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Electron Dynamics Associated with Stochastic Magnetic and Ambipolar Electric Fields

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A conservative transport operator in $(r, v_\parallel, v_\perp)$ space and moment equations are derived which describe the combined effects of a stochastic magnetic field and a radial electric field on the electrons. The transport operator is coupled with Fokker-Planck and Ohmic-heating terms to compute the distribution function. A physical picture emerges which emphasizes the possible importance of the ambipolar potential ($\sim T_e/e$) and plasma boundary physics in determining overall plasma confinement.

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The transport of electron energy out of tokamaks is known to be orders of magnitude greater than predicted by the classical and neoclassical theories, whereas ion energy transport is approximately neoclassical. Theoretically, it has been found that very small nonaxisymmetric perturbations to the flux surfaces can lead to magnetic field lines which, after many circuits around the torus, randomly wander out to the plasma edge. The fast transport of electrons parallel to the magnetic field is thus coupled to radial transport.[1,2] Radial magnetic perturbations of magnitude $\delta B/B \sim 10^{-4}$ have been measured,[3,4] and are consistent with experimentally observed electron-energy transport rates. It has been suggested that the rapid transport of electrons relative to ions gives rise to a positive radial ambipolar electric field,[1,2,5] adjusting the electron flow rate to equal the ion rate. Contrary to this, measurements of the plasma potential in the core of the ST tokamak indicated a negatively charged plasma so as to expel the electrons[6]; however, a positive potential was inferred from T3. Positive plasma potential of $\sim 10\,T_e/e$ has been measured in the Caltech tokamak at minor radius $\approx 2a/3$.[4]

Mathematical descriptions of the combined effect of the stochastic magnetic field and ambipolar electric field on the electron distribution function have been proposed.[7,8] In this Letter we derive such a transport operator from the guiding-center kinetic equation, obtain moment equations, and computationally solve for the electron distribution. Considering the electron distribution as a function of $\epsilon = \frac{1}{2}mv_\parallel^2/2 + \mu B - e\varphi$, $\mu = \frac{1}{2}mv_\perp^2/B$, and r, the guiding-center kinetic equation[9] is simply $\partial f/\partial t + \vec{v}\cdot\nabla f = 0$, where $\vec{v}$ is the guiding-center velocity and the gradient is taken at constant ϵ, μ. Neglecting drifts perpendicular to the magnetic field and using slab geometry for notational convenience, the equation $\partial f/\partial t + v_z(\partial f/\partial z + b_x \partial f/\partial x + b_y \partial f/\partial y) = 0$ is obtained, where $\vec{B} = \vec{B}_z + \vec{B}'$, $b_x = B_x'/B_z$, and $b_y = B_y'/B_z$. Letting $f = \langle f\rangle + f'$, where $\langle f\rangle$ is an average at constant x, and taking the y, z coordinates to be ignorable for $\langle f\rangle$ and $\vec{B}_z$, it follows that

$$\partial\langle f\rangle/\partial t + v_z\langle b_x \partial f'/\partial x + b_y \partial f'/\partial y\rangle = 0 + O(\delta^3), \quad (1)$$

where $\delta = |\vec{B}'|/B_z$, and

$$\partial f'/\partial t + v_z\,\partial f'/\partial z + v_z b_x\,\partial\langle f\rangle/\partial x = 0 + O(\delta^2), \quad (2)$$

where $v_z = \text{const} + O(\delta^2)$. Perturbations are assumed to be time independent. Integrating Eq. (2)

over the unperturbed orbit $z = z_0 + v_z t$, taking the initial-value term $f'(z_0)$ equal to zero, and substituting the result into Eq. (1) gives

$$\frac{\partial\langle f\rangle}{\partial t} = v_z \langle b_x(z) \frac{\partial}{\partial x} \int_{z_0}^{z} dz'\, b_x(z') + b_y(z) \frac{\partial}{\partial y} \int_{z_0}^{z} dz'\, b_x(z')\rangle \frac{\partial\langle f\rangle}{\partial x}. \quad (3)$$

