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Parity Nonconservation in ^{18}F and Meson-Exchange Contributions to the Axial Charge Operator

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A simple relationship between the one-body and exchange-current contributions to the axial charge operator may allow a separation of these amplitudes in the β-decay transition $^{18}\mathrm{Ne}(J^\pi I = 0^+1) \to {}^{18}\mathrm{F}(J^\pi I = 0^-0)$. Since the exchange-current operator is related by isospin rotation to the pion-exchange component of the parity-nonconserving nucleon-nucleon potential, this suggests that the parity-nonconserving pion coupling can be extracted, with minimal dependence on nuclear models, from circular-polarization measurements for the analog γ decay in ^{18}F.

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Strong arguments have been made for the importance, in the study of exchange currents in nuclei, of transitions between $J^\pi = 0^+$ and 0^- states. First, because of the weakness of "background" terms involving isobars and heavy mesons, soft-pion theorems provide a model-independent description of the leading exchange-current contribution to the axial charge operator.[1] Second, the range of the resulting pionic currents is sufficient so that conventional nuclear physics treatments, with the effects of short-range correlations included perturbatively, should be adequate. And third, since the single-nucleon contribution to the axial charge operator enters only in order $(\vec{p}/M)$, the exchange-current influence on observed $0^+ \to 0^-$ β decay and muon-capture rates can be unusually large.

In this Letter, I consider the β decay of the $J^\pi T = 0^+1$ ground state of ^{18}Ne leading to the 0^-0 1.08-MeV state in ^{18}F. A great deal of effort has been expended in searching for this weak-decay branch,[2] culminating in the measurement of Adelberger *et al.* reported in the preceding Letter.[3] The reason for this strong interest is that the dominant amplitude responsible for the parity-nonconserving (PNC) mixing of the 0^-0 and analog 0^+1 (1.04-MeV) levels in ^{18}F is related to the exchange-current contribution to the β-decay operator by isospin rotation. Thus, on the simplest level, this β-decay branch provides an important test of nuclear wave functions used in estimating PNC effects in ^{18}F. Wave functions used in recent calculations[4,5] fail this test for reasons that will be clarified in this Letter. More importantly, a simple relationship depending only on gross features of the nucleus is shown to exist between the strengths of the leading term in the one-body β-decay amplitude and the exchange current. The reported measurement[3] thus provides a direct estimate of the PNC potential matrix element. A constraint on the PNC pion coupling independent of nuclear models then follows from the current upper limit on the γ-ray circular polarization in ^{18}F.

The differential rate for a nucleus to β-decay via a $0^+ \to 0^-$ transition is determined by multipole matrix elements of the axial charge and longitudinal current operators.[6] These multipole operators can be written

$$\hat{\mathfrak{M}}_0^5 = (4\pi)^{-1/2} \int d^3x j_0(\kappa x) \hat{J}_0^5(\vec{x}),$$
$$\hat{\mathfrak{L}}_0^5 = (4\pi)^{-1/2} \int d^3x [\nabla j_0(\kappa x)] \cdot \hat{\vec{J}}^5(\vec{x}), \quad (1)$$

with κ the magnitude of the nuclear three-momentum transfer and with $\hat{\mathfrak{J}}_\mu^5(\vec{x}) = (\hat{J}_0^5(\vec{x}), \hat{\vec{J}}^5(\vec{x}))$ the axial hadronic current density. The longitudinal current multipole is first forbidden. The hadronic current density consists of one-, two-, and, in principle, up to A-body amplitudes operating in the nuclear Hilbert space. In the present work,

$$\hat{J}_0^5(\vec{x}) \approx [\hat{J}_0^5(\vec{x})]_{(1)} + [\hat{J}_0^5(\vec{x})]_{(2)},$$
$$\hat{\vec{J}}^5(\vec{x}) \approx [\hat{\vec{J}}^5(\vec{x})]_{(1)}. \quad (2)$$

That is, two-body exchange-current contributions to the dominant axial charge operator are included.

