eventual pions.

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The fact that there are several experiments in progress to try to detect proton decay[1] makes it worthwhile to look at possible mechanisms for this process, particularly since theoretical estimates in grand unified theories[2-5] with the presently favored values[6] of the masses of the leptoquark X particles responsible for the decay in the SU(5) model[7] give rates inside the experimental capabilities.

Proton decay may proceed through any of the diagrams of Fig. 1. Diagram (a) is strongly suppressed by phase space; the usual calculations take the two-quark fusion diagram (b) to be the one responsible for the decay. Nevertheless, it is clear that, from phase space alone, we would expect diagram (c) to dominate by a factor $\sim 2\pi$. Of course, the problem with diagram (c) is that it cannot take place for an isolated proton because of energy-momentum conservation. However, this argument is invalid for a proton inside a nucleus as energy-momentum may be absorbed by the rest of said nucleus.

The process (c) was considered in Ref. 3, but only under the hypothesis of coherent interaction with the rest of the nucleus; then it is suppressed by a factor $\sim m_\pi^2/m_p^2$. Here we discuss the *inclusive* decay in which we sum over all possible states of the final nuclear fragments so that the suppression is absent. For a nucleus like ^{16}O, relevant for experiments carried out in water, we find that process (c) is dominant by a factor ~3 (not quite the 2π from phase space) over the sum of the usual two-quark fusion channels. (A related process, $3q \rightarrow e^+ + \text{gluon}$, may be seen to be relatively depressed by a factor ~10.) The uncertainties are larger (an extra factor ~2). It should be stressed that, in view of all the theoretical uncertainties to any calculation of p decay, the important result is not that we raise the expected decay rate, but the fact that the dominant experimental signature will not be that corresponding to two-quark fusion, viz., a decay $p \rightarrow e^+(\pi, \rho, \omega, \eta)$, but the inclusive one induced by Fig. 1(c): a positron track and a corresponding star of nuclear fragments, with eventual slow pion(s).

Let us consider the SU(5) model for definiteness; the analysis for more general theories[8]

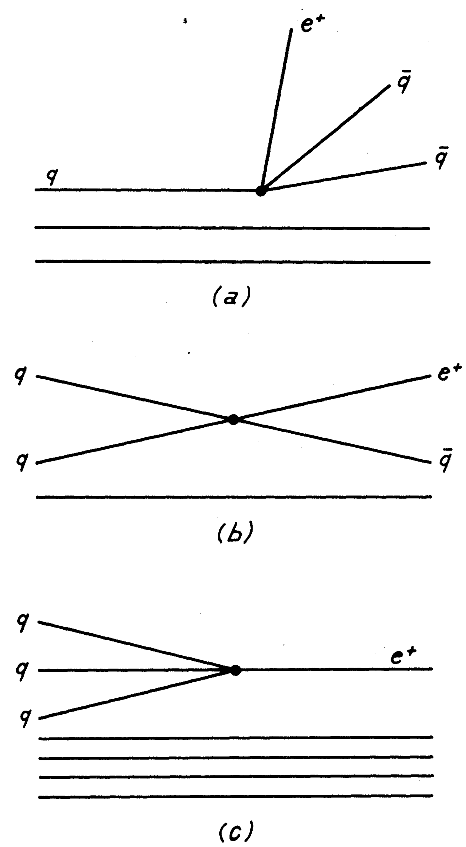

FIG. 1. Four-fermion effective interactions leading to p decay. (a) and (b) should be interpreted as exclusive processes; (c) should be interpreted as the pointlike part of an exclusive process.

may be carried out along similar lines. The effective Lagrangian for the decay is

$$\mathcal{L}_{\rm eff}(x) = (g_{\rm GUT}^2/2M_x^2)\bar{e}^+(x)p(x), \tag{1}$$

where $g_{\rm GUT}$ is the coupling constant at grand unified energies, $g_{\rm GUT}^2/4\pi \sim 0.02$, and we have defined the composite operator

$$p_\alpha(x) = \sum_{i,j,k} \epsilon_{ijk} u_i^{\,c}(x)\gamma^\mu \frac{1-\gamma_5}{2} u_j(x)\left[\gamma_\mu \frac{3-\gamma_5}{2} d_k(x)\right]_\alpha ; \tag{2}$$

i, j, and k are color indices, α is a Dirac one. The total decay rate for a proton inside a nucleus A with atomic number A, $A \to e^+(k,\lambda) + \text{all}$, is

$$\Gamma_A = \frac{(2\pi)^8}{4\pi m_A} \int \frac{d^3k}{2k^0} \sum_{F,\lambda} \delta(p_F + k - p_A) |\langle F, e^+(k,\lambda)| \mathcal{L}_{\rm eff}(0)|A\rangle|^2. \tag{3}$$