Since $\nabla \cdot \vec{B} = 0$, the partial derivatives before the integrals can be moved through b_x and b_y, respectively. In cylindrical coordinates, the rate of change of $\langle f\rangle$ due to transport is

$$\left(\frac{\partial\langle f\rangle}{\partial t}\right)_{\rm tr} = \frac{|v_\parallel|}{r} \mathcal{L} r D_{\rm st} \mathcal{L}\langle f\rangle, \quad (4)$$

where $\mathcal{L} \equiv (\partial/\partial r)|_{\epsilon,\mu} = (\partial/\partial r)|_{v_\parallel, v_\perp} - (eE_A/mv_\parallel)(\partial/\partial v_\parallel)|_{v_\perp, r}$, $E_A = -(\partial\varphi/\partial r)$, the stochastic magnetic field diffusion coefficient $D_{\rm st} = \langle b_r(l)\int_{l_0}^{l} dl'\, b_r(l')\rangle$, $b_r = B_r'/\langle B\rangle$, and l' is the distance along an unperturbed magnetic field line.

A critical point not previously recognized with regard to a conservation form for $(\partial\langle f\rangle/\partial t)_{\rm tr}$ is that the diffusion coefficient $D_{\rm st}$ goes to zero as $v_\parallel \to 0$, since $l \to l_0$. If $D_{\rm st}$ does not have this property, a spurious source is in general introduced at $v_\parallel = 0$. To cut off $D_{\rm st}$ as $v_\parallel \to 0$, we have added a Krook collision term to Eq. (2) to obtain $D_{\rm st} = \pi R_{\rm eff}\langle b_r^2\rangle$, where $R_{\rm eff}^{-1} \approx L_A^{-1} + \lambda_{\rm mfp}^{-1}$, L_A is the autocorrelation length of the magnetic field $L_A = \langle b_r(l)\int_{-\infty}^{\infty} dl'\, b_r(l')\rangle$, and $\lambda_{\rm mfp} = v_\parallel/v_{ei}$ is the mean free path for collisions. This dependence of $D_{\rm st}$ on $v_\parallel$ has been studied in detail by Krommes.[10]

By taking the zeroth and second moments of Eq. (4), assuming the distribution is locally Maxwellian, new particle and energy transport relations are obtained: $\partial n/\partial t = -r^{-1}\partial(r\Gamma_r)/\partial r$ and $\partial(\frac{3}{2}nT)\partial t = -eE_A\Gamma_r - r^{-1}\partial(rQ_r)/\partial r$, where

$$\Gamma_r = -\left(\frac{2}{\pi}\right)^{1/2} D_{\rm st} v_T \left(\frac{1}{n}\frac{\partial n}{\partial r} + \frac{1}{2T}\frac{\partial T}{\partial r} + \frac{eE_A}{T}\right) n, \quad (5)$$

$$Q_r = -2\left(\frac{2}{\pi}\right)^{1/2} D_{\rm st} v_T \left(\frac{1}{n}\frac{\partial n}{\partial r} + \frac{3}{2T}\frac{\partial T}{\partial r} + \frac{eE_A}{T}\right) nT, \quad (6)$$

and $v_T = (T/m)^{1/2}$.[11] We have assumed $\lambda_{\rm mfp} \gg L_A$ for all but the lowest $v_\parallel$ particles. In order to complete the particle and energy equations appropriate sources may be added. Several noteworthy features arise. At constant T, the relation between particle diffusion and mobility η in Eq. (5) is the Einstein relation $\eta/D = e/T$. If it is assumed that the electron particle flux in Eq. (5) is the difference between the relatively large gradient and ambipolar field terms, the small difference being equal to the ion-particle flux, then setting $\Gamma_r \approx 0$ shows that E_A is $-(T/e)(\partial/\partial r) \times[\ln(nT^{1/2})]$ with no explicit dependence on $D_{\rm st}$, and gives the particularly simple energy transport term $Q_r = -2(2/\pi)^{1/2} D_{\rm st} v_T n(\partial T/\partial r)$.

