the spectator electron.

In conclusion, I have observed the positronium negative ion, thus making available for study a new purely leptonic system with many interesting measurable properties.

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Range and Spectral-Density Measurements of Very Slow Muons

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Range and spectral density of very slow negative muons have been measured with novel techniques. The intensity around and below 1 keV agrees with semiclassical calculations but contradicts some quantum-mechanical theories of the Coulomb capture.

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The slowing down and Coulomb capture of negative mesonic particles has found widespread interest since the pioneering work of Fermi and Teller.[1] The per-atom capture rate at energies W between W_0 and W_1 with $W_0 > W_1$ is given by

$$R(W_0, W_1) = \int_{W_1}^{W_0} n(W)\sigma(W)\,dW, \tag{1}$$

where $n(W)$ is the spectral flux density, that is, the number of particles per unit energy and unit time entering a small sphere divided by the cross section of the sphere, and $\sigma(W)$ is the capture cross section. With neglect of capture, scattering, and straggling at $W > W_1$, $n(W_1)$ is correlated to $n(W_0)$ by

$$n(W_1)S(W_1) = n(W_0)S(W_0), \tag{2}$$

where $S(W)$ is the stopping power.[2] Straggling is, in our experiment, compensated for by the spread of the energy spectrum at any value taken for W_0. Scattering can be corrected for. Hence $S(W)$ can be measured with the help of $n(W)$ for energy values W_1 high enough that capture is negligible. Particles captured between W_0 and W_1 will be missing at W_1; hence, given a way to calculate $S(W)$, one can measure $R(W_0, W_1)$ and, with $W_0 \approx W_1$, also $\sigma(W_1)$.

Both slowing down and capture can be treated either semiclassically or quantum mechanically. The spectral flux density $n(W)$ was semiclassically calculated to be proportional[3] to W or to be constant[4] at low energy W. Most semiclassical calculations predict capture from energies of the order of 100 eV, depending on the atomic number Z of the capturing element, while most quantum-mechanical calculations predict capture predominantly from substantially higher energies which rise strongly with Z; for example, in Li at 240 eV[5] and in C at 8 keV.[6] The subject is reviewed in Refs. 7 and 8.

No direct experimental information on $n(W)$ and $\sigma(W)$ at very low energies has been available up to now. It is the aim of this Letter to report on the first measurement of the two quantities for μ^- down to energies of some hundred electron-volts.

The experiment was performed at the Schweizerisches Institut für Nuklearforschung. Figure 1 shows the setup. sc2–sc8 are scintillation counters. The beam with incident energy and spread [full width at half maximum (FWHM)] of 11.2 and 2.6 MeV, respectively, is simultaneously used for measurements of muonic x rays emitted from target 1. This parasitic experiment will not be described here. Part of the beam passes through the slits in sc3, target 1, and sc4, is registered in sc5, and passes through a thin Al window to the vacuum chamber of a spectrometer magnet (one slice of an "orange"). Muons with a typical energy of 520 keV are focused onto a wedge-shaped degrader. The deflecting magnet and the wedge transform a thin beam of large energy spread into a wide beam of small energy spread. The wedge degrader thickness is such that the maximum of the μ stopping distribution is on the downstream surface of the thin counter sc6 (3 mg/cm^2), mounted directly behind the degrader. This counter gives the start signal of a time-of-flight measurement of muons between sc6 and target 2. In some runs there was a thin metal foil, for example, 160 μg/cm^2 Ag, immediately behind sc6. The stop signal comes from the Ge detector, Ge, measuring muonic x rays emitted from target 2 [in the experiment for $n(W)$, 40 μg/cm^2 Cu on Al]. The energy of muons incident on target 2 ranges from almost zero to 200 keV (FWHM). Counters sc7 and sc8 are in anticoincidence in order to reduce the background. Muon time-of-flight and x-ray energy are measured individually for each event.