The one-body current densities are[6]

$$[J_0^5(\vec{x})]_{(1)} = F_A \sum_{i=1}^{A} \tau_-(i)\vec{\sigma}(i) \cdot \{[\vec{p}(i)/M]\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)[\vec{p}(i)/M]\} - i[\hat{H}, \hat{\varphi}_{PS}],$$
$$[\hat{\vec{J}}^5(\vec{x})]_{(1)} = \hat{\vec{A}}(\vec{x}) + \nabla \hat{\varphi}_{PS}(\vec{x}), \quad (3)$$

where $\hat{\vec{A}} = F_A \sum_{i=1}^A \tau_-(i)\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i)$ and $\hat{\varphi}_{PS} = (F_p/2MF_A)\nabla\cdot\hat{\vec{A}}$. Here $F_A = -1.23$, $F_p = 2MF_A/m_\pi^2$, $\hat{H}$ is the nuclear Hamiltonian, and $\tau_- = \frac{1}{2}(\tau_1 - i\tau_2)$. Doubly reduced matrix elements of the resulting one-body components of the operators in Eq. (1) can be expressed conveniently in terms of the one-body density matrix $\psi_{\alpha\beta}$

$$\langle 0^- 0 \| \hat{O}_{(1)} \| 0^+ 1\rangle \equiv \sum_{\alpha\beta} \psi_{\alpha\beta} \langle \alpha \| O_{(1)} \| \beta\rangle. \quad (4)$$

This relation is exact if the sums extend over complete sets of single-particle wave functions. Choosing an harmonic-oscillator basis with oscillator parameter b (=1.7 fm) yields[7] (with $\|$ denoting matrix elements reduced in spin only)

$$\langle 0^- \| (\mathfrak{M}_0^5)_{(1)} \| 0^+\rangle = (-2i/bM)[F_A/(4\pi)^{1/2}][B + O(y^{1/2})],$$
$$\langle 0^- \| (\mathfrak{L}_0^5)_{(1)} \| 0^+\rangle = O(y^{1/2}), \quad (5)$$

where $y^{1/2} = \kappa b/2 \lesssim 0.01$ is small because of the low-energy release in the ^{18}Ne β decay. Though complete calculations of these matrix elements have been performed, this dominant coefficient B provides a convenient basis for comparing different treatments of the nuclear structure in mass 18. Note that if a shell-model approximation to Eq. (4) is employed in which nuclear wave functions of arbitrary complexity are allowed in the $1s$-$1p$-$2s1d$-$2p1f$ space, then

$$B = \tfrac{1}{2}[\sqrt{3}\,\psi^-_{1p_{1/2}1s_{1/2}} - \sqrt{2}\,\psi^-_{2s_{1/2}1p_{1/2}} + (10)^{1/2}\psi^-_{1d_{3/2}1p_{3/2}} + \sqrt{5}\,\psi^-_{2p_{1/2}2s_{1/2}} - 2\psi^-_{2p_{3/2}1d_{3/2}} + (21)^{1/2}\psi^-_{1f_{5/2}1d_{5/2}}],$$

where $\psi^-_{\alpha\beta} \equiv \psi_{\alpha\beta} - \psi_{\beta\alpha}$. The very simplest model of the ^{18}Ne β decay would retain only the $\psi_{2s_{1/2}1p_{1/2}}$ contribution.

As was mentioned previously, the one-body axial charge operator is intrinsically weak because of the dependence on $\vec{p}/M$. I now discuss two sets of nuclear wave functions similar to those used in recent studies[4,5] of parity nonconservation in ^{18}F which show that nuclear-structure effects generate additional suppressions of the β-decay amplitude. The first allows all possible $0\hbar\omega$ and $1\hbar\omega$ configurations to contribute in the descriptions of the positive- and negative-parity states, respectively. Thus a large model space ($1p$-$2s1d$-$2p1f$) and nonspurious wave functions are employed, but the effects of multiple-particle-hole correlations are omitted entirely (so that $\psi^-_{\alpha\beta} = \psi_{\alpha\beta}$). The effective interaction for the $2s1d$ shell is taken from Kuo and Brown,[8] while cross-shell matrix elements are taken from Millener and Kurath and from the bare-g-matrix results of Kuo.[9] The resulting $B = 0.140$ is considerably reduced over that obtained from $\psi_{2s_{1/2}1p_{1/2}}$ alone ($B^{0\hbar\omega+1\hbar\omega}_{2s_{1/2}1p_{1/2}} = 0.275$), because all of the smaller density matrix elements interfere destructively with that dominant term. The second model calculation is performed with Zuker pds wave functions[10] in which all configurations within the limited (and spurious) $1p_{1/2}$-$2s_{1/2}$-$1d_{5/2}$ space are allowed. Although the only nonzero $\psi_{\alpha\beta}$ are $\psi_{2s_{1/2}1p_{1/2}}$ and $\psi_{1p_{1/2}2s_{1/2}}$, this calculation does provide an estimate of the influence of multiple-particle-hole configurations. The resulting $B = B^-_{2s_{1/2}1p_{1/2}} = 0.102$ is again significantly smaller than the naive estimate $B^{\text{Zuker}}_{2s_{1/2}1p_{1/2}} = 0.284$.

