the "impurity" uncertainty $1/\Lambda$ of any k^x being small compared to quantum distance between $k_n{}^x$ with different n's, and thus the approximate conservation of k^x between impurity collisions). This implies $l \gg n_t(aw)^{1/2}$ in the most favorable (because of the corresponding magnet field) case and $l \gg n_t(adw^2)^{1/4}$ in the least favorable case; Eq. (19) and $l \gg n_t(aw)^{1/2}$ lead to Eq. (3c). Unfortunately, the r multiplicativeness in Ref. 9 was proved only formally. That is why, when Eqs. (3c) are valid, Eqs. (13) and (19) determine the localization length L_0 and therefore[2] the temperature condition $l_i > L_0{}^2/l$ of a strong localization, but the physical implications of $l < n_t(aw)^{1/2}$ remain unknown.

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[12]Suppose $L \to \infty$, and $R \propto \exp[L\lambda(c_i)]$. Then $\langle R \rangle \propto \int \exp[L\lambda(c_i)] \exp[-\frac{1}{2}\mu L(c_i - \langle c_i \rangle)^2] dc_i \propto \exp[L \times \tilde{\lambda}(\langle c_i \rangle)]$, where $\tilde{\lambda}(\langle c_i \rangle) = \lambda(\tilde{c}_i) - \frac{1}{2}\mu(\tilde{c}_i - \langle c_i \rangle)^2$ and $\lambda'(\tilde{c}_i) = \mu(\tilde{c}_i - \langle c_i \rangle)$; and $\tilde{\lambda}(\langle c_i \rangle)$ is known from $\langle R \rangle$ dependence on c_i. If, e.g., $\mu \ll \lambda$, then $\langle c_i \rangle \simeq \tilde{c}_i - \tilde{\lambda}'(\langle c_i \rangle)/\mu$, and $\lambda(\tilde{c}_i) \simeq \tilde{\lambda}(\tilde{c}_i) - \frac{1}{2}\mu\tilde{\lambda}'(\tilde{c}_i)$. Note that the major contribution into $\langle R \rangle$ may come from exponentially rare fluctuations $\tilde{c}_i - \langle c_i \rangle$, which may not manifest themselves in experiments or computer simulations.

Experimental Verification of Scaling Relations for Electron-Hole Liquid Condensation

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Detailed experimental results for the phase diagrams, ground states, and critical points for electron-hole liquid condensation in unstressed Si and in Si stressed along the ⟨111⟩, ⟨110⟩, and ⟨100⟩ directions are given and are shown to be in good agreement with microscopic calculations. These results for a set of systems with widely varying band structure provide, for the first time, a quantitative test for, and a verification of systematic scaling relations between the ground state and the critical properties for electron-hole liquid condensation.

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Electron-hole liquid (or "droplet") condensation from a high density of carriers in semiconductors has been the subject of intensive study in large part because this system provides a unique opportunity to test our understanding of interacting electronic systems by making quantitative comparisons between experiment and theory.[1] Recently, the number of systems in which this phenomenon has been observed has been increasing. For indirect-gap materials quantitative re-

sults have been reported for Ge and Si[1,2]; less detailed results have been obtained for Ge under uniaxial stress, for the low-temperature properties of stessed Si[3] and for various compound semiconductors. As a result, this unique Fermi system now can be studied under conditions for which its density and condensation energy vary widely.

An important issue at present is the possible existence of general relations ("scaling relations") between the ground state and the critical properties for electron-hole liquid (EHL) condensation in all semiconductors.[1,4,5] The existence of such relations is not only an important question in principle, but these relations also serve as a very useful guide to experiment. To date, however, experimental results have not been sufficiently extensive or accurate to verify their existence or to distinguish between the proposed forms.[4,5]

In the present work, the first quantitative results for the systematic variations of the critical points, the ground-state properties, and the phase diagrams for EHL condensation in a set of well characterized systems with substantially wide variations in EHL energetics are presented. From these results we successfully test and verify the existence of scaling relations for EHL condensation and give their forms. In addition, these results are used to distinguish between two models which have been proposed to calculate the critical point for condensation.

The present studies have been made on unstressed Si and on Si with large uniaxial stress along the ⟨111⟩, ⟨110⟩, and ⟨100⟩ directions. These stresses decrease the conduction- and valence-band degeneracies and give rise to systems denoted by Si[6; 1], Si[4; 1], and Si[2; 1], respectively, where the numbers in brackets give the conduction- and valence-band degeneracies. The unstressed case is denoted by Si[6; 2]. The band structures of these systems are well known.[1,6] The variations in the band structure give rise to substantial systematic variations in the EHL properties, and thus the systems studied here provide a particularly good set of model systems in which to study the systematics of EHL condensation in all semiconductors.