Range-energy relations were measured for thin layers covering target 2. The energy distribu-

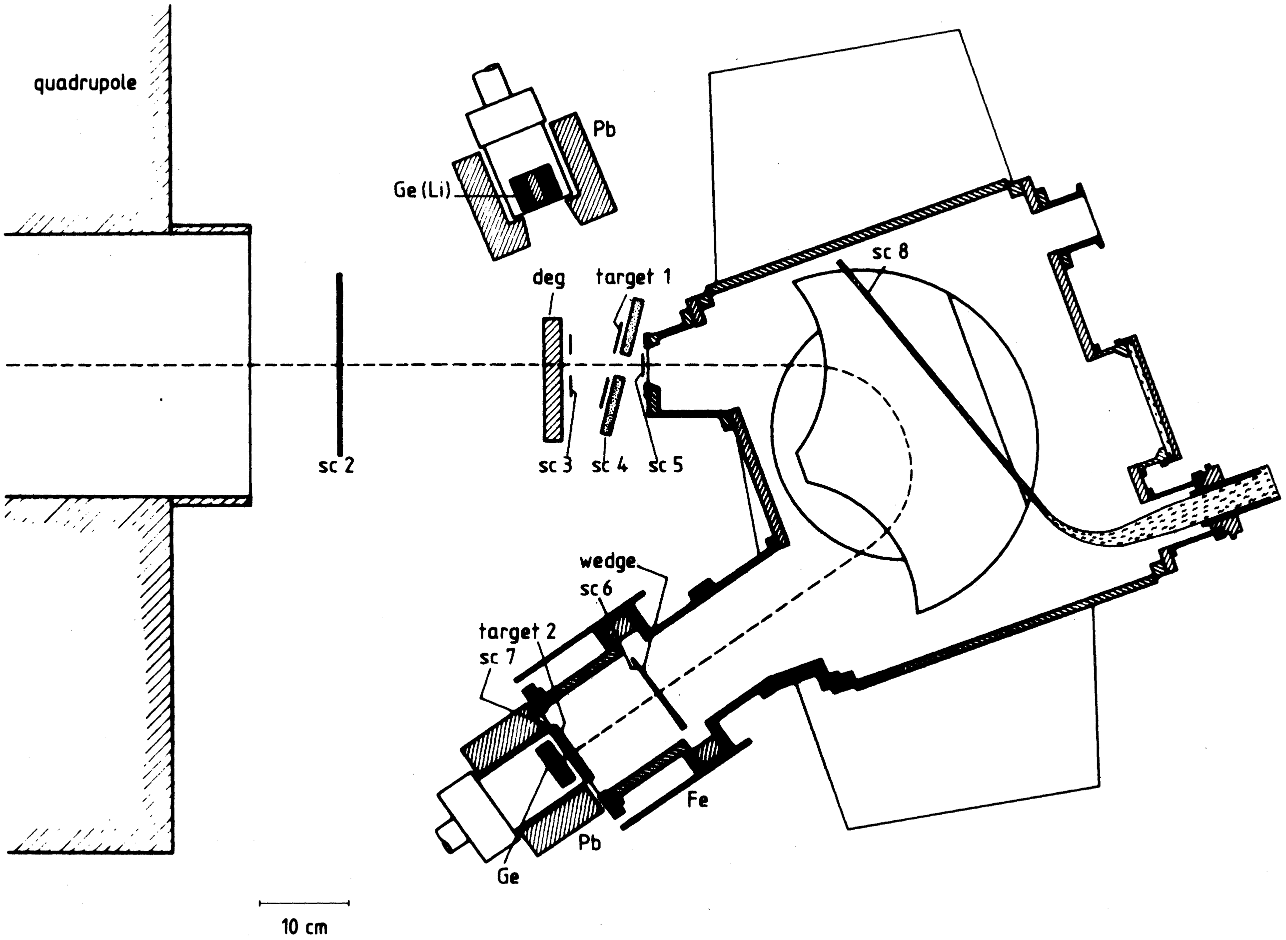

FIG. 1. Setup for slow muons. sc2–sc8 are scintillation counters. Ge(Li) and Ge are germanium detectors. deg is the degrader. Events (sc2, $\overline{\mathrm{sc4}}$, sc5, sc6, $\overline{\mathrm{sc7}}$, $\overline{\mathrm{sc8}}$, Ge) are registered for the spectral flux density experiment. The μ time of flight between sc6 and target 2 is measured for each event individually.

tion of muons penetrating the thin layer was obtained from their time-of-flight distribution. In this case the muons are detected with the help of their x rays from the target backing. For comparison the energy distribution of muons with the layer removed was separately measured (with only the backing in place). From the two distributions the energy at which half of the muons penetrate the thin layer was derived. The results are shown in Table I.

