conversion using SRS is potentially a very efficient process. The generation of useful radiation in the sidebands requires suppression of the cascade process, and the conversion efficiency to the nth order is limited only by the shift ratio (ω_R/ω_0) and the input spectrum. For example, if this limit $(\nu_R/\nu_0 \sim 0)$ is not taken, then one can show that the parametric gain is bounded[14]:

$$z|\beta| < (\nu_0/\nu_R)[\sum_r E_r E_{r-1}^*/\sum_r |E_r|^2]. \tag{19}$$

Thus for large bandwidth (e.g., vibrational SRS in hydrogen) the parametric gain and hence the conversion efficiency to high orders is limited. A fuller account of the process and further exact solutions for more general molecular dynamics and propagation where ω_R/ω_0 is not small will be published elsewhere.[14]

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Possibility of Achieving Ignition in a High-Field Ohmically Heated Tokamak

C. E. Wagner[(a)]
Laboratory for Applied Plasma Studies, Science Applications, Inc., La Jolla, California 92037
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There exists a regime in parameter space where a small high-field Ohmically heated tokamak may be capable of reaching thermonuclear ignition. Results of numerical simulations of the minimum ignition conditions are presented, including empirical, sawtooth, and magnetic-field ripple diffusion and the effects of impurities. An ignition condition is derived and compared with the results of the numerical simulations.

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It has been suggested that thermonuclear ignition may be achievable in a small high-field tokamak[1] with or without auxiliary heating provided by compression, *rf*, and/or neutral-beam auxiliary heating.[2] Here, with full transport-model numerical simulations, I confirm the accuracy of these analytic estimates and point out that recent engineering calculations[3] and experimental and theoretical results suggest the feasibility of achieving ignition with Ohmic heating alone. This would have vast advantages in terms of the simplicity of a proof-of-principle experiment and in terms of the economic viability of reactor concepts emerging from this approach.[3]

The severe engineering constraints of materials stress and heat removal in the toroidal field and Ohmic heating magnets dictate a window in size for the major radius $0.6 \lesssim R \lesssim 0.9$ m. Within these dimensions, toroidal magnetic fields of 16 to 20 T appear feasible.[3] Accordingly, I have investigated numerically the minimum ignition conditions at radii lying in this "window."

For this analysis, I have used the latest version[4] of the MAK1 tokamak reactor code. This code solves conservation equations for hydrogen (deuterium and tritium), electron and ion energy, and toroidal and poloidal flux for plasmas with circular or elliptical cross sections. It follows the evolution of the hot-α-particle population with nonlocal time-delayed deposition of the α-particle energy. The impurity charges are computed from the coronal equilibrium "average-ion" model.[5] A two-species neutral transport model[6] is used which computes plasma recycle and impurity sputtering self-consistently with the plasma efflux. The code incorporates a transport model which includes "mixed-collisionality" neoclassical diffusion,[7] Ware-pinch, bootstrap current, empirical scaling of electron heat conductivity, sawtooth diffusion, and ripple diffusion. This model gives an accurate representation when benchmarked with Princeton Large Torus, Alcator-*A*, or Alcator-*C* data.

To gain insight into the dominant physics of a high-field tokamak, consider first a simple idealized model of ignition. Assume that diffusion due to the sawtooth instability within the $q=1$ surface, r_s, is rapid enough that $T=T_e=T_i$ and $j=(2B/\mu_0 R)(1+\kappa^2)/2\kappa$ are constant out to r_s. Here κ is the ellipticity. I will evaluate four basic energy fluxes within r_s, namely Ohmic heating P_Ω, α-particle heating P_α, bremsstrahlung P_r, and electron thermal conduction Q_c. With a thermal conduction of the form

$$Q_c \simeq (3nkT\chi_e/a)(1+\kappa^2/2\kappa^2)^{1/2},$$

there exists a temperature T_1 at which the Ohmic heating within r_s,

$$\int_{V_s} P_\Omega\, dV = j^2 \int_{V_s} \eta_\parallel\, dV \equiv C_\Omega T^{-3/2},$$

equals the conduction loss through r_s. Solving $\int_{V_s} P_\Omega\, dV = \int_{S_s} Q_c\, dS$ at $r_s = a/2$, we obtain

$$n\hat{\chi}_e \equiv \frac{n\chi_e}{(1.8)10^{18}(B/A)^2[\frac{1}{2}(1+\kappa^2)]} = \left(\frac{T_0}{T_1}\right)^{5/2} \qquad (1)$$

in MKS units (T in kiloelectronvolts), where $A = R/a$ is the aspect ratio and $T_0 = 4.3$ keV, the ideal ignition temperature. In Eq. (1), the numerical coefficient includes a neoclassical trapped-particle correction to the Spitzer-Harm resistivity

$$\frac{\eta_\parallel}{\eta_s} = \{1 - [1.95(r/R)^{1/2} - 0.95 r/R]/(1+\nu_*)\}^{-1} \qquad (2)$$

averaged over V_s. ν_* is the usual collisionality parameter. At typical ignition collisionalities $\int(\eta_\parallel/\eta_s)dV \approx 2.0$. Equation (1) assumes that the sawtooth turbulence neither enhances nor decreases the neoclassical resistivity over this value.

