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[13]$\mathcal{S}(\mathscr{I})$ is the space of C^∞ fields such that they and all their derivatives fall off faster than any power of u as $u \rightarrow \pm\infty$ on $\mathscr{I}$ where u is an affine parameter along n^a.

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Masses and Widths of Odd-Parity N and Δ Resonances

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The masses and widths of $S=0$ resonances in the $(\underline{70}, 1^-)$ and $(\underline{56}, 1^-)$ multiplets have been fitted with use of a quark model with hyperfine interactions. Three models for the decay of these and other states have been examined, and it is concluded that the usual spectator model for the decays must be modified. No positive evidence is found for a tensor force, while conflicting evidence is found for a three-body spin-orbit force, and the $(\underline{56}, 1^-)$ mass is lower than expected.

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The quark-shell model of baryons explains most qualitative features of the baryon spectrum. There is a gross level structure which can be identified with the harmonic-oscillator spectrum and which is split into SU(6)⊗O(3) multiplets.[1] Splittings within SU(6)⊗O(3) multiplets have been ascribed to quark masses and quantum-chromodynamics- (QCD) motivated flavor-independent hyperfine interactions.[2] Koniuk and Isgur, using a simple meson-emission model, have provided a qualitative fit to many partial widths.[3] Here we reexamine more quantitatively, using newer data, properties of the $M<2$ GeV odd-parity $S=0$ baryons along with the elastic widths of several other leading-trajectory baryons.

In agreement with the earlier work, we conclude that the QCD-perturbed quark-shell model is qualitatively successful. In detail, however, our results are significantly different in several respects: (1) The $(\underline{56}, 1^-)$ multiplet is lower than predicted. (2) We find no evidence for a tensor force. (3) There is conflicting information about a possible three-body spin-orbit force. (4) Spectator-independent decay models fail to describe the elastic widths.

We use as input data results from the Carnegie-Mellon University–Lawrence Berkeley Laboratory partial-wave analysis[4] (see Table I). The main difference from previous work is that we include components of the $(\underline{56}, 1^-)$ multiplet. The observed $(\underline{56}, 1^-)$ is so low that its octet components mix strongly with the $(\underline{70}, 1^-)$, profoundly altering the pattern of perturbations involving the $(\underline{70}, 1^-)$. It is therefore important to establish whether any simple extension of current ideas is able to accommodate the extra states. The masses of the one-star $D_{13}(1880)$ and an unseen S_{11} are unconstrained in our fits, but we include upper bounds on their elastic widths. In most previous models, the D_{33}-S_{31} mass difference has not been considered.[5] This difference may indicate a need for three-body spin-orbit forces.[6]

The S_{11} partial widths in Table I are quite different from those adopted in previous work or from Particle Data Group averages.[7] There are smaller differences in other widths. Significant

TABLE I. Experimental masses and widths along with fitted widths from models 1–3. Masses in parentheses are fitted values. Widths off by more than two standard deviations are marked by x. All units are in megaelectronvolts.

State	Mass	$\Gamma_{\pi N}$	Γ_1	Γ_2	Γ_3
S_{11} $\underline{70}$	1550	120 ± 47	5x	115	84
$\underline{70}$	1650	98 ± 30	2x	107	99
$\underline{56}$	(1803)	10 ± 20	4	16	17
D_{13} $\underline{70}$	1525	70 ± 9	106x	95x	73
$\underline{70}$	1675	10 ± 7	36x	22	21
$\underline{56}$	(1804)	15 ± 15	48x	24	5
D_{15} $\underline{70}$	1675	61 ± 8	39x	29x	54
S_{31} $\underline{70}$	1620	35 ± 7	1x	18x	35
$\underline{56}$	1890	17 ± 7	2x	9	12
D_{33} $\underline{70}$	1710	34 ± 13	36	27	31
$\underline{56}$	1940	10 ± 6	5	5	9
D_{35} $\underline{56}$	1940	45 ± 15	29	20	56
P_{11} $\underline{56}$	938	66 ± 2	84x	93x	64
P_{33} $\underline{56}$	1232	120 ± 10	82x	85x	125
F_{17} $\underline{70}$	1970	21 ± 10	1x	12	5
F_{37} $\underline{56}$	1950	133 ± 24	61x	87	101
G_{19} $\underline{70}$	2250	48 ± 22	13	12	18
G_{39} $\underline{56}$	2300	17 ± 12	22	37	37
Model error			30	19	5
Model error (P_{11} omitted)			31	16	6

information about the structure of S_{11} states is also obtained from the ηN channel. Instead of fitting the ηN width for the $S_{11}(1550)$, which is strongly correlated with the mass and with the background parametrization, we fit the ratio $R_{\eta\pi} = \alpha(\eta N)/\alpha(\pi N)$ of reduced widths α (coefficients of the center-of-mass momentum). Our value of $R_{\eta\pi} = 1.5 \pm 0.2$ is estimated by expressing the threshold cross-section slope[8] $B = 21.2 \pm 1.8$ mb/GeV in terms of the Breit-Wigner formula. For the $S_{11}(1650)$ we take $\Gamma(\eta N) = 15 \pm 15$ MeV. The η is interpreted as Isgur's equal mixture of strange and nonstrange quarks.[9]

