Quasilinear Mechanism of High-Energy Ion-Tail Formation in the Ion-Acoustic Instability

Osamu Ishihara and Akira Hirose
Department of Physics, University of Saskatchewan, Saskatoon, Saskatchewan S7N 0W0, Canada
(Received 24 November 1980)

It is shown that quasilinear diffusion in velocity space can fully account for the formation of a high-energy ion tail in current-driven ion-acoustic instability. The mechanism is effective for a relatively small turbulence level, $W/nT_e \simeq 10^{-3}$.

PACS numbers: 52.35.Dm, 52.35.Py

In current-driven turbulent-heating experiments, it is commonly observed that a high-energy tail in the ion velocity distribution is formed.[1] The ion population in the tail amounts to almost 10% of the whole population, and the characteristic temperature of the high-energy ions is comparable to the electron temperature. Several mechanisms have been proposed to explain this phenomenon, including resonance broadening[2] and ion trapping,[3] which are higher-order nonlinear effects than the quasilinear effects. Little attention has been paid in the past to this lowest-order nonlinear effect. Recently, we have shown[4] that even the strong Buneman instability is successfully explained by the "generalized" quasilinear theory, provided that mode coupling is properly included.

In this note, we reexamine the mechanism of high-energy ion-tail formation strictly in terms of quasilinear theory. The results are in excellent agreement with those found in a recent computer simulation.[5] We consider a uniform, collisionless, unmagnetized plasma in which a constant current generates ion-acoustic instability.

For simplicity, we assume that the electrons maintain a Maxwellian structure

$$f_e(v,t)=\left(\frac{m}{2\pi T_e(t)}\right)^{1/2}\exp\left(-\frac{m(\vec{v}-\vec{V}_d)^2}{2T_e(t)}\right), \tag{1}$$

where $\vec{V}_d$ (a constant) is the drift velocity and T_e is the electron temperature. This Maxwellian assumption[6] can be justified if we note that the electron correlation time in ion-acoustic waves is practically the electron plasma period, $1/\omega_{pe}$, which makes it impossible for the electrons to be trapped by the ion-acoustic waves, and the distribution of electrons can become rapidly Maxwellian. The time-dependent electron temperature takes into account the turbulent heating caused by the anomalous resistivity. It is straightforward to show that for the electrons the nonresonant interaction with waves is negligible compared with the resonant interaction if the growth rate of the unstable mode γ_k satisfies the condition $\gamma_k \ll kv_e$, where $v_e=(2T_e/m)^{1/2}$ is the electron thermal velocity. Thus, using the quasilinear equation

$$\frac{\partial f_e}{\partial t}=\frac{8\pi^2e^2}{m^2}\frac{\partial}{\partial v}\sum_k \mathcal{E}_k(t)\,\delta(\omega_k-\vec{k}\cdot\vec{v})\frac{\partial f_e}{\partial v},$$

we obtain the equation which governs the temporal change of the electron temperature in a form

$$n_0\frac{dT_e}{dt}=(32\pi)^{1/2}\left(\frac{V_d}{v_e}\right)^2\omega_{pe}\sum_k\frac{k_{De}}{k}\mathcal{E}_k(t)\left(1-\frac{\omega_k}{kV_d}\right)^2, \tag{2}$$

where $\mathcal{E}_k=|E_k|^2/16\pi$ is a spectral energy density associated with the electric field fluctuation, k_{De} is the electron Debye wave number $k_{De}=(4\pi n_0e^2/T_e)^{1/2}$, and $\omega_{pe}=(4\pi n_0e^2/m)^{1/2}$ is the electron plasma frequency. The oscillation frequency ω_k can be determined from the real part of the dielectric response function,

$$\epsilon_r(k,\omega_k)=1+\frac{k_{De}^2}{k^2}\left(1-\frac{2V_d^2}{v_e^2}\right)-\frac{\omega_{pi}^2}{\omega_k^2}=0. \tag{3}$$