We normalize states to $\langle p|p'\rangle = 2p_0\delta(\vec{p}-\vec{p}\,')$, spinors to $\bar{u}u = 2m$. The quantity to be compared with the result of the usual calculations is the lifetime per nucleon, $\tau_p = A/\Gamma_A$. Reducing the positron and using completeness we may rewrite (3) as

$$\Gamma_A = \frac{g_{\rm GUT}^4}{8m_A M_x^4} \int \frac{d^3k}{2k^0} \int d^4x\, e^{ikx} \langle A|[\bar{p}(0), \not{k}p(x)]|A\rangle . \tag{4}$$

The commutator in (4) may be related to the total cross section for antiproton-nucleus annihilation at rest. Indeed, let $\psi(x)$ be a field operator for the proton, normalized to

$$\langle 0|\psi(x)|p,\sigma\rangle = u(p,\sigma)/(2\pi)^{3/2}. \tag{5}$$

Using the Lehmann-Symanzik-Zimmermann reduction formulas we get for the imaginary part of the forward $\bar{p}A$ scattering amplitude

$$\mathrm{Im}\,T_{\sigma\sigma}(p,A) = \pi[\bar{v}(p,\sigma)(\not{p}-m_p)]_\alpha \int d^4x\, e^{-ixp}\langle A|[\bar{\psi}_\beta(0), \psi_\alpha(x)]|A\rangle[(\not{p}-m_p)v(p,\sigma)]_\beta . \tag{6}$$

To be able to connect (6) with (4) we first have to clear up a few points. In (4) the momentum of the positron is k, $k^2 \simeq 0$; but in $\bar{p}A$ scattering we deal with real antiprotons. We assume the dependence of $\mathrm{Im}\,T$ on p^2 to be negligible. In fact, we can estimate this dependence by taking it of the form

$$\Gamma((m_p^2 - p^2)/(m_*^2 - m_p^2))(m_p^2 - p^2)/(m_*^2 - m_p^2),$$

a form that works well when it has been tested. Here m_* is the mass of the $N(1400)$ resonance: We find the dependence on p^2 to be of ~15%. Next, (4) also has projection over other processes [like $\bar{N}(1400)A$ annihilation]. The same argument suggests that such contributions will *increase* the decay rate by a small amount, m_p^2/m_*^2. Finally, if, as in the case for ^{16}O, the nucleus has spin 0, the combinations of amplitudes that enter (4) and (6) are slightly different. To be precise, letting

$$\int d^4x\, e^{-ixp}\langle A|[\bar{\psi}_\beta(0), \psi_\alpha(x)]|A\rangle = \frac{-1}{2\pi m_p}\left\{\frac{1}{\not{p}-m_p}\left[\mathcal{A} - \frac{\not{p}_A}{m_p}\mathcal{B}\right]\frac{1}{\not{p}-m_p}\right\}, \tag{7}$$

and sitting at threshold we see that (6) involves the combination $\mathcal{A} + (m_A/m_p)\mathcal{B}$, and (4) contains $\mathcal{A} + \frac{9}{5}(m_A/m_p)\mathcal{B}$. If, as is likely, $\mathcal{A}$ and $\mathcal{B}$ are positive, identification of the two will underestimate Γ_A by a factor 1.5 ± 0.5.

Lastly, we require the projection of $p(x)$ on $\psi(x)$. We find

$$p_\alpha(x) = \sum_\beta F_{\alpha\beta}\psi_\beta(x),$$

$$F = \frac{\Psi(0,0)}{\sqrt{2}}\begin{pmatrix} 0 & 5 & 0 & -7 \\ -5 & 0 & 7 & 0 \\ 0 & 7 & 0 & -5 \\ -7 & 0 & 5 & 0 \end{pmatrix}, \tag{8}$$

where $\Psi(0,0)$ is the wave function for the three quarks in the proton to be at the same point.

Substituting into (4), and using the optical theorem, we find

$$\Gamma_A = \frac{37N}{6}\frac{g_{\rm GUT}^4}{M_x^2}\frac{\lambda^{1/2}({\rm th}, m_p^2, m_A^2)}{m_A\pi^2}\sigma_{\rm tot}({\rm th}, \bar{p}A)|\Psi(0,0)|^2,$$

where $N = 1$ to 3, and th means that we take λ, $\sigma_{\rm tot}$ at threshold. The total cross section, $\sigma_{\rm tot}({\rm th}, \bar{p}A)$, is measurable; but we may also relate it to the cross section $\bar{p}p \to$ all at rest using the well-known scaling law[9] $\sigma_{\rm tot}(\bar{p}A) = A^{2/3}\sigma_{\rm tot}(\bar{p}p)$, and the known values of the last.[10] Introducing also the enhancement factor[2,3]

$$As(f) = [g_s^2(1\ {\rm GeV})/g_{\rm GUT}^2]^{12/(33-2nf)},$$

we obtain

$$\Gamma_A = \frac{37NA^{2/3}}{6}\frac{g_{\rm GUT}^4}{m_p M_x^4}\Phi As(f)|\Psi(0,0)|^2,$$

and we have defined Φ by

$$\lambda^{1/2}({\rm th}, m_p^2, m_A^2)\sigma_{\rm tot}({\rm th}, \bar{p}A)/\pi^2 = A^{5/3}\Phi.$$

