show different relaxation times in echo experiments,[13] i.e., at short times, cannot be separated on long time scales.

In addition to the results reported here, each cooldown of our samples from room temperature to low temperatures was accompanied by large thermal relaxation effects.[14] Because of the ill-defined cooling procedure, however, no quantitative results are presented.

In summary, we have observed substantial and reproducible thermal relaxation at low temperatures on a time scale of 10^3 s in vitreous silica. We attribute these effects to a slow relaxation of low-energy excitations in the amorphous state. The method presented seems to be well suited for such investigations. On the basis of the standard tunneling model we have tentatively derived a density of states for these excitations, which is comparable in magnitude to that of the more strongly coupled, faster-relaxing excitations, which contribute to phonon scattering. We have found no upper limit for the relaxation times in contrast to recent experiments on the time dependence of the specific heat.

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[1]R. C. Zeller and R. O. Pohl, Phys. Rev. B 4, 2029 (1971); R. B. Stephens, Phys. Rev. B 8, 2896 (1973).

[2]P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. 25, 1 (1972).

[3]W. A. Phillips, J. Low Temp. Phys. 7, 351 (1972).

[4]J. L. Black, Phys. Rev. B 17, 2740 (1978).

[5]W. M. Goubau and R. A. Tait, Phys. Rev. Lett. 34, 1220 (1975).

[6]R. B. Kummer, R. C. Dynes, and V. Narayanamurti, Phys. Rev. Lett. 40, 1187 (1978).

[7]M. T. Loponen, R. C. Dynes, V. Narayanamurti, and J. P. Garno, Phys. Rev. Lett. 45, 457 (1980).

[8]Vacuumschmelze GmbH, Hanau, Germany.

[9]A. Berton, J. Chaussy, J. Odin, R. Rammal, J. Souletie, and R. Tournier, J. Phys. (Paris) Lett. 40, L-391 (1979), performed a similar experiment to measure the heat flux in a spin-glass.

[10]J. C. Lasjaunias, B. Picot, A. Ravex, D. Thoulouze, and M. Vandorpe, Cryogenics 17, 111 (1977).

[11]J. C. Lasjaunias, A. Ravex, M. Vandorpe, and S. Hunklinger, Solid State Commun. 17, 1045 (1975).

[12]B. Golding and J. E. Graebner, Phys. Rev. Lett. 37, 852, 1105(E) (1976).

[13]B. Golding, M. v. Schickfus, S. Hunklinger, and K. Dransfeld, Phys. Rev. Lett. 43, 1817 (1979).

[14]D. S. Greywall, Phys. Rev. B 18, 2127 (1978), even had to remove parts made of PMMA from his apparatus for the measurement of the specific heat of ^{4}He, because PMMA released heat with a time constant of about two weeks.

Dislocation Motion in hcp ^{4}He

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Low-frequency measurements of the shear modulus and internal friction of pure and ^{3}He-doped crystals of hcp ^{4}He are reported. Previously unobserved features are seen in both quantities. With use of the parameters obtained from our measurements, it is shown for the first time that all the existing dissipation measurements spanning five decades in frequency are consistent with each other and the Granato-Lücke model of overdamped dislocation motion.

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Dislocation dynamics can be studied by measuring the shear modulus and the internal friction of crystals at different frequencies, impurity levels, and temperatures. Solid ^{4}He is nearly an ideal crystal for dislocation studies because of the fact that ^{3}He is the only impurity and can be purified down to 10^{-9} level. Ultrasonic attenuation measurements in hcp ^{4}He have shown that only basal-plane ($\perp \hat{c}$ axis) dislocations contribute to the shear modulus by gliding in the basal plane itself.[1] Recent attenuation measurements as a function of amplitude have demonstrated the pinning of dislocations to ^{3}He impurities at lower temperatures[2] but the effect is absent above 1.5 K.[3] The ultrasound velocity measurements show a dislocation contribution[4] which is difficult to analyze and whose interpretation is still controversial.[4-6] Dissipation measurements in the kilohertz region are much easier to perform because of a larger effect, but unfortunately previous measurements in ^{4}He were done without a simultaneous study of the shear modulus.[7] Finally, previous low-frequency measurements[8] did not include a determination of the quantity of ^{3}He impurities present.

