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Long-Range Spin-Spin Forces in Gauge Theories

J. Kogut

Istituto di Fisica "Guglielmo Marconi," Università degli Studi di Roma, I-00185 Rome, Italy, and Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

and

G. Parisi

Istituto Nazionale de Fisica Nucleare, Laboratori Nazionali di Frascati, I-00044 Frascati, Italy

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If the long-distance properties of pure gauge theories are described by thin strings, then quantum fluctuations induce a long-range spin-spin potential between widely separated heavy quarks. This potential falls off as the fifth power of the distance between the quarks and may be important in heavy-quark phenomenology.

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Recently the flux-tube (string) picture of confinement[1] has been refined significantly. On the basis of classical field-theory models, the original flux-tube picture suggested that when a quark and an antiquark are separated a distance R in a pure gauge theory, i.e., when quark-induced vacuum polarization is neglected, a static flux tube of finite intrinsic width will form between them. The quarks would then be confined by a linearly rising potential. However, studies in string models[2] and lattice gauge theories[3] have shown that when quantum fluctuations are accounted for, the transverse spatial distribution of the flux tube grows without bound when $R \to \infty$. It has been argued in all dimensions[2] and verified in $(2+1)$-dimensional models[4] that the mean squared transverse width of the spatial distribution grows as $\ln R$. One says that the flux tube "wanders without bound" or is "rough."[5]

This divergent spatial distribution can be related to the fact that the excitation spectrum of an infinite-length flux tube has a vanishing mass gap. The absence of a mass gap leads to power-law corrections to the linear confining potential $V(R)$. String-model calculations have shown that[2]

$$V(R) = \kappa R - \gamma/R + \cdots, \tag{1}$$

where κ is the string tension, $\gamma = (d-2)\pi/24$, and d is the number of space-time dimensions. Thus, quantum fluctuations give rise to a universal, attractive R^{-1} potential. The fact that γ does not depend on the gauge group or quark charges can be understood as a consequence of the energy scales in the problem. One pictures the string as having a small *intrinsic* width of the order of the reciprocal of the glueball mass M_g but a divergent spatial distribution. The R^{-1} contribution in Eq. (1) is generated by the long-wavelength vibrations of the string which do not excite the internal structure of the string itself. In fact, γ can be computed from the Casimir effect in a $(1+1)$-dimensional scalar field theory with $d-2$ degrees of freedom.[6] Equation (1) is expected to apply to gauge theories when $R \gg M_g^{-1}$.

It is very tempting to pursue this physical picture further and search for additional power-law corrections to the static flux-tube model. We are particularly interested in the possible existence of long-range *spin-spin* interactions mediated by the fluctuating flux tube. Recall that such effects are absent in the static model because the tube contains only static electric flux and because magnetic fields which could be induced by perturbing the tube cannot propagate significantly. It is plausible, however, that if the vanishing mass gap of the flux tube is accounted for, then magnetic effects could become long range. We shall compute the magnetic-magnetic correlation function below and verify this guess. Accounting for the spins of the heavy quarks at the ends of the flux tube, we shall find a contribution to the heavy quark potential which behaves as

$$\vec{\sigma}^{(a)}\cdot\hat{R}\,\vec{\sigma}^{(b)}\cdot\hat{R}/R^5 \tag{2}$$

for $R \gg M_g^{-1}$ where $\hat{R}=\vec{R}/|\vec{R}|$. Although the spin and spatial structure of this contribution to the heavy-quark potential will be found, the strength of the effect is model dependent. However, we compute its sign (positive) and find that it falls off as the fourth power of the heavy-quark mass. This potential may be comparable in importance to short-range single-gluon exchange and it appears to improve our theoretical expectations for ψ/J spectroscopy.

The magnetic-magnetic correlation function on the string.—In the static flux-tube approximation the effect of any local disturbance falls off exponentially with a decay length that is proportional to the radius of the string. Magnetic and electric correlations are short range. Consider how this picture is modified by fluctuations of the string. Now one describes the long-distance properties of a flux tube (string) with a free, massless field theory in $1+1$ dimensions. Let the heavy quarks be separated a distance R along the z axis. Then the string is described by a two-component field $\vec{\xi}(z,t)$ whose dynamics in the long-wavelength limit are controlled by the Gaussian action,