If we suppose that the wave function $\Psi(\vec{r}_1, \vec{r}_2)$ can be separated, as is the case in the more usual proton constituent models, we have the relation $|\Psi(0,0)|^2 = |\Psi(0)|^4$, where $|\Psi(0)|^2$ is the probability that two quarks be at the same point. So finally,

$$\tau_p = \frac{6}{37N}\frac{m_p M_x^4}{g_{\rm GUT}^4}\frac{A^{1/3}}{As(f)\Phi|\Psi(0)|^4},$$

to be compared with the standard two-quark fusion, $qq \to e^+\bar{q}$. Taking $M_X \simeq (4.5 \pm 1.5)\times 10^{14}$ GeV, the value[11] $|\Psi(0)|^2 \simeq 8\times 10^{-3}$ GeV^3, the value of Ref. 10 for Φ, $\Phi \simeq 16$, and[2,3] $A_s(6) \simeq 5$, we find

$$\tau_p = 1.2\times 10^{30}\ {\rm to}\ 7\times 10^{28}\ {\rm yr}. \qquad (9)$$

We see that the process we have considered dominates by a factor ~3 over the standard mechanism.[3] It should be noted, however, that the errors in our calculation are larger, and may very well decrease or increase the estimate by a factor 2 to 3. In view of these uncertainties we believe that the important result is not the decrease in the proton lifetime, but the realization that the dominant experimental signal (if protons decay at all) is likely to be the one produced by our mechanism. So we devote the last part of this Letter to discussing the experimental signal expected.

Unlike in the exclusive proton decay, where one expects as final state a positron and a hadron (π, ρ, ω, or η), monochromatic and back to back, here we have a positron with energy ranging from zero to m_p, the most likely value being around 0.6 GeV. Recoiling against this, we shall have a star of slowly moving nuclear fragments, among which there is some chance that we should find a pion. In fact, if a sizable part of the recoil energy is absorbed by a single spectator nucleon, the process will be dominated by the Δ_{33} resonance which will then decay into a nucleon and a pion with $E_\pi \simeq 350$ MeV; but the direction of the e^+ and this pion will not be strongly correlated. For neutron decay the only difference is the absence of the positron track, replaced by an unobservable neutrino. If the proton decays within the experimental acceptance, the inclusive three-quark fusion should provide an interesting complementary insight into the underlying grand unified structure [on which the matrix F of Eq. (8) depends strongly] apart from information on $\Psi(0, 0)$.

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[1]Experiments are being performed, or have been put forward, by the following institutions: Tata Institute for Fundamental Research, University of California at Irvine, Michigan State University, Brookhaven National Laboratory, Harvard University, Purdue University, University of Wisconsin, University of Minnesota, Laboratori Nazionali di Frascati, Università di Milano, Università di Torina, Centre d'Etudes Nucléaires de Saclay, and CERN. For a review see, e.g., the Proceedings of the Summer Institute on Particle Physics, Stanford Linear Accelerator Center Report No. 239, 1981 (unpublished).

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Higher-Twist Ambiguities in the Determination of $\sin^2\theta_W$

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It is shown how present measurements of $\sin^2\theta_W$ are hampered by contributions of higher-twist operators, which are even more important than recently calculated electroweak radiative corrections. The resulting limit of accuracy for measurements of $\sin^2\theta_W$ is estimated to be of the order of 5% at presently available energies.

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Interest in the accurate determination of $\sin^2\theta_W$ is largely motivated nowadays by the fact that this quantity is predicted in grand unified theories (GUT's). Therefore a great effort has recently gone into improving both the accuracy of the predictions of GUT's and the theoretical analysis of the experiments. We refer the reader to Marciano and Sirlin[1] for a summary of the relevant activity. In particular the theoretical analysis of the data was improved to the level of radiative QED corrections which were found to modify $\sin^2\theta_W$ by at most 5%. On the other hand no corrections for possible departures from the quark-parton model are available except for the parity-nonconserving left-right asymmetry in deep-inelastic electron-deuteron scattering.[2] It is our purpose to extend this investigation to the more accurate neutrino data presently available[3] and thus draw some conclusions on the ultimate accuracy expected for measuring $\sin^2\theta_W$ at presently available energies.

It should be stressed at this point that while the nonperturbative $O(\mu^2/Q^2)$ corrections due to the higher-twist contributions are of minor importance, even with $\mu^2 \simeq 0.5$ GeV^2, in studying quantum chromodynamic scaling violations[4] where they are to be compared essentially with $\alpha_s \simeq 0.2$, they can, however, turn out to be quite important when compared to quantities pretended to be theoretically understood to the level of $\alpha \simeq 1/137$.

To understand the appearance of higher-twist terms in neutrino reactions, recall that the Gla-

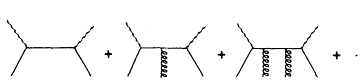

FIG. 1. Handbaglike diagrams.

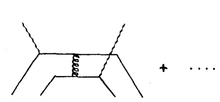

FIG. 2. Flavor-changing non-handbaglike higher-twist diagrams contributing to ϵ.