These results suggest that a realistic calculation must include the effects of both a large model space and multiple-particle-hole correlations. Previous work indicates that the principal correlations affecting the states of interest are the four-particle, two-hole (4p-2h) admixtures (≈25%) in the 0^+ level.[11] Thus the first model described above has been expanded to include all $2\hbar\omega$ configurations. Those configurations involving one particle (hole) in the $3s2d1g$ ($1s$) shells are included only for the purpose of exactly projecting spuriousity. (Such projection is essential as small spurious components can drastically alter nonspurious components of the wave function when effective interactions violating translational invariance are employed.) Cohen and Kurath (8-16) matrix elements are used in the $1p$ shell, and additional matrix elements involving $2p1f$ particles are taken from Kuo.[9] Single-particle energy splittings are determined from analogous $2\hbar\omega$ fits to the levels in ^{15}O and ^{17}O. The resulting value for B is 0.027; correlations and the minor components of the density matrix conspire to yield a value much smaller than one would expect from any of the simpler treatments discussed earlier.

Before a comparison to experiment can be made, the two-body amplitude must be calculated. The pion-exchange contribution to the axial charge operator can be computed from a low-energy theorem based on partial conservation of axial-vector current and current algebra. The leading contribution is given by the seagull term[1]

$$[\hat{J}_0^5(\vec{x})]_{(2)} = -\frac{1}{2}\frac{m_\pi^2 g_{\pi NN}^2 F_1^v}{8\pi M^2 F_A}\sum_{\substack{i,j\\ i\neq j}}[\tau(i)\otimes\tau(j)]_-[\vec{\sigma}(i)\cdot\hat{r}_{ij}\delta(\vec{x}-\vec{x}_j)+\vec{\sigma}(j)\cdot\hat{r}_{ij}\delta(\vec{x}-\vec{x}_i)]\varphi_\pi(m_\pi r_{ij}), \tag{6}$$

where $\varphi_\pi(x)=(e^{-x}/x)(1+1/x)$, $[\ ]_-=\frac{1}{2}([\ ]_1-i[\ ]_2)$, and the vector form factor $F_1^v=1$. In analogy with the treatment of the one-body operator, this expression can be used in Eq. (1) to generate $(\mathfrak{M}_0^5)_{(2)}$, and matrix elements of this operator can be evaluated once the two-body density matrices have been determined for the three sets of wave functions discussed above. Note that in the $\kappa\to 0$ limit appropriate for the present calculation, $(\mathfrak{M}_0^5)_{(2)}$ becomes proportional, apart from an isospin rotation, to the PNC isovector pion potential given in Ref. 4. The effects of short-range correlations are introduced by multiplying the shell-model two-particle wave functions by $f(r_{ij})=1-(1-\beta r_{ij}^2)\exp(-\alpha r_{ij}^2)$, with $\alpha=1.1\ \mathrm{fm}^{-2}$ and $\beta=0.68\ \mathrm{fm}^{-2}$ (see Ref. 4).

The results are summarized in Table I. Both sets of simpler wave functions discussed earlier overestimate the experimental β-decay rate by approximately a factor of 10. In contrast, the third calculation, which one could argue is the minimal realistic treatment of the structure of the relevant levels in the sense of incorporating both multiple-particle-hole correlations and an adequate model space, does predict a much smaller decay rate, $\omega=4.8\times10^{-6}$/s, in good agreement with the experimental result $\omega=(5.4\pm1.3)\times10^{-6}$/s.[3]