We also include the widths of the P_{11}, P_{33}, F_{37}, F_{17}, G_{19}, and G_{39} resonances in our fits. [$\Gamma(P_{11})$ is given by $g_{\pi NN}{}^2$.] These essentially fix the overall scale of the decay Hamiltonian and the radius parameter in the baryon wave functions. These extra states are especially simple, because they cannot mix with other states in the same harmonic-oscillator band. However, we have included a 0.25 admixture of the $(\underline{70}, 0^+)$ into the nucleon.[10,11]

The Hamiltonian in the sector considered is $H = H_0 + H_C + H_T + H_{LS\,(3)}$, where H_0 is SU(6)$\otimes$O(3) invariant, and H_C, H_T, and $H_{LS\,(3)}$ are contact, tensor, and three-body spin-orbit terms, respectively. Here H_0 is represented by M_{70} and M_{56}, the central $(\underline{70}, 1^-)$ and $(\underline{56}, 1^-)$ masses, taken as free parameters. Harmonic-oscillator wave functions are used in evaluating matrix elements.

We have extended previous calculations of H_C and H_T matrix elements to include the $(\underline{56}, 1^-)$. It turns out that H_C strongly mixes the $\underline{56}$ and $\underline{70}$ doublet spin components, causing the 70-plet mass splittings to increase. We take the strengths C and T of these forces as free parameters, independent of the value $C = T = 250$ MeV, which reproduces the P_{33}-P_{11} mass difference and which would be satisfactory for the $(\underline{70}, 1^-)$ alone.

We omit the two-body spin-orbit force $H_{LS\,(2)}$, which is known to be small.[2] Vector and scalar contributions tend to cancel in $H_{LS\,(2)}$, but not in $H_{LS\,(3)}$.[6] We approximate the form of $H_{LS\,(3)}$ by an expression in which the orbital part involves O(6) generators and does not mix different harmonic-oscillator bands. Matrix elements between 56-plet states vanish. The $(\underline{70}, 1^-)$ matrix elements are proportional to those given by Isgur and Karl.[2] Our strength parameter F_3 is the first-order D_{33}-S_{31} mass difference.

We consider three models for the decay of a resonance. In all three models, we imagine that the decay proceeds via the creation of a $q\bar{q}$ pair which then combines with the three original quarks to form the final baryon and meson. We take our decay Hamiltonian to be of the form $H = \vec{S} \cdot \vec{V}$, where the total spin $\vec{S}$ of the $q\bar{q}$ pair is correlated with some vector $\vec{V}$.[12] The partial width for channel i is then $\Gamma_i = (k/M)|\langle i|H|\mathrm{Res}\rangle|^2$, where k is the center-of-mass momentum for channel i. The meson is treated as a point.

Model 1 is essentially the pion-emission scheme of Horgan[13] in which we take $\vec{V} = g_1 \vec{P}_{\bar{q}} + g_2 \vec{P}_q$, where $\vec{P}_{\bar{q}}$ and $\vec{P}_q$ are the antiquark and quark momenta in the meson. When we set $g_2 = 0$, we get the ${}^3P_0$ model,[14] while if we set $g_2 = g_1$, we get the model used by Faiman and Hendry.[15] The matrix elements are typically of the form $\langle \cdots \| V \| \cdots \rangle \sim Z^{1/2} e^{-Z} P_K(Z)$, where $Z = \frac{1}{6} k^2 b^2$, P is a polynomial of order K, and b is the harmonic-oscillator radius parameter; $K = 0$ for states with $J = N + \frac{1}{2}$ or $J = N + \frac{3}{2}$ (N is the harmonic-oscillator band), while the other states have $K > 0$. The predictions for states with $K = 0$ are independent of g_1/g_2 and are equivalent to those derived from a string-breaking model.[12] This model gives a poor overall fit to the data, primarily because of its inability to fit the "structure-dependent" amplitudes, that is, those with $K > 0$.

Model 2 is an extension of the scheme used by

Koniuk and Isgur,[3] in which each distinct polynomial factor P_K in model 1 is replaced by an independent adjustable constant. For the states considered here, there are four parameters. Although this model greatly improves the S-wave fit, the D-wave amplitudes are helped only slightly. In addition, the P_{11} "width" cannot be fitted.

In model 3, we break the usual spectator picture and allow the values of g_1 and g_2 to depend on the spin and orbital state of the two "spectator" quarks. The $\boldsymbol{P}$-wave spectators enter only through the $(\underline{70}, 0^+)$ component of the nucleon. This generalization is sufficiently flexible to fit all the widths.