$$S=\int dz\,dt\,\partial_\mu\vec{\xi}\cdot\partial_\mu\vec{\xi}, \tag{3}$$

subject to the boundary conditions $\vec{\xi}(0,t)=\vec{\xi}(R,t)=0$. It is convenient to work in Euclidean space-time and we shall do so in all that follows. With use of Eq. (3) the correlation functions of $\vec{\xi}(z,t)$ are simple to compute. For example, for z near 0 and z' near R,

$$\langle\xi_i(z,t)\xi_j(z',t')\rangle \sim \frac{z(R-z')}{R^2}\,\frac{\delta_{ij}}{\cosh^2[\pi(t'-t)/2R]}, \tag{4}$$

where i and j run over the two transverse directions. Equation (4) follows from a simple exercise in two-dimensional electrostatics.

Since the electric flux is confined to the string and points along it, we have

$$E_i(z,t)\sim\partial\xi_i/\partial z \tag{5a}$$

and

$$\langle E_i(z,t)E_j(z',t')\rangle \sim \frac{\delta_{ij}}{R^2}\,\frac{1}{\cosh^2[\pi(t'-t)/2R]}, \tag{5b}$$

where $E_i(z,t)=\int dx\,dy\,E_i(x,y,z,t)$. Here and in all that follows we shall use the notation of Abelian flux. We believe that the SU(N) character of the theory does not affect our considerations in a serious way, and so we describe the fields with Maxwell's equations. The Maxwell equation $\vec{\nabla}\times\vec{B}=\dot{\vec{E}}$ implies that a magnetic field will be induced by fluctuations of the string. The fields $\vec{E}$ and $\dot{\vec{E}}$ in the vicinity of the fluctuating string are shown in Fig. 1. We imagine that the string fluctuates upward with the heavy quark fixed. Then a divergenceless field of $\dot{\vec{E}}$ occurs roughly as shown. A chromomagnetic field is induced perpendicular to the figure. From Maxwell's equation and the string relation Eq. (5a), the magnetic field in the vicinity of the heavy quark is

$$B_i\sim\epsilon_{ij}\,\partial\dot{\xi}_j(z,t)/\partial z\,. \tag{6}$$

Therefore, the magnetic-magnetic correlation function between the ends of the string is obtained

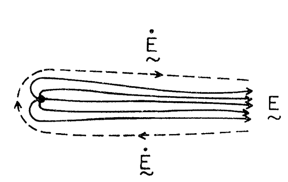

FIG. 1. The fields $\vec{E}$ and $\dot{\vec{E}}$ in the vicinity of the fluctuating string. The string is rotating upward in the plane of the figure about the heavy quark.

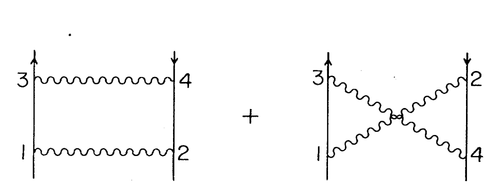

FIG. 2. Fourth-order contributions to the spin-spin interaction. The wavy lines indicate a magnetic-magnetic correlation.

by differentiating Eq. (4):

$$\langle B_i(0,t)B_j(R,t')\rangle \sim (\delta_{ij}-\hat{R}_i\hat{R}_j)\frac{1}{R^4}\left(\frac{2\sinh^2[\pi(t'-t)/2R]-1}{\cosh^4[\pi(t'-t)/2R]}\right). \tag{7}$$

The tensor structure of Eq. (7) enforces the constraint $\vec{R}\cdot\vec{B}=0$. It is also convenient to write Eq. (7) as

$$\langle B_i(z,t)B_j(z',t')\rangle \sim (\delta_{ij}-\hat{R}_i\hat{R}_j)\langle\dot{\varphi}_{,z}(z,t)\dot{\varphi}_{,z'}(z',t')\rangle, \tag{8}$$

where $\varphi_{,z}$ denotes $\partial\varphi/\partial z$ and φ satisfies $\Box\varphi(z,t)=0$ subject to the boundary conditions $\varphi(0,t)=\varphi(R,t)=0$.

Equation (7) applies at long distances $R \gg M_g^{-1}$. It has been assumed in the past that magnetic correlations are short ranged in gauge theories and that the single-gluon exchange approximation is valid. This assumption has been applied to studies of hadron spectroscopy.[7] In light of Eq. (7), these calculations are incomplete and could be misleading.