The ion distribution function $f_i(v,t)$ evolves according to the quasilinear equation[7]

$$\frac{\partial f_i}{\partial t}=\frac{8\pi e^2}{M^2}\sum_k\mathcal{E}_k(t)\frac{\partial}{\partial v}\frac{\gamma_k(t)}{[\omega_k(t)-kv]^2+\gamma_k^2(t)}\frac{\partial f_i}{\partial v}, \tag{4}$$

with

$$\frac{1}{2\mathcal{E}_k}\frac{\partial\mathcal{E}_k}{\partial t}=\gamma_k=\sum_s\gamma_s=\frac{\pi}{\partial\epsilon_r/\partial\omega_k}\sum_s\frac{\omega_{ps}^2}{k^2}\int dv\,k\frac{\partial f_s}{\partial v}\,\delta(\omega_k-kv), \tag{5}$$

where ω_{ps} is the plasma frequency of particle species s.

Equations (1)–(5) complete the description of the time development of the system. We have numerically solved these simultaneous differential equations assuming that initially, electrons and ions obey Maxwellian distribution functions,

$$f_e(t=0)=\left(\frac{m}{2\pi T_{e0}}\right)^{1/2}\exp\left[-\frac{m(v-V_d)^2}{2T_{e0}}\right],$$

$$f_i(t=0)=\left(\frac{M}{2\pi T_{i0}}\right)^{1/2}\exp\left(-\frac{Mv^2}{2T_{i0}}\right),$$

where T_{s0} is the initial temperature of species s. Figures 1–3 show the results obtained for a typical set of plasma parameters, $T_{i0}/T_{e0}=0.01$, $V_d/v_e=0.2$, $\sum \mathcal{E}_k/nT_{e0}=10^{-5}$, $M/m=1836$, and twenty modes (k values) equally spaced between $k_1=k_0/10$ and $k_{20}=2k_0$, where $k_0^2=k_{De0}^2/2$. The wave numbers are chosen such that the several modes can be included around the fastest-growing mode. As the instability evolves, the electrons are heated, and the growth-rate profile can vary with time. The time evolution of the ion distribution function is shown in Fig. 1. The high-energy ion tail formation can be clearly seen. The tail extends toward higher energy with time and it is apparent that a significant number of ions diffuse toward the ion-acoustic velocity by $\omega_{pi}t=60$. The development of a two-temperature distribution is characterized by its relatively cold bulk ions ($T_{ic}=0.022\,T_e \ll T_e$) and high-energy tail whose temperature is almost the electron temperature, $T_{ih}=0.82T_e$. The evolution of the k spectrum is shown in Fig. 2. The spectrum grows around $k=k_0=k_{De0}/\sqrt{2}$, the initial fastest growing mode, with a slight shift to a larger value of k, since the Debye wave number k_{De} becomes smaller because of turbulent electron heating. The cutoff of the spectrum at larger k values is due to the ion Landau damping. We expect that from the formation of the ion tail the interaction between waves and ions starts to play an important role and to cause the quenching of the instability. As is shown in Fig. 3, the growth of the instability becomes slower (no saturation state is attained in our computation time), although the electrons are continuously heated.

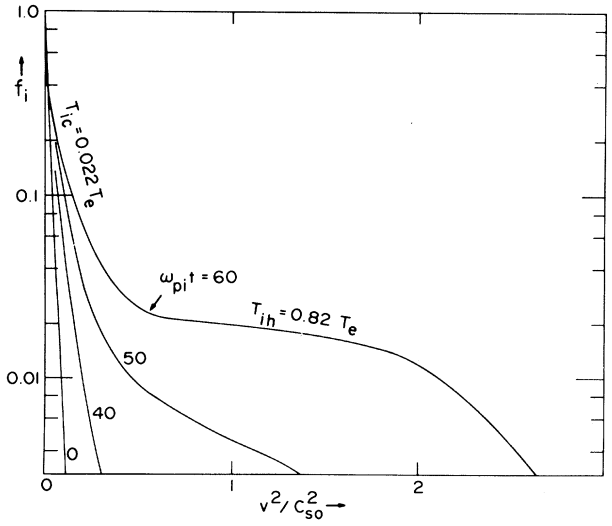

FIG. 1. Time development of ion energy distribution. $M/m=1836$, $(T_i/T_e)_{t=0}=0.01$, $V_d=0.2(2T_{e0}/m)^{1/2}$, and $W(0)/nT_{e0}=10^{-5}$.