The computational model solves for the steady-state solution of the equation

$$\frac{\partial \bar{f}(r,v,t)}{\partial t} = \frac{1}{rv}\mathcal{L}' r v^2 \bar{D}_{\rm st}\mathcal{L}'\bar{f} + \mathcal{C}(\bar{f}) + p_\Omega(v), \quad (7)$$

where $\bar{f}$ is assumed to be independent of $\xi = v_\parallel/v$. The first term of the right-hand side is then obtained from Eq. (4) by averaging over a sphere in velocity space at constant v (i.e., $\frac{1}{2}\int_{-1}^{+1} d\xi \equiv \langle\ \rangle_\xi$), giving $\mathcal{L}' = \partial/\partial r - (eE_A/mv)\partial/\partial v$, and $\bar{D}_{\rm st} = \langle D_{\rm st}\rangle_\xi$. $\mathcal{C}$ is the ξ average of the full nonlinear electron-electron and electron-ion collision operator,[12] and $p_\Omega(v) = \langle\langle (e\vec{E}_\Omega/m)\cdot\partial f_{\rm SH}/\partial\vec{v}\rangle_\xi$, where $f_{\rm SH}$ is the Spitzer-Härm[13] distribution function and $\vec{E}_\Omega$ is a steady Ohmic-heating electric field.

The boundary conditions employed are zero flux through the boundaries at $r = 0$ and $v = 0$, and $f(r, v\approx\infty) = 0$, and a low-temperature Maxwellian distribution at the plasma boundary $r = a$. It is assumed that the ions are immobile, thereby fixing the density profile, and E_A is adjusted to obtain quasineutrality, $n_e(r) = n_i(r)$. E_{DC} is chosen according to a specified equivalent total plasma current. $\bar{D}_{\rm st}(r)$ is chosen in conformity with experimental values of total energy confinement times and $T_e(r)$ profiles ($T_e \equiv \frac{1}{3}\int d^3v\, mv^2\bar{f}$).

Equation (7) is treated fully implicitly, and the resulting nine-point difference scheme is solved by the conjugate gradient algorithm with an incomplete LU preconditioning of the matrix.[14] The ambipolar electric field is adjusted to obtain quasineutrality by setting $(4\pi/r)(\partial/\partial r)\{r\int_0^\infty dv \times[(\partial/\partial r)(D_{\rm st}v^3\bar{f}) - (eE_A/m)D_{\rm st}v^2\partial\bar{f}/\partial v]\}$, which gives $\partial n/\partial t$, equal to zero. Since $E_A = 0$ at $r = 0$, this allows the recursive calculation of E_A at all radii.

A moderate density ($n_{e0} = 2.0\times 10^{14}$ cm^{-3}) Alcator-A discharge is modeled. The density profile is chosen to be parabolic with edge density equal to $0.1 n_{e0}$. $T_i(r)$ is parabolic squared with $T_{i0} = 700$ eV. The radial variation of the particle diffusion coefficient $v_T\bar{D}_{\rm st}$ is made proportional to $(n_eT_e)^{-1}$ with a central value of $\delta B/B = 10^{-4}$ and the autocorrelation length $L_A = \pi R$, R being the major radius. The total plasma current is 250 kA. The resultant T_e profile ($T_{e0} = 1300$ eV) is more peaked than the density profile, as is typical of the experimental results.[15] Figure 1 shows

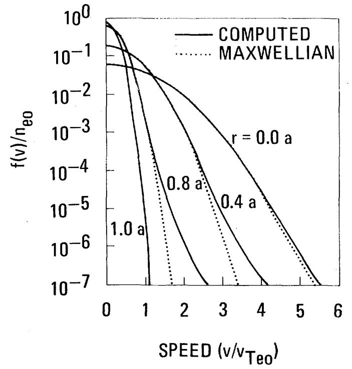

FIG. 1. Electron velocity distribution function at various plasma radii.

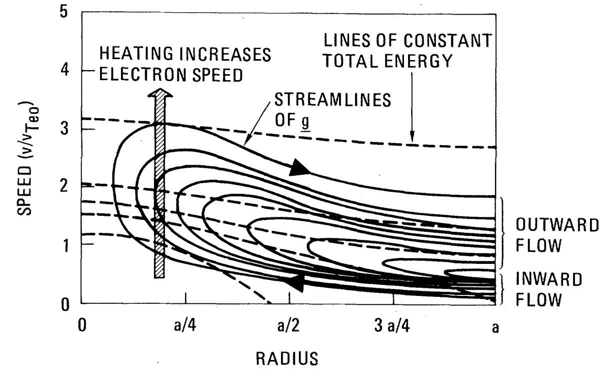

FIG. 2. Streamlines of electron flow in (r,v) space. Also illustrated are characteristics of the transport operator, i.e., lines of constant total energy.

the resulting electron velocity distribution at various radii. At the plasma center ($r=0$) the distribution is closely Maxwellian, whereas towards the boundary an electron tail builds up due to transport of the hot electrons from the plasma center through the cooler edge plasma.[7] The moment equations have been compared with the code results; the results for the ambipolar potential and the energy flux agree within $\approx 10\%$ for these parameters.