Cylindrical samples ($r = 0.6$ mm, $l = 10$ mm) of high-purity p-type Si (7000 Ω cm) were mounted in a variable temperature dewar and excited by a cavity-dumped Ar^+-ion laser at $\lambda = 514.5$ nm every 10 μs. The luminescence was detected by a S1 photomultiplier tube and processed by a fast single-photon–counting system (time resolution better than 10 ns). Stress homogeneity was checked by time-resolved measurements of the free-exciton (FE) luminescence. No change of the exciton peak position was observed for delay times between 10 ns and 1 μs after the excitation pulse. This, as well as the clearly resolved transverse-optical and longitudinal-optical (TO/LO) replicas of the low-temperature free-exciton luminescence,[7] indicated highly homogeneous stress conditions.

Results for Si[2; 1], Si[4; 1], and Si[6; 1] were obtained at stresses corresponding to valence-band splittings of 17, 29, and 31 meV, and conduction-band splittings of 30, 25, and 0 meV, respectively. In each case the stress applied was sufficiently high to approximate "infinite"-stress conditions[8] in which the valence-band splitting is much larger than the hole Fermi energy, and hence only the upper ($m_j = \frac{1}{2}$) valence band was populated. For Si[4; 1] and Si[2; 1], a conduction-band splitting sufficient to allow for a fast thermalization between the nondegenerate minima via intervalley phonon scattering[8,9] was selected.

Figure 1 shows a typical luminescence spectrum for Si[2; 1]. At low temperatures the broad EHL band, the biexciton (BI),[3] and the free-exciton emission can be seen. For temperatures above 14 K for Si[2; 1] only two components are found. They correspond to FE and electron-hole plasma[10] (EHP) emission. A detailed discussion of the band structure and temperature dependence of the EHP is given elsewhere.[11]

In order to obtain the equilibrium densities of

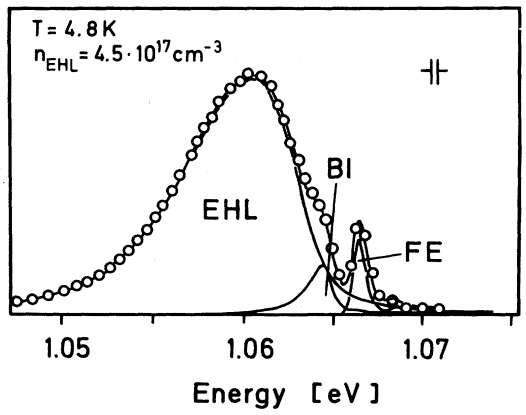

FIG. 1. Luminescence spectrum for Si[2; 1]. Circles are experimental data and the solid lines are the fit as described in the text.

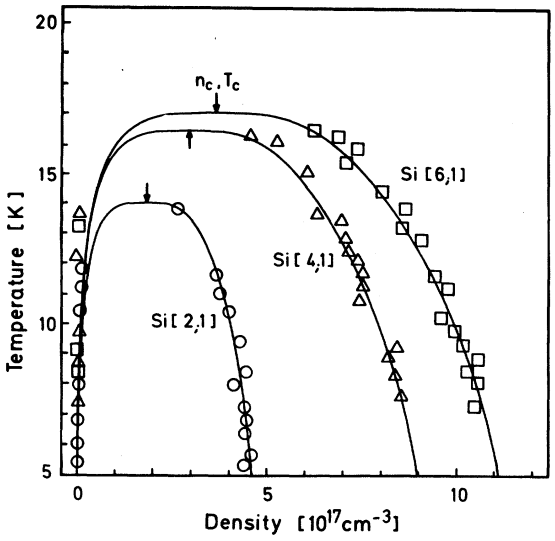

FIG. 2. Phase diagrams for Si under high uniaxial stress in the ⟨111⟩, ⟨110⟩, and ⟨100⟩ directions. The size of the symbols corresponds to error bars in the low-temperature region. Near the critical point the error bars are about twice as large. The solid curves are from phase diagram fits with use of the droplet fluctuation model (see text).