Long-range spin-spin interactions in gauge theories.—Using Eq. (7) we can calculate the spin-spin interaction between two heavy quarks a and b. The quarks couple locally to the chromomagnetic field through the interaction $\vec{\sigma}\cdot\vec{B}$. The second-order contribution to the heavy quark potential is

$$V^{(2)}(R) = -\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt_1\int_{-T}^{T}dt_2\,\sigma_i^{(a)}\sigma_j^{(b)}\langle B_i(R,t_1)B_j(0,t_2)\rangle. \tag{9}$$

It is easy to see that Eq. (9) vanishes identically. Consider the integral over t_2 and the fact that $B_i(0,t_2)\sim\epsilon_{ij}\,\partial\dot{\xi}_j(0,t_2)/\partial z$. Thus Eq. (9) is proportional to the mean velocity of a bit of string. But the string is just an ensemble of simple harmonic oscillators, and so this time average is zero.

To obtain a nonvanishing result we must proceed to fourth order and calculate the graphs in Fig. 2. With use of slightly abbreviated notation for clarity, the fourth-order shift in the action is

$$\begin{aligned}&\sigma_i^{(a)}\sigma_k^{(a)}\sigma_j^{(b)}\sigma_l^{(b)}\int dt_1\cdots dt_4\,\langle B_i(1)B_j(2)\rangle\langle B_k(3)B_l(4)\rangle\,\theta(3-1)\theta(4-2)\\&\quad+\sigma_i^{(a)}\sigma_k^{(a)}\sigma_l^{(b)}\sigma_j^{(b)}\int dt\cdots dt_4\langle B_i(1)B_j(2)\rangle\langle B_k(3)B_l(4)\rangle\,\theta(3-1)\theta(2-4).\end{aligned} \tag{10}$$

With use of the identity $\sigma_l\sigma_j=\sigma_j\sigma_l+[\sigma_l,\sigma_j]$ and the fact that $\int dt_4\langle B_k(3)B_l(4)\rangle=0$, Eq. (10) becomes

$$\sigma_i^{(a)}\sigma_k^{(a)}[\sigma_l^{(b)},\sigma_j^{(b)}]\int dt_1\cdots dt_4\,\langle B_i(1)B_j(2)\rangle\langle B_k(3)B_l(4)\rangle\,\theta(3-1)\theta(2-4). \tag{11}$$

Using the tensor structure of Eq. (8) one easily sees that only the antisymmetric part of $\sigma_i^{(a)}\sigma_k^{(a)}$ contributes to Eq. (11). So we now have

$$\begin{aligned}&[\sigma_i^{(a)},\sigma_k^{(a)}][\sigma_l^{(b)},\sigma_j^{(b)}](\delta_{ij}-\hat{R}_i\hat{R}_j)(\delta_{kl}-\hat{R}_k\hat{R}_l)\\&\qquad\times\int dt\cdots dt_4\,\langle\dot{\varphi}_{,z}(1)\dot{\varphi}_{,z}(2)\rangle\langle\dot{\varphi}_{,z}(3)\dot{\varphi}_{,z}(4)\rangle\,\theta(3-1)\theta(2-4).\end{aligned} \tag{12}$$

The t_3 and t_2 integrals are trivial:

$$\int_{t_1}^{T}\dot{\varphi}_{,z}(3)\,dt_3=-\varphi_{,z}(1),\quad \int_{t_4}^{T}\dot{\varphi}_{,z}(t_2)dt_2=-\varphi_{,z}(4), \tag{13}$$

and the Pauli algebra can be simplified to give

$$-\epsilon_{ikp}\sigma_p^{(a)}\epsilon_{ljq}\sigma_q^{(b)}(\delta_{ij}-\hat{R}_i\hat{R}_j)(\delta_{kl}-\hat{R}_k\hat{R}_l)\int dt_1\cdots dt_4\langle\dot{\varphi}_{,z}(1)\varphi_{,z}(4)\rangle\langle\varphi_{,z}(1)\dot{\varphi}_{,z}(4)\rangle. \tag{14}$$