Since P_α and P_r are proportional to n^2 and both P_Ω and Q_c are independent of density (if Alcator scaling, $\chi_e \propto 1/n$, is assumed), I note that ignition occurs if, for all T,

$$W \equiv \int_{V_s} P_\Omega\, dV + \int_{V_s} P_\alpha\, dV - \int_{V_s} P_r\, dV - \int_{S_s} Q_e\, dS > 0 = C_\Omega T^{-3/2} + C_\alpha n^2 T^{7/2} - C_r n^2 T^{1/2} - C_c T > 0, \qquad (3)$$

where I have made the approximation $\langle\sigma v\rangle_{\rm DT} \propto T^{7/2}$. Here $C_\Omega = C_c T_1^{5/2}$ and $C_r = C_\alpha T_0^3$, from the definitions of T_0 and T_1. This implies that a sufficient condition for ignition is

$$n\hat{\chi}_e < 1 \qquad (4)$$

with the density in the range

$$n_{\max,\min} = 1.1\times 10^{20}(B/R)[(1+\kappa^2)/2\kappa] f(\eta\hat{\chi}_e)_{\max,\min}, \qquad (5)$$

where a numerical evaluation of $f(n\hat{\chi}_e)_{\max,\min}$ is presented in Fig. 1.

The maximum density restriction arises when $T<T_0$, as here the radiation loss outweighs the α-particle gain. For $T>T_0$, the density must be above a minimum value for the difference of the α-particle heating less the radiation loss to increase faster with temperature than the difference of the conductive losses and the Ohmic heating. At values of n satisfying Eq. (5), no programming of the density is required to keep $dW/dt>0$ for all T as the temperature rises to ignition. Note that at the limiting conductivity $n\hat{\chi}_e$

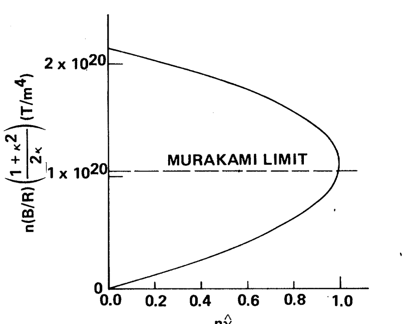

FIG. 1. Density limits for ignition.

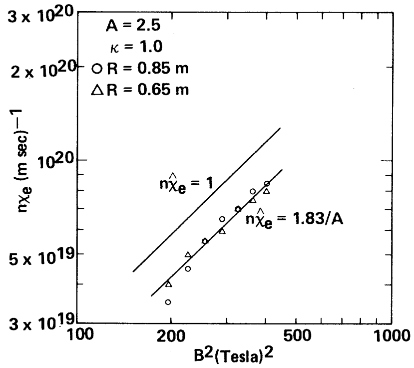

FIG. 2. Minimum ignition conditions ($n\chi_e$ vs B^2).

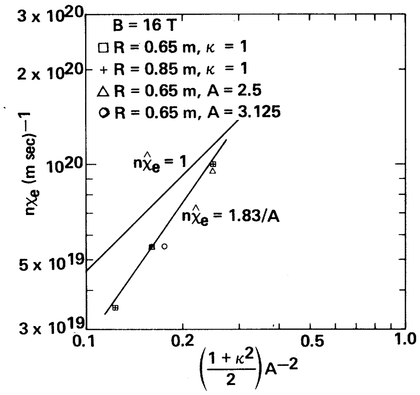

FIG. 3. Minimum ignition conditions [$n\chi_e$ vs $A^{-2}\frac{1}{2}(1+\kappa^2)$].

$=1$, the density must lie virtually at the Murakami limit[8] (adjusted for recent experimental data),[9] which gives the highest density at which tokamaks are currently able to operate.

One assumption of this model is the neglect of neoclassical ion heat conduction. At typical ignition parameters, $\chi_{nc}/\chi_e \sim 0.2$. Other physically reasonable assumptions are the neglect of synchrotron radiation and the local deposition of the α-particle energy. The latter assumption requires $IA > 7.5$ MA.[10] Equations (4) and (5) are consistent with the more detailed energy balance derived in Ref. 2.

Present understanding of tokamak transport theory does not give a definitive value of $n\chi_e$ at the ignition parameters. Values of $n\chi_e = (5-7) \times 10^{19}$ (m sec)$^{-1}$ have been inferred from experimental data in Ohmically heated tokamaks.[11] Recent results from beam-heated plasmas in the Princeton Large Torus have suggested a temperature dependence of $n\chi_e \propto T^{-\sigma}$, with $0 \leq \sigma \leq 1$.[12] Until the precise dependence of $n\chi_e$ is the known, the ignition criteria define the toroidal field and geometry requirements of an ignition tokamak only in terms of this parameter.