The analytical model can be checked by following the phase trajectories of test ions placed in the growing field obtained from the analytical model. This method has been used in our previous work on Buneman instability.[4] The equation of motion

$$\frac{d^2x}{dt^2}=\sum_k \frac{eE_k(0)}{M}\exp\left(\int_0^t \gamma_k\,dt'\right)\cos\left(kx-\int_0^t \omega_k\,dt'+\varphi_k\right),$$

where φ_k is a random phase, can be integrated to follow the ion trajectory in the phase space. The results are shown in Fig. 4 for 10^3 particles. During the early time, up to $t=40\omega_{pi}^{-1}$, ion heating can hardly be seen. However, after $t=50\omega_{pi}^{-1}$, a small fraction of particles in the space-averaged ion distribution extends to larger velocities, reaching or even exceeding the ion acoustic velocity C_{s0}. Since we have employed a rather

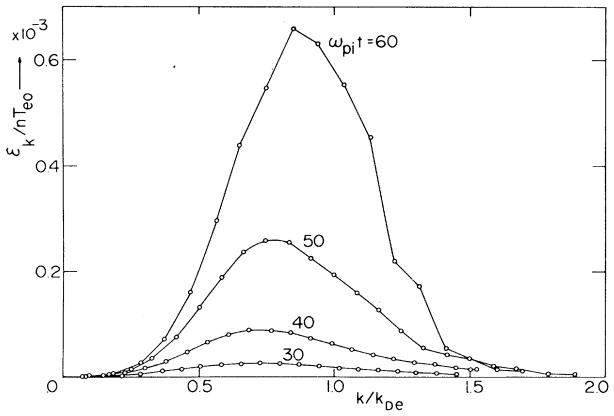

FIG. 2. Time evolution of wave spectrum.

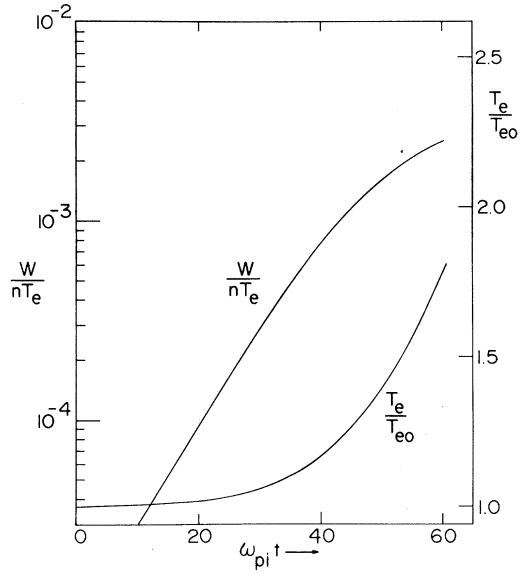

FIG. 3. Wave energy and electron temperature as a function of time.

small number of test particles, it is not possible to do a quantitative comparison between the analytic model and the trajectory calculation. However, the behavior of test particles is in qualitative agreement with the analytical model as can be clearly seen.