Physical insight can be obtained by considering electron flow in (r,v) space. In steady state, Eq. (7) can be written $\nabla_{rv}\cdot\vec{g}=0$, where the divergence operator is in (r,v) space and the flux vector $\vec{g}$ contains the transport, collisional, and Ohmic terms. Then there exists a stream function, shown in Fig. 2, whose contours are streamlines of $\vec{g}$. Near the plasma center the Ohmic heating moves particles to higher velocity. The enhanced population of tail electrons diffuses outward to the plasma boundary approximately along lines of constant total energy. The depleted population of low-velocity electrons near the plasma center is replenished by colder plasma diffusing inward from the boundary, and the time for replacement of the electrons is approximately $2\tau_{Ee}$. At the plasma center the ambipolar potential value is $\approx T_e/e$, and this potential results in an electrostatic energy exchange between the outgoing and incoming electrons.

The soft-x-ray (SXR) spectrum resulting from $\bar{f}(r,v)$ is shown in Fig. 3. The minimum plasma radius of the SXR viewing cone is $r_{\min}$. At $r_{\min}=0$, the experimentally observed enhancement of the SXR count rate beyond 3 keV is not reproduced. However, for larger $r_{\min}$, the nonthermal enhancement arising from the hotter central plasma diffusing through the cooler plasma increases markedly in agreement with the theory of Ref. 7 and with experiment.[16]

That this theory fails to produce a nonthermal tail at $r_{\min}=0$ can be inferred from Fig. 2. At the plasma center the transport operator acts only to remove particles from the tail, as indicated by the outward flow lines of $\vec{g}$ at high v. The observed nonthermal spectra at $r_{\min}=0$ occur during the rising density phase of the Alcator discharge,[16] and therefore may be further associated with time-dependent aspects of the discharge.

Speculating on some possible implications of this work, we think that the positive ambipolar potential may be sufficient to drive out the impurities. By applying the above theory to impurity ions rather than electrons[17] and using $\delta B/B = 3\times 10^{-4}$ at the plasma center, the neoclassical inward convection of impurities is reversed and results consistent with the Alcator-A Si-injection[18] experiments are obtained. Also, it is seen from Eq. (5), assuming $\Gamma_r=0$, that decreasing

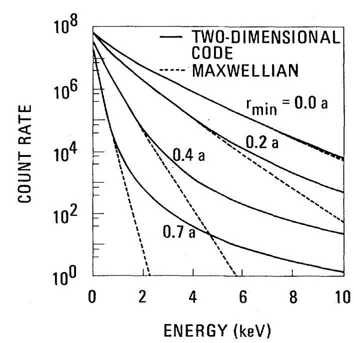

FIG. 3. Soft-x-ray spectra computed from $f(r,v)$ with use of the bremsstrahlung cross section.

the edge density or increasing the central T_e will increase E_A. To the extent that E_A controls outward transport of the main ion species, the plasma density may be decreased consistent with gas puffing[15] and electron cyclotron heating.[19] Finally, this theory suggests a need for further plasma potential measurements.

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$$E_A' = E_r + \langle B\rangle \sum_{\vec{k}} D_{\mathrm{st}\,\vec{k}} (\omega_{\vec{k}}/k_\theta c) / \sum_{\vec{k}} D_{\mathrm{st}\,\vec{k}},$$

where E_r is the actual radial field,

$$D_{\mathrm{st}\,\vec{k}} + |v_z| |b_{r\vec{k}}|^2 R(k_z v_z - c k_\theta E_r / \langle B\rangle - \omega - i\nu),$$

and $R(\cdots)$ is a resonance function.

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