Figure 2 shows the spectral density $N(W)$ at target 2 where $N(W)$ means $n(W)$ integrated over the detector area (layer on target 2) and summed up over the accumulation time (16 h in this case), for muons from the 160 $\mu g/cm^2$ Ag foil, as measured by the time distribution of the CU 4-3 line emitted from the 40 $\mu g/cm^2$ Cu on target 2. The open circle is a normalization point at the energy of the muons before entering sc6, measured by the total number of muon events in counter sc6 and the measured width of the stopping distribution behind the wedge (by varying the magnet current). The two points with dashed range and error bars are somewhat too low because the Cu foil was not thick enough to reliably stop all muons (cf. Table I).

Figure 2 also shows calculated spectral density curves, based on Eq. (2). In order to transform this equation into the corresponding relation for $N(W)$, $n(W)$ must be corrected for losses due to directional scattering (negligible at the normalization point). The dashed line shows $N(W)$ without this correction, the full line labeled *a* was obtained by calculating the multiple scattering in Gaussian approximation, and the full line labeled *b* by applying Lambert's law, that is, a cosine distribution, valid at low energies where the particles have lost memory of their original direction. In the Gaussian approximation the angular distribution of the muons leaving the Ag foil is given by

$$f(\theta)\,d\Omega \propto \exp(-\theta^2/\langle\theta^2\rangle)\sin\theta\,d\theta,$$

where $d\Omega$ is the differential solid angle, θ the angle between particle trajectory and beam axis, and $\langle\theta^2\rangle^{1/2}$ the mean-square angle of multiple scattering. The contribution to $\langle\theta^2\rangle$ between W_0 and W_1 was calculated with[9]

$$\langle\theta^2\rangle = \frac{E_S^2}{4X_0}\int_{W_1}^{W_0}\frac{dW}{S(W)W^2},$$

where X_0 is the radiation length and E_S the characteristic energy ($E_S = 21.2$ MeV). $S(W)$ is taken from proton data[10] for $W \geqslant 1$ keV. This is justified as the effective charge of protons in metals is unity, even at zero velocity.[11] For $W \lesssim 1$ keV the shape of $S(W)$ was numerically calculated by applying a semiclassical theory and taking condensed-matter effects properly into account.[12] The absolute value was matched around $W = 1$ keV. Capture is not taken into account.

TABLE I. Experimental range-energy relations.

Layer (mg/cm^2)	Energy (keV)
Al 0.040 ± 0.004	17 ± 3
Cu 0.040 ± 0.004	4.2 ± 0.7
Au 0.120 ± 0.010	20 ± 6
Au 0.192 ± 0.008	29 ± 6

Because of poor statistics, the error bars of the measured points in Fig. 2 are large. However, experiment and our calculation agree well. It is important that there are muons around and below 1 keV. This is in agreement with the semiclassical theory of the Coulomb capture, predicting capture starting well below 1 keV. On the other hand, it would be very hard to reconcile our experimental results with quantum-mechanical treatments of the kind given in Refs. 5 and 6. This is corroborated by other runs, also with no

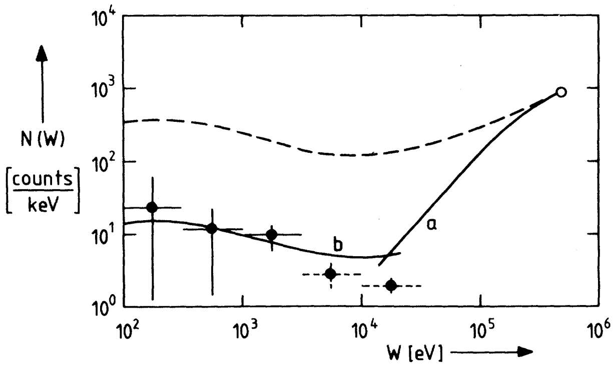

FIG. 2. Spectral density $N(W)$ at target 2 for muons from Ag in counts per kiloelectronvolt vs muon energy W, as measured by the Cu 4-3 line and calculated for the conditions of the experiment. Open circle, normalization point; dashed line, no multiple-scattering correction. Solid lines, multiple scattering taken into account: segment *a* in Gaussian approximation; segment *b*, with Lambert's law. Target 2: 40 $\mu g/cm^2$ Cu on Al; accumulation time, 16 h. The two points with dashed error and range bars are somewhat too low because the Cu layer was not thick enough to stop all muons.

foil behind sc6, that is, for muons emerging from CH_2. The statement is by no means meant to be made against the quantum-mechanical method in general. Indeed, new quantum-mechanical calculations seem to indicate much lower capture energies.[13]

A full description of apparatus, experimental method, calculation of energy loss, and experimental and theoretical results, for other elements also, will be given elsewhere.

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