We have performed simultaneous dissipation and shear modulus measurements at 331 Hz with torsional standing waves. To determine the contribution due to dislocations we have studied heavily doped (430 ppm ^{3}He), commercial (0.3 ppm ^{3}He) and ultrapure (0.0024 ppm ^{3}He) samples of ^{4}He at different wave amplitudes as a function of temperature using the high-Q torsional oscillator technique developed by Reppy and co-workers at Cornell University. The oscillator had a hollow torsion rod 10 mm long with a 0.4 mm i.d. and a 1.0 mm o.d. The helium crystals were grown inside the torsion rod. The oscillator was operated in a feedback loop and the shear modulus and dissipation of a ^{4}He sample were determined from the difference between empty- and full-cell period and amplitude. The period stability measured

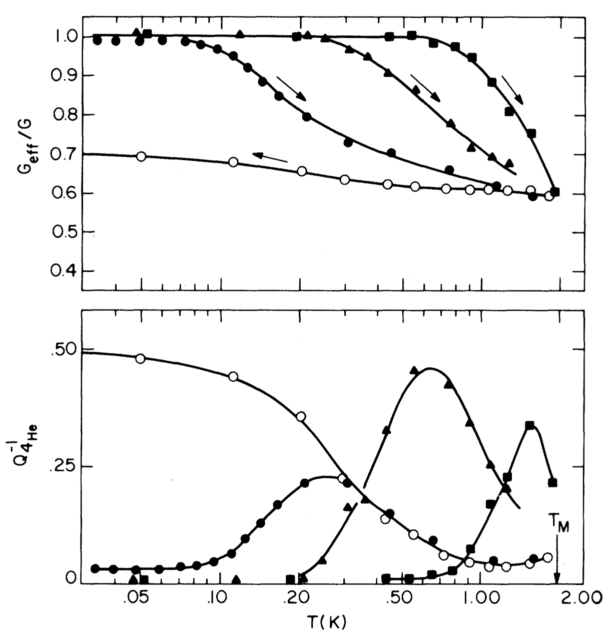

FIG. 1. Temperature dependence of the effective shear modulus G_{eff} and dissipation Q^{-1} of a ^{4}He sample with 0.3 ppm of ^{3}He at 37 bars. Different symbols correspond to different maximum strain amplitudes ϵ. Because of the capacitive detection system the *absolute* magnitudes of the amplitudes are only known within a factor of 2. Squares, $\epsilon = 10^{-7}$; triangles, $\epsilon = 6\times10^{-7}$; open circles and closed circles, $\epsilon = 10^{-5}$.

over one month was better than 10^{-7} and the Q of the oscillator was $\approx 5\times10^5$. The solid ^{4}He samples were grown by the blocked-capillary method, with the formation of solid in the rod observed by a drop in the oscillation amplitude. The samples were carefully annealed and, at each temperature, $\approx$1 h was required for the amplitude to stabilize.

The commercial purity ^{4}He was studied most extensively and data from one of the crystals are shown in Fig. 1. At all amplitudes we observed a broad maximum in the dissipation and a simultaneous decrease in the shear modulus from the constant low-temperature value G to a limiting high-temperature value, which in this case was $\approx$60% of G. We studied 15 commercially pure ^{4}He samples at different pressures and found the decrease of G_{eff} to vary from 20% to 40% of G and the transition region to change slightly from sample to sample. In contrast for a crystal grown from ^{4}He gas containing 430 ppm ^{3}He the transition region moved above 1 K and for ultrapure ^{4}He samples we observed no transition region to 17 mK (Fig. 2).

These features can be explained by the dislocation theory of Granato and Lücke[9] in which the dislocation line between two stable nodes can bend when stress is applied to the crystal. This elastic dislocation motion reduces the shear modulus. ^{3}He impurities tend to bind to the dislocation lines at low temperatures and pin this dislocation motion. When the temperature is increased ^{3}He unbinds from the dislocation lines and a breakaway occurs. The critical stress τ_c for this breakaway is

$$\tau_c = \pi F_{max}/4aL_c(T), \tag{1}$$

where F_{max} is the maximum binding force of a dislocation to a ^{3}He atom, a is the lattice parameter, $L_c(T) = (a/x)\exp(-E_0/kT)$ is the average distance between ^{3}He impurities on the dislocation line, E_0 is the activation energy of ^{3}He trapped on the dislocations, and x is the concentration of ^{3}He atoms in the crystal.[9] Equation (1) is valid for a constant L_c over the whole crystal. The broad absorption peak suggests that a large distribution of lengths L_c is present in real crystals. From the amplitude dependence of the breakaway temperature (Fig. 1) we obtain $E_0 = 0.7$ K. This is somewhat smaller than a simple theoretical estimate of 3 K,[1] but agrees with the previously measured value of 0.6 K.[2]