To clarify the role played by the ions through resonant and nonresonant interaction with waves, we replace Eq. (4) by

$$\frac{\partial f_i}{\partial t} = \frac{\partial}{\partial v}(D_R + D_{NR})\frac{\partial f_i}{\partial v}, \tag{6}$$

where

$$D_R = \frac{8\pi^2 e^2}{M^2}\sum_k \mathcal{E}_k(t)\,\delta(\omega_k - kv)$$

in the resonant region ($\omega_k \simeq kv$) of velocity space and

$$D_{NR} = \frac{8\pi e^2}{M^2}\sum_k \mathcal{E}_k(t)\,\frac{\gamma_k}{(\omega_k - kv)^2}$$

in the nonresonant region ($|\omega_k - kv| \gg \gamma_k$) of velocity space. The set of equations (1)–(3), (5), and (6) has been solved numerically by using the approximations

$$\delta(x) \approx \frac{1}{\pi}\frac{\eta}{x^2+\eta^2}\ (\nu \to 0)$$

and

$$\frac{1}{x^2} \approx \frac{x^2-\eta^2}{(x^2+\eta^2)^2}\ (\eta \to 0).$$

The small parameter η is chosen to join the diffusion coefficients of the resonant part and the nonresonant part at $|\omega/k - v| = 2\eta$. First, we suppress the resonant term artifically to see the effect caused by the nonresonant diffusion. The solution, which includes both effects of resonant

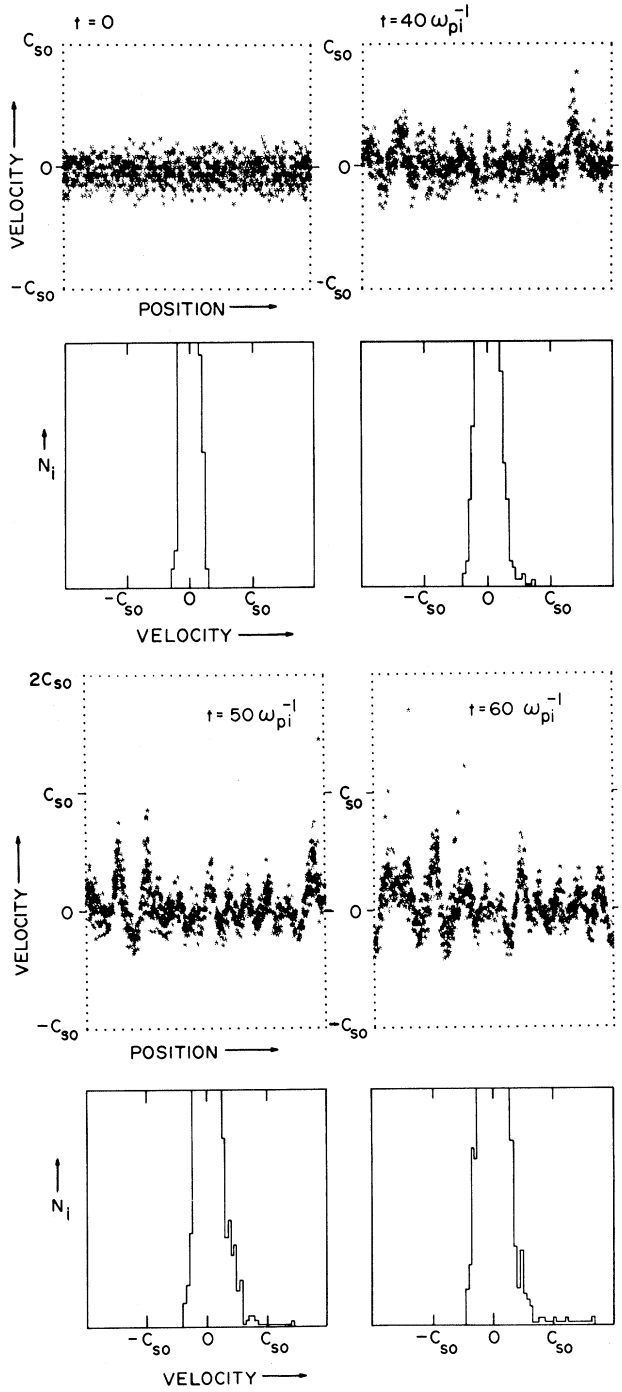

FIG. 4. Ion distribution in phase space and spatially averaged distribution function.

and nonresonant diffusion, is then calculated. It should be noted that the wave spectrum employed in our analysis contains a wide range in k space and there exist waves with small phase velocity, e.g., $v_{ph} = \omega_k/k \approx 0.6C_s$ for $k = k_{20}$, which can directly interact with ions. We find that the nonresonant diffusion is essential for the ion-tail formation. It is the nonresonant quasilinear diffusion that brings a small portion of ions into the ion-acoustic velocity range. Once an ion tail is formed by the nonresonant diffusion, the resonant interaction pulls more ions from the bulk into the tail.