the EHL phase and gas phase for these systems, we have fitted the spectra (solid line in Fig. 1) as a sum of EHL, BI,[3] and FE luminescence and have introduced an EHP contribution[9] in the temperature region near the critical point. The liquid line shape was fitted as the convolution of the joint densities of state of electrons and holes by using the valence-band parameters of Hensel and Feher[6] and including a temperature-dependent[12] broadening to account for finite intraband relaxation times. The experimental values of the gas densities were obtained from Richardson-type equations as the sum of FE, BI, and trion densities[13] assuming ideal gas constituents and a parabolic density of states for each species.[13] This approach gives reasonable results for the temperatures ($0.2T_c < T < 0.8T_c$) we are considering here. Finally, because of the high excitation necessary for liquid formation even at bath temperatures of 1.7 K, carrier temperatures only above 4.5 K could be reached.

The experimental results for the phase diagrams for Si[6; 1], Si[4; 1], and Si[2; 1] are shown in Fig. 2. The values of the ground-state densities and energies (n_0 and ϵ_0) and the critical temperatures

TABLE I. Ground-state and critical parameters from (a) present experiments and (b) present calculations. Part (c), various scaling ratios formed from experimental values. In the last line, the values of κ and μ do not vary for these systems. The parameters are defined in the text.

		Si [2;1]	Si [4;1]	Si [6;1]	Si [6;2]
(a)	n_0 [10^{17}cm^{-3}]	4.8±0.1	9.3±0.2	11.3±0.2	35.0±1.0
experiment	ε_0 [meV]	14.95±0.1	17.65±0.2	18.15±0.2	22.15±0.2
	n_c [10^{17}cm^{-3}]	1.8±0.3	2.9±0.6	3.6±0.8	12.0±2.0
	T_c [K]	14.0±0.5	16.4±0.5	16.9±0.5	23.0±1.0
(b)	n_0 [10^{17}cm^{-3}]	4.51	6.96	8.33	32.5
theory	ε_0 [meV]	14.7	15.8	16.6	22.1
	n_c [10^{17}cm^{-3}]	1.44	2.04	2.10	9.51
	T_c [K]	14.1	15.8	16.5	23.3
(c)	ϕ_0/k_BT_c	1.7	3.4	3.6	4.7
Scaling	$T_c/n_0^{1/2}$ [10^{-9}Kcm$^{3/2}$]	20.2	17.0	15.9	12.3
relations	$\lvert\varepsilon_0\rvert/k_BT_c$	12.4	12.5	12.5	11.2
from	n_c/n_0	0.35	0.31	0.32	0.34
experiment	$\kappa T_c/n_0^{1/4}(\kappa/\mu)^{1/4}$[$10^{-2}Kcm^{3/4}$]	1.88	1.87	1.84	1.88

and densities (T_c and n_c) obtained from experiment are given in Table I(a). The values of n_0, T_c, and n_c were obtained by fitting the experimental results for the phase diagrams with the equations from the droplet fluctuation model[14] in which these parameters are adjustable (solid curves in Fig. 2). The data for n_0 and T_c vary by less than 5% for widely varying fit conditions. Because of the flat top of the phase diagrams, the uncertainty in the n_c determination is ~20%. The values of $\epsilon_0 = \varphi_0 + R_0$ are obtained from low-temperature data for the EHL binding energy φ_0 which are extrapolated[1] to $T = 0$ and by using a single exciton Rydberg R_0 of 12.85 meV (Ref. 1) for all four systems.

Microscopic calculations of ϵ_0 and n_0 [Table I(b)] were made from the sum of the electron and hole kinetic energies and the exchange-correlation energy $\epsilon^{xc}(n)$ as a function of density n with use of band parameters corresponding to infinite stress as above.[1,6] $\epsilon^{xc}(n)$ when expressed in units of the appropriate exciton Rydberg and Bohr radius depends only very weakly on band structure.[5,8] For Si[6; 2] and Si[2; 1] we have used the detailed results of Vashishta, Das, and Singwi[15] for $\epsilon^{xc}(n)$, and for Si[6; 1] and Si[4; 1] we have used values obtained from the average of their (very similar) results for $\epsilon^{xc}(n)$ for Si[6; 2], Si[2; 2], and Si[2; 1]. Variations due to different choices of $\epsilon^{xc}(n)$ are small (<3% for n_0).