The observed dissipation and shear modulus had a large temperature hysteresis at every am-

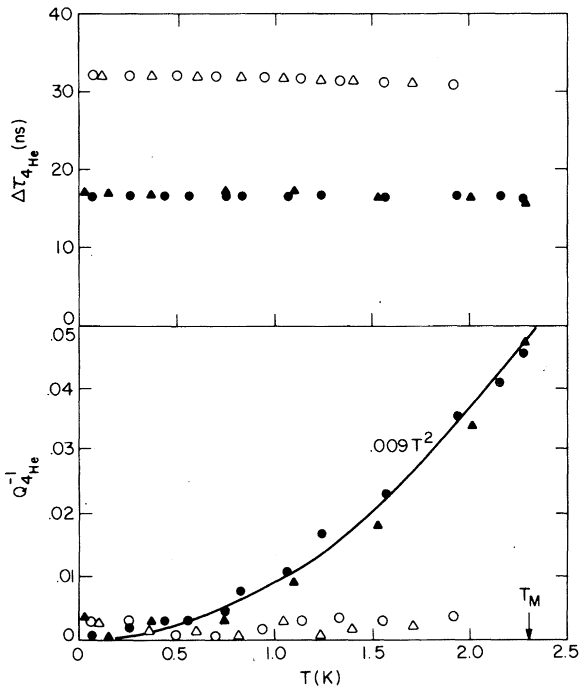

FIG. 2. Data at 48 bars from two ultrapure ^{4}He samples showing the effect of different crystal orientation. For one sample (open triangles, $\epsilon = 6\times10^{-7}$; open circles, $\epsilon = 10^{-5}$) the dislocation motion has little effect on the shear modulus [$\propto (\Delta\tau)_{^4\mathrm{He}}$ = period change] and the dissipation Q^{-1} and we assume the orientation angle $\theta \approx 45°$. In the second crystal (solid triangles, $\epsilon = 6\times10^{-7}$; solid circles, $\epsilon = 10^{-5}$) the dislocation motion reduces the shear modulus and produces a large temperature dependent dissipation. In this crystal, $\theta \approx 0°$.

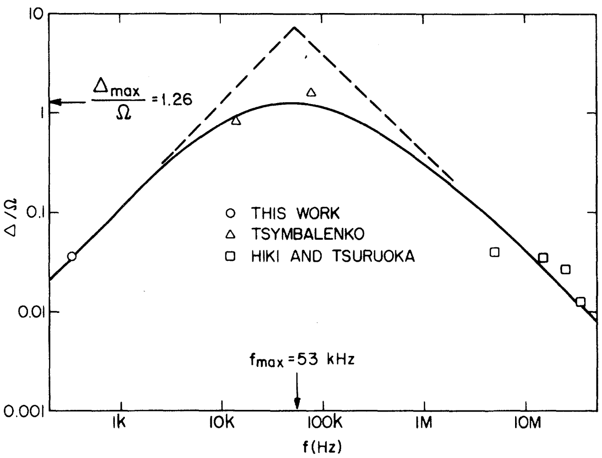

FIG. 3. Comparison of all the existing dissipation measurements ($\Delta = 1/2Q$) at $\approx$32 bars and 1.70 K. The solid curve is calculated from our dislocation parameters at 32 bars [$\Lambda L^2 = 2.0$, $BL^2 = 1.5\times10^{-13}T^2$ (in cgs units)] (Ref. 12). We assume an orientation factor $\Omega = \frac{1}{2}$ for Tsymbalenko's measurements (Ref. 7), which will give a lower limit for Δ/Ω. We have used data for crystal $n = 28$ ($\theta \approx 28°$) from the work of Hiki and Tsuruoka and estimated $1/\Omega(\theta) \approx 16$ (Refs. 1 and 13).