The importance of small concentrations of low-z (fully stripped) impurities may be estimated by taking $P_\Omega \propto Z_{\rm eff}$ and $P_r \propto \langle z\rangle^2 Z_{\rm eff}$, where $\langle z\rangle = n_e/n_i$ and $Z_{\rm eff} = \sum_i n_j z_j^2/n_e$. The ideal ignition temperature then increases, $T_0 \propto \langle z\rangle^{2/3} Z_{\rm eff}^{1/3}$. Here the limiting conductivity is

$$n\hat{\chi}_e < Z_{\rm eff}^{1/6}/\langle z\rangle^{5/3}. \tag{6}$$

I have investigated the minimum ignition conditions in tokamaks with circular and elliptic cross sections. The results of one-dimensional simulations are plotted in Figs. 2 and 3 at fixed aspect ratio and fixed major radius, respectively. The safety factor at the plasma edge was held constant at $q_a = 2$ in these runs. The plots represent the smallest value of $n\chi_e$ at which global ignition

$$W_\alpha > W_r + W_c \tag{7}$$

occurred. Here W_α is the α-particle power delivered to the plasma, W_r the radiation losses, and W_c the conductive and convective losses. The transport model included anomalous electron conductivity as a parameter, neoclassical (including banana-plateau effects) convection and conduction, Ware-pinch, bootstrap current, ripple diffusion (1% edge ripple from 180 coils), and sawtooth dif-

TABLE I. MAK1 benchmark of low-q Alcator-A discharge. Alcator-A parameters are $B = 6$ T, $I = 2.35 \times 10^5$ A, and $n\chi_e = 7\times10^{19}$ (m sec)$^{-1}$, with a deuterium working gas.

	n_0 (m^{-3})	T_{e0} (keV)	T_{i0} (keV)	τ (sec)
Expt.	5.2×10^{20}	0.97	0.80	1.96×10^{-2}
Comp.	5.4×10^{20}	0.92	0.85	2.0×10^{-2}

TABLE II. MAK1 computations of future Alcator-C parameters.

$n\chi_e$ [(m sec)$^{-1}$]	I (A)	B (T)	n_0 (m^{-3})	T_{e0} (keV)	T_{i0} (keV)
5×10^{19}	1×10^6	14	1.46×10^{21}	1.87	1.85
5×10^{19}	1.6×10^6	14	1.90×10^{21}	2.39	2.38

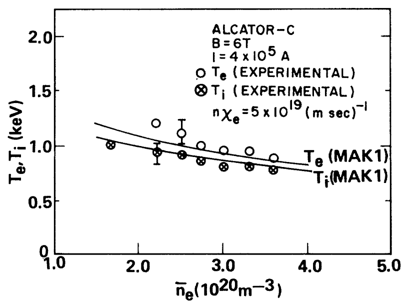

FIG. 4. Benchmark of MAK1 vs Alcator-C data.

fusion. The latter was modeled by 10% of Bohm diffusion inside the $q=1$ surface. Results with full Bohm diffusion in this region are similar, as this level is sufficient essentially to flatten the profiles.

Benchmarks of this model with current experimental results[13,14] and predictions for future experiments are shown in Fig. 4 and Tables I and II. The results of the simulations shown in Figs. 2 and 3 show that the data can be fitted well by an empirical ignition criterion

$$n\hat{\chi}_e < 1.83/A. \qquad (8)$$

The aspect ratio was varied over the range $2 \leqslant A \leqslant 3.125$. It is not surprising that this criterion is somewhat more restrictive than Eq. (4), as many effects neglected in the idealized model—ion neoclassical conduction, ripple diffusion, synchrotron radiation, nonlocal α-particle energy deposition, and the presence of He ash—work toward a more stringent criterion. Typical optimal central densities for ignition are found near $n_0 = 2\times10^{21}$ m^{-3}.

The apparent dependence of the criterion Eq. (8) on aspect ratio is due to two effects. At larger aspect ratios the ion neoclassical terms become nonnegligible. Also the trapped-particle corrections to $\eta_\parallel$ have some dependence on A, though not quite a linear dependence. The plasma profiles and the location of r_s do not appear to vary significantly with A.

The effects of impurities were investigated with a model which allowed the plasma efflux (including the hot alphas) to sputter impurities from the wall. These then were introduced into the plasma as 5-eV neutrals. Blistering effects were neglected. A marginal ignition case was chosen, i.e., $B=16$ T, $A=2.5$, $\kappa=1$, and $n\chi_e = 5\times10^{19}$ (m sec)$^{-1}$. With beryllium walls the plasma still ignited, while with copper walls it did not, albeit failing narrowly ($T_{i0}=5.7$ keV). Note that these ignition parameters do not represent an extrapolation of q_a, ν_*, or β_{max} beyond currently achieved tokamak results. The extrapolation in B/R is less than a factor of 2.

I have developed idealized theoretical [Eqs. (4)–(6)] and slightly more restrictive empirical [Eq. (8)] ignition criteria for Ohmically heated tokamaks. These criteria identify a regime in parameter space where it may become feasible to attain thermonuclear ignition in a small Ohmically heated tokamak.

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