For each model, we adjust the parameters by a somewhat unconventional scheme—we minimize the residual model error τ. The model error is the rms discrepancy between the model values and experimental values, which are allowed to float within a range given by the experimental errors. We construct the quantity $\chi^2(\tau) = \sum(\text{data} - \text{model})^2/[(\text{error})^2 + \tau^2]$. If $\chi^2(0)$ exceeds the number of degrees of freedom, ν, we define the model error to be the value of τ for which $\chi^2(\tau) = \nu$; then we minimize τ. [If $\chi^2(0) < \nu$, $\tau = 0$ and we minimize χ^2.] The scale of mass differences here is about twice that of widths, and so we use $\tau(\text{mass}) = 2\tau(\Gamma)$. No model error was allowed in fitting $\boldsymbol{R}_{\eta\pi}$—this was taken as a constraint.

Independent of fits to the widths, we find from our fits to the odd-parity masses that $\boldsymbol{C}$ should be about 75% of the value needed in the ground state. In agreement with Horgan's estimate,[16] M_{56} is about 150 MeV lower than the value predicted in models with simple confining forces.[17] The S_{31} and D_{33} mass shifts are consistent with the 2:1 ratio given by the model with $H_{LS\ (3)}$ and with $F_3 \sim 90$ MeV. The signs are such, however, that this would make the problem of the $\Lambda(1520)$-$\Lambda(1405)$ masses more severe than in the Isgur-Karl model.[2] The masses alone favor a small value of T.

The mixing of states caused by hyperfine interactions influences the observed widths. In previous work, the $\boldsymbol{S}_{11}$ widths, especially $\boldsymbol{R}_{\eta\pi}$, have provided evidence for a strong tensor force.[2,3] Our calculations lead to a different conclusion, partly because the $S_{11}(1550)$ is primarily mixed with the $(\underline{56}, 1^-)$, and partly because the $(\underline{70}, 0^+)$ part of the nucleon brings in additional terms. As shown in Table II, the best fits have small $\boldsymbol{T}$; however, if $T = C$ the model error is only increased by a few megaelectronvolts. The N^* widths favor $F_3 \approx 0$. These conclusions are not significantly changed if Particle Data Group values for the S_{11} widths are used; the main effect is on the g_i for $\boldsymbol{P}$-wave spectators.

The fitted elastic widths (Table I) show that, of the models considered, only model 3 is satisfactory. We do not have a fundamental explanation for the phenomenological parameters listed in Table III, and it may be possible to find alternative parametrizations of the data. However, the ratio $\Gamma(\boldsymbol{P}_{11})/\Gamma(\boldsymbol{P}_{33})$ alone shows that the pair-creation constants must be spectator dependent. In fact, the fits are essentially unchanged if the $\boldsymbol{P}_{11}$ is left out, so the odd-parity widths and the value of ${g_{\pi NN}}^2$ provide independent support for model 3.

We have also looked at other decay channels. These are generally reasonable in model 3, except that several $\pi\Delta$ and $\rho\Delta$ widths are too large. This may be related to the fact that the models considered here do not have explicit crossing symmetry, and suggest that the nonrelativistic spin-recombination model may need further adjustment.

In model 3, polynomial factors $\boldsymbol{P}_K(\boldsymbol{Z})$ occur in many predicted widths. These factors, which were omitted in model 2, make it possible to fit the relatively small $(\underline{56}, 1^-)$ widths and the larger $(\underline{70}, 1^-)$ widths with the same parameters. Thus, our ability to obtain a satisfactory fit provides indirect support for the quark-shell model, and, in particular, for the interpretation of the $(\underline{56}, 1^-)$ as

TABLE II. Parameters used in the mass calculation, in megaelectronvolts. M_{70} and M_{56} are the central masses of the $(\underline{70}, 1^-)$ and $(\underline{56}, 1^-)$ multiplets. C, T, and F_3 are the strengths of contact, tensor, and three-body spin-orbit forces. τ is the model error when these parameters were used with model 3 for the widths.

M_{70}	M_{56}	C	T	F_3	τ
1624	1836	194	38	42	5
1636	1814	194	194	66	6
1639	1822	250	250	78	12
1620	1839	191	44	0	7
1622	1838	183	183	0	9

TABLE III. Values of g_1 and g_2 vs spectator configuration in units of $\text{GeV}^{-1/2}$. A value of $\boldsymbol{b} = 2.6\ \text{GeV}^{-1}$ was used.

	^3S	^1S	^3P	^1P
g_1	18	20	31	18
g_2	14	-6	-35	-44

a normal three-quark excitation having a specific spatial wave function.

In summary, we find that the widths from Ref. 4 are consistent with a simple model. The model can be tested by applying it to other states in the $N=2$ and $N=3$ bands, but this is limited by scarcity of data and by the increased complexity of mixing effects. It is now clear that mixing with higher bands is important both for $N=0$[2,10] and for $N=1$; it is likely that such mixing will be even more important for higher states. The fact that Δ, N, and Λ states give different indications about F_3 suggests that some of the masses may have substantial contributions from dynamical effects outside the simple hyperfine model. The low mass of the $(\underline{56},1^-)$ remains a puzzle. Perhaps the normal three-quark state is weakly mixed with a higher state having excited gluons[17]; if the gluonic component did not couple to the elastic channel, it would have a minor effect on the matrix elements used here.

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