plitude of oscillation as shown for the largest amplitude in Fig. 1. After cooling to 17 mK while oscillating at the largest amplitude, and then waiting at that temperature for one hour at zero amplitude, the shear modulus would increase back to the maximum value. This is explained by the fact that freely vibrating dislocations (a line of length 5×10^{-4} cm has a vibration amplitude of $>100a$ when $\epsilon = 10^{-5}$) cannot trap a ^{3}He atom and it takes $\approx$1 h for ^{3}He to diffuse 5×10^{-4} cm,[10] which is a reasonable estimate for the distance between dislocation lines.[4,11]

The reduction of shear modulus due to dislocation motion shown in Fig. 1 is one of the largest found in the dislocation literature. If we assume an exponential distribution of dislocation line lengths L, the theory of Granato and Lücke[9,12] gives an effective shear modulus for low frequencies,

$$G_{\mathrm{eff}}/G = (6\Omega\Delta_0\Lambda L^2/\pi + 1)^{-1}, \tag{2}$$

where Ω is the orientation factor, $\Delta_0 = 4(1-\nu)/\pi^2$, where ν is Poisson's ratio, Λ is the total length of dislocation lines per unit volume, and L is the average length between nodes. The orientation factor Ω is a function of the angle θ between the $\hat{c}$ axis of the sample and the axis of rotation. For standing shear waves[9,13] $\Omega(\theta = 0°) = \frac{1}{2}$, and Ω has a minimum at $\theta \approx 45°$ due to the transformation properties of the shear stress tensor. Because of the large effect of dislocation motion on G and a 10% agreement of the measured G with calculated $G(0°)$[14] we estimate that $\theta \lesssim 10°$ for the crystal in Fig. 1 and set $\Omega = \frac{1}{2}$ and obtain $\Lambda L^2 = 2.0$.

Ultrapure ^{4}He samples allowed study of the internal dissipation of dislocation motion which is described by a friction parameter B.[9] In Fig. 2, we show data from two samples at 48 bars. The large difference between the samples can be fully understood by assuming $\theta \approx 0°$ for the ^{4}He crystal with high dissipation and $\theta \approx 45°$ for the other sample. The friction coefficient B is determined from the equation[9,12]

$$\Delta = 1/2Q = (120\Omega\Delta_0\Lambda L^2)(\omega BL^2/\pi^2 C), \tag{3}$$

where Δ is the logarithmic decrement and $C = 2Ga^2/\pi(1-\nu)$. At 48 bars we find $BL^2 = 1.3$

$\times 10^{-13} T^2$ (in cgs units). Notice the simple T^2 temperature dependence.

Assuming an exponential distribution of dislocation lengths[15] and using our dislocation parameters we can construct the whole dissipation curve as a function of frequency[9,12] (Fig. 3). For comparison we have plotted in the same figure results of Tsymbalenko at kilohertz frequencies[7] and of Hiki and Tsuruoka at megahertz frequencies.[1] These are all of the available direct attenuation data at 32 bars and 1.7 K. The agreement is excellent when the different sample geometries and annealing procedures are taken in consideration. We can also compare our results with Wanner, Iwasa, and Wales,[4] who measured the ultrasound velocity in hcp ^{4}He as a function of temperature and calculated $\Lambda L^2 \geqslant 0.1$ and $BL^2 = (1-3) \times 10^{-13} T^n$ (in cgs units) where $n = 2 \pm 0.5$. Iwasa, Araki, and Suzuki[6] obtain $\Lambda L^2 \geqslant 0.25$ and $BL^2 = 8 \times 10^{-14} T^3$ (in cgs units). ΛL^2 as measured by ultrasound is a factor of 10 less than our low-frequency result. Because ΛL^2 should be independent of average line length L, the difference might be due to the deviations of length distribution from the assumed exponential distribution. Calder and Franck[5] have measured ultrasound attenuation at 124.4 bars. By reanalyzing their results we find $BL^2 \propto T^2$ within the scatter of their data. Hiki and Tsuruoka explained their ultrasound attenuation results by jog pinning of dislocation lines.[1] We saw no evidence for jog pinning above 1.5 K, in agreement with Hikata, Kwun, and Elbaum.[3]