In conclusion, we have found that quasilinear diffusion in velocity space is sufficient to explain the mechanism of high-energy ion-tail formation in current-driven ion-acoustic instability. The nonresonant quasilinear diffusion plays an important role in the early stage, and later both nonresonant diffusion mechanisms become important.

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

[1]C. Wharton, P. Korn, D. Prono, S. Robertson, P. Auer, and C. T. Dum, in *Proceedings of the Fourth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, 1971* (International Atomic Energy Agency, Vienna, Austria, 1972), Vol. II, p. 25; H. de Kluiver, C. J. Barth, H. J. B. M. Brocken, J. J. L. Caarls, B. de Groot, H. W. Kalfsbeek, H. W. Piekaar, A. Ravestein, W. R. Rutgers, B. de Stigter, H. W. H. Van Andel, and H. W. Van der Ven, in *Proceedings of the Seventh International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Innsbruck, Austria, 1978* (International Atomic Energy Agency, Vienna, Austria, 1979), Vol. II, p. 639.

[2]M. Z. Caponi and R. C. Davidson, Phys. Fluids 17, 1394 (1974); T. Hatori and R. Sugihara, J. Phys. Soc. Jpn. 39, 808 (1975); D. Choi and W. Horton, Jr., Phys. Fluids 18, 858 (1975).

[3]W. M. Manheimer and R. W. Flynn, Phys. Fluids 17, 409 (1974).

[4]O. Ishihara, A. Hirose, and A. B. Langdon, Phys. Rev. Lett. 44, 1404 (1980).

[5]C. T. Dum, R. Chodura, and D. Biskamp, Phys. Rev. Lett. 32, 1231 (1974).

[6]The electron distribution function revealed in computer simulations (see, for example, Ref. 5) is not pure Maxwellian and flattening at lower velocities takes place. This effect slightly reduces the growth rate of the instability and consequent complete stabilization which was not observed in the present study. As long as the mechanism of high-energy ion-tail formation is concerned, the small change in the growth rate does not appreciably affect the ion dynamics, and the Maxwellian assumption does not cause significant errors. Non-Maxwellian electron distribution, however, does govern the period of wave growth and final saturation. We are currently working on this problem by removing the Maxwellian constraint, and assuming a two-dimensional plasma. Results will be reported in a separate paper.

[7]W. E. Drummond and D. Pines, Nucl. Fusion, Suppl. 3, 1049 (1962).

Vibrational Excitations in *a*-Si:F and *a*-Si:(F,H) Alloys

Bal K. Agrawal

Department of Physics, University of Allahabad, Allahabad 211002, Uttar Pradesh, India

(Received 4 September 1980)

The vibrational excitations, both in the Si-bulk band mode region and above it at the impurity centers in the various configurations for the *a*-Si:F and *a*-Si:(F,H) alloys, have been obtained by using a cluster Bethe-lattice method. The predicted frequencies are in excellent agreement with the available experimental data.

PACS numbers: 63.50.+x, 72.80.Ng

Amorphous silicon (*a*-Si) is well known to be a very promising material for low-cost solar cells and field-effect transistors (FET's) for display panels. Hydrogenated *a*-Si can be substitutionally doped, in contrast to *a*-Si, but the doping efficiency is much smaller because of, in part, the presence of the appreciable density of localized states in the energy gap of[1] *a*-Si; however, this density is reduced by the incorporation of hydrogen, but not completely removed. Madan and coworkers[2] have reported that the addition of fluorine to *a*-Si:H increases the doping efficiency of the material, and also that the *a*-Si:(F,H) alloys have much better thermal and mechanical stabil-