But

$$\langle\dot{\varphi}_{,z}(1)\varphi_{,z}(4)\rangle=-\langle\varphi_{,z}(1)\dot{\varphi}_{,z}(4)\rangle=\frac{1}{R^3}\,\frac{\sinh[\pi(t'-t)/2R]}{\cosh^3[\pi(t'-t)/2R]} \tag{15}$$

and

$$\tfrac{1}{2}\epsilon_{ikp}\sigma_p^{(a)}\epsilon_{ljq}\sigma_q^{(b)}(\delta_{ij}-\hat{R}_i\hat{R}_j)(\delta_{kl}-\hat{R}_k\hat{R}_l)=-\vec{\sigma}^{(a)}\cdot\hat{R}\,\vec{\sigma}^{(b)}\cdot\hat{R}, \tag{16}$$

so that Eq. (12) becomes

$$-\vec{\sigma}^{(a)}\cdot\hat{R}\,\vec{\sigma}^{(b)}\cdot\hat{R}\int dt_1\,dt_4\langle\dot{\varphi}_{,z}(1)\varphi_{,z}(4)\rangle^2 \tag{17}$$

which behaves as

$$-(\vec{\sigma}^{(a)}\cdot\hat{R}\,\vec{\sigma}^{(b)}\cdot\hat{R}/R^5)T. \tag{18}$$

Dividing by the time interval T and changing the sign gives the potential

$$V(R)_{\text{spin}}=g^2\vec{\sigma}^{(a)}\cdot\hat{R}\vec{\sigma}^{(b)}\cdot\hat{R}/R^5, \tag{19}$$

where g^2 is a positive coupling constant whose magnitude is model dependent.[8] We note that $V(R)$ favors antialigned spins. It falls off as R^{-5} and can be understood as a van der Waals interaction. Note also that $g^2\sim m^{-4}$, where m is the heavy-quark mass, since Fig. 2 contains four magnetic moments.

Heavy-quark phenomenology.—It is interesting to see how $V(R)_{\text{spin}}$ effects heavy-quark phenomenology. Unfortunately Eq. (19) only applies when $R\gg M_g^{-1}$, and so its detailed form may not be revealed in heavy-quark spectroscopy which requires a potential for distances $\lesssim\frac{1}{3}f$. However, if $M_g\sim1.5$ GeV an application to ψ/J spectroscopy may not be too unreasonable and so we consider it briefly. First we need the matrix element of Eq. (19) in states $|JLS\rangle$. If we use the identity[9]

$$\hat{R}_i\hat{R}_k-\tfrac{1}{3}\delta_{ik}=-(L_iL_k+L_kL_i-\tfrac{2}{3}\delta_{ik}L^2)/(2l-1)(2l+3), \tag{20}$$

which is valid in a subspace of angular momentum l, and the identity[10]

$$s_i^{(a)}s_j^{(b)}+s_j^{(a)}s_i^{(b)}=-\tfrac{1}{2}\delta_{ij}+\tfrac{1}{2}(S_iS_j+S_jS_i), \tag{21}$$

where $\vec{s}^{(a)}=\frac{1}{2}\vec{\sigma}^{(a)}$, $\vec{s}^{(b)}=\frac{1}{2}\vec{\sigma}^{(b)}$, and $\vec{S}=\vec{s}^{(a)}+\vec{s}^{(b)}$, then Eq. (19) becomes

$$V(R)_{\text{spin}}=-\frac{g^2}{R^5}\left[(\vec{L}\cdot\vec{S})^2+\tfrac{1}{2}(\vec{L}\cdot\vec{S})-\tfrac{1}{2}\vec{L}^2-\vec{s}^{(a)}\cdot\vec{s}^{(b)}(2l^2+2l-1)\right]/(4l^2+4l-3). \tag{22}$$

It would be interesting to include Eq. (22) into a potential-model calculation of the ψ/J family of $L=0$, 1, and 2 states. We will limit our discussion to a few observations. Consider the χ family of ${}^3P_J$ states. Equation (22) gives a splitting pattern

$$r\equiv[m(^3P_2)-m(^3P_1)]/[m(^3P_1)-m(^3P_0)]=-\tfrac{2}{5} \tag{23}$$

if $\langle R^{-5}\rangle$ is taken to be the same for all members of the multiplet. Pure $\vec{L}\cdot\vec{S}$ coupling expected from the classical spin-orbit effect gives $r=2$ while pure one-gluon exchange gives $r=\frac{14}{25}$.[11] Since the experimental value is $r_{\text{exp}}=0.506\pm0.018$, some admixture of V_{spin} as suggested by Eq. (22) is welcome. Perhaps the unexpectedly small value of r_{exp} is evidence of the dynamical character of the quark-confining mechanism of gauge theories.

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