The critical points were calculated with use of a method motivated from the droplet fluctuation model in which the critical temperature is that for which the temperature-dependent surface tension vanishes and stable droplets can no longer form,[14] and the results are given in Table I(b) [with use of $\epsilon^{xc}(n)$ as above]. The theoretical phase diagrams formed from these parameters are in good agreement with the solid curves in Fig. 2 [compare Tables I(a) and I(b)]. The agreement between experiment and theory for this broad range of ground state and critical parameters is very good, and it tends to confirm the experimental results.

We can compare the present theoretical results for the critical point which were obtained from the droplet fluctuation model[14] with those from the alternative uniform plasma model[1,16] in which the critical point is obtained from an inflection point in the chemical potential of a dense uniform EHL. This latter approach gives for T_c for Si[2; 1], Si[4; 1], and Si[6; 1], and Si[6; 2] the values 17.4, 20.8, 21.4, and 29.0 K, respectively. Thus we see that the uniform plasma model gives a consistent reliable upper bound for T_c but that the results from the droplet fluctuation model are more accurate. The overestimate in the results from the uniform plasma model arises from its neglect of statistical fluctuations.[14]

The present work gives the first quantitative experimental results for the variations of the critical points and ground-state properties for EHL condensation in a set of systems with substantial systematic variations in the band structure (wide systematic variations in EHL energetics). We can now examine the question of the existence of systematic relations between these properties. Several proposals have been made previously. They include $\varphi_0/k_B T_c \simeq$ const, which was based on an analogy with classical gas condensation,[1] and $T_c/n_0^{1/2} \simeq$ const and $n_c/n_0 \simeq$ const from previous experiments.[1,4] Recently, an alternative set was proposed by Reinecke and Ying[5] based on theoretical consideration of the critical point as a dense plasma. This set is $|\epsilon_0|/k_B T_c \simeq$ const, $n_c/n_0 \simeq$ const, and $(\kappa k_B T_c/n_0^{1/4})(\kappa/\mu)^{1/4} \simeq$ const, where κ is the static dielectric constant and μ is the total reduced optical mass of the electron and hole.[1] To date, however, no complete derivation of such rules has been possible, and experimental results have not been sufficiently accurate or extensive to test their existence or to distinguish between them.[4,5]

In Table I(c) these five ratios are formed from the present experimental results. The first two vary systematically and unambiguously and therefore cannot be used as scaling rules. The latter three are remarkably constant and therefore are verified as scaling relations by the present results. In general, variations in EHL parameters for all semiconductors are determined in the main by variations in the effective band structure. Therefore, on the basis of the present results we argue that these scaling relations are expected to hold for EHL in all semiconductors.[17]

The present experimental values for the scaling ratios are in good agreement with those for unstrained Ge. It is the only system for which accurate experimental values for the critical point and ground-state parameters were available previously. For it,[1,5,13] $|\epsilon_0|/k_B T_c = 10.3 \pm 0.7$, $n_c/n_0 = 0.31 \pm 0.07$, and $(\kappa k_B T_c/n_0^{1/4})(\kappa/\mu)^{1/4} = (2.01 \pm 0.2) \times 10^{-2}$ K cm$^{3/4}$. Note that the values of κ and μ in the last relation change appreciably in going from the Si systems to Ge, thus substantiating the present form of this relation.

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Isotopic Anomalies in the Galactic Cosmic-Ray Source

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High-resolution ($\sigma_M < 0.25$ amu) observations have been made of the isotopic composition of oxygen, neon, and magnesium in the galactic cosmic rays (~100–300 MeV/amu). Derived source abundances shown enhancements of $^{22}Ne/^{20}Ne$, $^{25}Mg/^{24}Mg$, and $^{26}Mg/^{24}Mg$ over solar-system values by factors of $4.1^{+0.8}_{-0.6}$, $1.6^{+0.3}_{-0.2}$, and $1.6^{+0.3}_{-0.2}$, respectively. These data demonstrate that the cosmic-ray source is compositionally distinct from the solar system and constrain nucleosynthesis models, several of which are considered.

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Observations of the nuclear composition of solar-system material have formed the basis for modern theories of the nucleosynthesis of heavy nuclides in stars. By analyzing the composition of a sample of matter from another source it should be possible to investigate the extent to which the products of nucleosynthesis are unique and possibly to study the variations in the nucleosynthetic products resulting from spatial and/or temporal evolution. The galactic cosmic rays (GCR) provide such a sample from outside the solar system.

It has been well established through a number of experiments[1-4] that the ratio $^{22}Ne/^{20}Ne$ is larger by a factor of ~3–4 at the cosmic-ray source than in solar-system material and attempts to