In conclusion we have measured the dislocation parameters of hcp ^{4}He at 331 Hz using crystals with differing concentrations of ^{3}He impurities. We have shown that measured parameters describe the dislocation dissipation in hcp ^{4}He over five decades in frequency. All the internal dissipation measurements over that frequency range are in good agreement with each other and the dislocation theory of Granato and Lücke. We grew ultrapure ^{4}He crystals having a shear modulus as small as 60% of the shear modulus determined from the sound velocity. This reduction of shear modulus by dislocation motion is one of the largest ever observed. We have shown that the friction parameter B is proportional to T^2. Although at present there is no theory to explain this temperature dependence, possible sources for the dissipation are phonons, residual ^{3}He impurities, and kinks on the dislocation line.

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[1]Y. Hiki and F. Tsuruoka, Phys. Lett. 56A, 484 (1976), and 62A, 50 (1977); F. Tsuruoka and Y. Hiki, Phys. Rev. B 20, 2702 (1979).

[2]I. Iwasa and H. Suzuki, in *Proceedings of the Third International Conference on Phonon Scattering in Condensed Matter, Providence, Rhode Island, 1979,* edited by H. Maris (Plenum, New York, 1980).

[3]A. Hikata, H. Kwun, and C. Elbaum, Phys. Rev. B 21, 3932 (1980).

[4]R. Wanner, I. Iwasa, and S. Wales, Solid State Commun. 18, 853 (1976).

[5]I. D. Calder and J. P. Franck, Phys. Rev. B 15, 5262 (1977), and J. Low Temp. Phys. 30, 579 (1977).

[6]I. Iwasa, K. Araki, and H. Suzuki, J. Phys. Soc. Jpn. 46, 1119 (1979).

[7]V. L. Tsymbalenko, Zh. Eksp. Teor. Fiz. 74, 1507 (1978) [Sov. Phys. JETP 47, 787 (1978)], and Zh. Eksp. Teor. Fiz. 76, 1690 (1979) [Sov. Phys. JETP 49, 859 (1979)].

[8]E. L. Andronikashvili, I. A. Gachechiladze, and V. A. Melik-Shakhnazarov, Fiz. Nizk. Temp. 1, 635 (1975) [Sov. J. Low Temp. Phys. 1, 305 (1975)].

[9]A. Granato and K. Lücke, J. Appl. Phys. 27, 583 (1956).

[10]A. S. Greenberg, thesis, Cornell University (unpublished).

[11]Equation (1) and the assumption $x = 0.3$ ppm give for the sample in Fig. 1 $L_c(T) = 4 \times 10^{-2}$ cm at the breakaway temperature for the smallest amplitude. This contradicts the generally accepted value $L = 5 \times 10^{-4}$ cm. However, Eq. (1) is an approximation for a δ-function distribution of line lengths. For an exponential distribution at low frequencies ~ 65% of the dislocation effect on shear modulus comes from lines longer than $3L$. Also because this sample had the highest breakaway temperatures of all our commercial ^{4}He samples, x was probably bigger than 0.3 ppm, but probably less than 3 ppm. We then get $L \gtrsim 10^{-3}$ cm for sample in Fig. 1.

[12]A. V. Granato and K. Lücke, in *Physical Acoustics*, edited by W. P. Mason (Academic, New York, 1966), Vol. IV, p. 225.

[13]R. E. Green, Jr., and T. Hinton, Trans. Met. Soc. AIME 236, 435 (1966).

[14]We are actually measuring an average shear modulus in the plane perpendicular to the rod axis $G = \frac{1}{2}(C_{3131} + C_{3232})$, where C_{lkmn} is the tensor of elastic moduli. G is a function of the angle θ between the crystal axis $\hat{c}$ and the rod axis: $G(0) = C_{44}$, $G_{\max}(\theta = 42°) = 1.20 C_{44}$, and $G_{\min}(\theta = 90°) = 0.89 C_{44}$.

[15]For a δ-function distribution our data gives $\Lambda L^2 = 12$ and $BL^2 = 3 \times 10^{-12} T^2$ (in cgs units). However, this gives $\Delta_{\max}/\Omega = 1.7$ and $f_{\max} = 32$ kHz in good agreement with the corresponding exponential distribution values shown in Fig. 3. Therefore, for overdamped dislocation motion the theoretical curve shown in Fig. 3 is reasonably insensitive to the selected distribution function.