In spite of this agreement, the prospects for constraining the PNC matrix element in ^{18}F would not appear too good: A high degree of cancellation is occurring in both the one- and two-body contributions to the β-decay strength, and thus it would appear difficult to argue, even with the present sophisticated structure treatment, that these amplitudes can be reliably disentangled. However, an important general feature of the various calculations has not yet been discussed: The exchange current *systematically tracks* the strength of the *allowed* (i.e., proportional to B) contribution to the one-body amplitude. Despite the very different structure assumptions made in the three calculations discussed here, the resulting β-decay amplitudes all satisfy the relation

$$\langle\|(\mathfrak{M}_0^5)_{(2)}\|\rangle = 0.67\langle\|[\mathfrak{M}_0^5(\kappa=0)]_{(1)}\|\rangle \pm 0.07\langle\|\mathfrak{M}_0^5\|\rangle_{\mathrm{av}}^{\mathrm{expt}}. \tag{7}$$

That is, the relative strength of the calculated one- and two-body amplitudes is constant to within an absolute error of 7% of the amplitude $\langle\|\mathfrak{M}_0^5\|\rangle^{\mathrm{expt}}$ derived from the experimental β-decay rate in the allowed approximation. This relation holds to this remarkable accuracy despite a spread of a factor of 16 in the calculated β-decay rates.

The origin of this relation is well known in a different context, that of representing two-body PNC potentials by equivalent one-body operators. The PNC potential (or exchange-current operator) consists of two sets of terms: true two-body components where the quantum numbers of a pair of nucleons are changed, and components which are semidiagonal in one quantum number. The latter, often viewed as the interaction of valence nucleons with an inert core, generally dominate when the valence particles are few. These semidiagonal

TABLE I. Calculated one-body (ω_1), two-body (ω_2), and full (ω_{1+2}) rates (in units of 10^{-6}/s) are compared with experiment.

	ω_1	ω_2	ω_{1+2}	$\langle V_{\rm PNC}\rangle$ (eV)
Zucker *pds*	23.0	5.9	52.2	$0.72\times 10^6 F_\pi$
$0\hbar\omega+1\hbar\omega$	29.5	10.5	75.3	$0.95\times 10^6 F_\pi$
$1\hbar\omega+2\hbar\omega$	1.84	0.70	4.78	$0.25\times 10^6 F_\pi$
expt			5.4 ± 1.3	$\leq -0.13\pm 0.36$

terms can be exactly represented by an equivalent one-body operator; in several limiting cases, however, one recovers simply an equivalent one-body potential of the form $\omega_\pi \tau_3 \vec{\sigma}\cdot\vec{p}$, with ω_π a constant, often determined empirically, which depends primarily on gross features of the nucleus.[2] The analogous operator for the present exchange-current work, $\omega_\pi \tau_- \vec{\sigma}\cdot\vec{p}$, is proportional to that term in Eq. (3) which generates B.

The experimental β-decay strength and Eq. (7) combine to place stringent and largely model-independent constraints on PNC in ^{18}F. In the long-wavelength limit these yield $\langle |V_{\rm PNC}{}^\pi|\rangle = (0.278 \pm 0.065\ {\rm MeV})F_\pi$, with F_π (Ref. 4) more conventionally given as $g_{\pi NN} f_\pi/(32)^{1/2}$.[13] The forbidden contributions to the β-decay amplitude are not, however, completely negligible, since they depend in part on the density-matrix-element combinations $\psi^+{}_{\alpha\beta}$ which are enhanced by the multiple-particle-hole correlations. Since these combinations are not subject to the delicate cancellations occurring for the allowed terms, values are taken from the third model calculation (full one and two $\hbar\omega$ bases). This gives $\langle |V_{\rm PNC}{}^\pi|\rangle = (0.236 \pm 0.065\ {\rm MeV})F_\pi$. Combining this with the current limit on the γ-ray circular polarization in ^{18}F, $P_\gamma \leq (-0.7\pm 2.0)\times 10^{-3}$,[14] constrains $\pm F_\pi \leq (-0.54 \pm 1.55)\times 10^{-6}$.[15] Some part of the Weinberg-Salam "reasonable range" for F_π specified in Ref. 13 is thus ruled out. Taking $F_\pi = 0.36\times 10^{-6}$ ($f_\pi = 1.52 \times 10^{-7}$) from Ref. 4, where this value was derived by comparing nuclear-model calculations with measurements of parity nonconservation in ^{19}F and ^{21}Ne, yields $\langle |V_{\rm PNC}{}^\pi|\rangle = 0.085$ eV. Thus it appears likely that the current experimental limit on P_γ in ^{18}F may have to be reduced by a factor between 2.8 and 5.8, depending on the sign of the $M1/E1$ ratio in ^{18}F, to obtain a definite measurement of parity nonconservation. This may be a difficult task.

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