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Asymptotic Quantization of the Gravitational Field

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A new approach to quantum gravity is proposed. The framework is sufficiently general to handle nontrivial space-time topologies required, e.g., in the black-hole formation and evaporation processes, and yet it incorporates notions such as energy-momentum, angular momentum, spin, mass, and discrete symmetries, normally associated with the Poincaré group.

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In this Letter, I shall outline a new approach to quantum gravity. The key idea is to use the framework describing gravitational radiation in *exact* general relativity to isolate the "true" gravitational degrees of freedom, and, symplectic methods, to quantize them. The analysis is geared to obtain a S-matrix description. However, it differs from other S-matrix approaches in a crucial way: At no stage do I introduce a background metric or linearize Einstein's equation. Thus, for example, gravitons emerge as "asymptotic entities" in the exact theory, rather than spin-2 quanta on a flat background.

Isolation of the radiative degrees of freedom.—Penrose's[1] null infinity, $\mathscr{I}$, is a powerful tool in the S-matrix theory of zero-rest-mass fields.[2] Consider, for example, Maxwell fields in Minkowski space. A regular source-free field F_{ab} is completely characterized by ϵ_a, the pullback to $\mathscr{I}$ of $F_{ab}n^b$, where n^b is the (null) normal to $\mathscr{I}$. This ϵ_a represents the radiative degrees of freedom of F_{ab} and can be quantized directly on $\mathscr{I}$, without any reference to the "interior" of space-time. Although equivalent, via field equations, to the usual quantization method, this procedure avoids altogether the use of momentum space and is therefore well suited for extension to the gravitational case. An analysis of Yang-Mills fields along these lines has recently been carried out by Newman[3] and by Hawking and Pope.[2]

In the gravitational case, quantum considerations were in fact among the main motivations for the original investigation of null infinity.[1,4] Unfortunately, the structure turned out to be substantially more complicated than that, say, in the Maxwell case: Several asymptotic fields, interrelated via nonlinear equations, were involved, making it difficult to single out, in an invariant way, the fundamental fields from which all "radiative aspects" can be derived directly. Recently, however, Geroch[5] recast this description in an intrinsic framework, thereby providing a badly needed stepping stone. In what follows, I use his notation.

Fix an asymptotically empty and flat[6] space-time $(\hat{M},\hat{g})$ and a Penrose completion (M,g) thereof, such that $\mathscr{I}$ is divergence-free. Then, the connection ∇ on (M,g) induces a connection D on $\mathscr{I}$, satisfying

$$D_a \underline{g}_{bc} = 0;\ D_a n^b = 0;$$
$$D_a V_b = 2D_{[a}V_{b]} + 2\mathcal{L}_{V^a}\underline{g}_{ab} \text{ if } V\cdot n = 0; \tag{1}$$

where, $\underline{g}_{ab}$ is the pullback to $\mathscr{I}$ of g_{ab}; n^b, the normal to $\mathscr{I}$; and, V^a, any vector field on $\mathscr{I}$ such that $V^a \underline{g}_{ab} = V_b$. Under a conformal rescaling, $g \to \omega^2 g$, (which keeps $\mathscr{I}$ divergence-free) we have

$$D_a K_b \to D_a K_b - 2\omega^{-1} K_{(a} D_{b)} \omega + (\underline{\omega}^m K_m)\omega^{-1} \underline{g}_{ab}, \quad (2)$$

where, $\underline{\omega}^m$ is the restriction to $\mathscr{I}$ of $\nabla^m \omega$. This D, together with pullbacks to $\mathscr{I}$ of various tensor fields and equations on (M, g) enables one to introduce various asymptotic notions of physical interest.[7]

It turns out, remarkably, that these connections on $\mathscr{I}$ represent precisely the radiative degrees of freedom of $\hat{g}_{ab}$! To see this, it is convenient to fix, *on* $\mathscr{I}$, a conformal frame $(\underline{g}_{ab}, n^a)$. Then, connections D on $\mathscr{I}$ corresponding to $\hat{g}_{ab}$ form an equivalence class $\{D\}$: Given D and $\tilde{D}$ in $\{D\}$, there exists an f on $\mathscr{I}$, such that, for all K_a,

$$(D_a - \tilde{D}_a) K_b = f(K_c n^c) \underline{g}_{ab}. \quad (3)$$

(I have $D \to \tilde{D}$ if $g \to \omega^2 g$, with $\omega = 1$ on $\mathscr{I}$.) Fix a connection D in $\{D\}$. Then, I claim that, because of Eq. (1), there exists a unique tensor field $\underline{S}_a{}^b$ on $\mathscr{I}$ such that: $S_a{}^b n^a = \sigma n^b$ for some function σ; $\underline{S}_{ab} := \underline{S}_a{}^c \underline{g}_{cb}$ is symmetric; $\underline{S}_{ab} \underline{g}^{ab} = \mathcal{R}$, the lift to $\mathscr{I}$ of the scalar curvature of the two-metric on the manifolds of orbits of n^a induced by $\underline{g}_{ab}$; and,

$$2D_{[a} D_{b]} K_c \equiv \mathcal{R}_{abc}{}^d K_d = (\underline{g}_{c[a} \underline{S}_{b]}{}^d - \underline{S}_{c[a} \delta_{b]}{}^d) K_d \quad (4)$$

for all K_c. Consequently, one can define

$$*K^{ab} := \tfrac{1}{4} \epsilon^{amn} D_m \underline{S}_n{}^b, \quad (5)$$

$$N_{ab} := \underline{S}_{ab} - \rho_{ab}, \quad (6)$$

where[7] ϵ^{amn} is the alternating tensor on $(\mathscr{I}, \underline{g}_{ab})$ and ρ_{ab} is the unique symmetric tensor field on $\mathscr{I}$ satisfying $\rho_{ab} n^b = 0$, $D_{[a} \rho_{b]c} = 0$, and $\rho_{ab} \underline{g}^{ab} = 0$. Then, it follows that $*K^{ab}$ and N_{ab} are symmetric and trace-free and that $D_a *K^{ab} = 0$. Hence, $*K^{ab}$ and N_{ab}, introduced here with use of only D, are the same fields as those introduced by Geroch in Ref. 5 by pulling back to $\mathscr{I}$ space-time fields: N_{ab} is the news-tensor while the five components of $*K^{ab}$ correspond in the Newman-Penrose[8] notation, to $\Psi_4{}^0$, $\Psi_3{}^0$, and $\mathrm{Im}\Psi_2{}^0$. Next, I claim that the two fields and their properties are unchanged if D is replaced by any $\tilde{D}$ in $\{D\}$. Thus, all information about "radiative aspects" of $\hat{g}_{ab}$ can be extracted from $\{D\}$ alone. Furthermore $\{D\}$ has "no other information": For example, $\mathrm{Re}\Psi_2{}^0$, which represents "longitudinal modes," cannot be obtained from $\{D\}$.[9]

Hamiltonian formulation.—I now wish to construct the phase space of *all* radiative modes. Since we do not have any background space-time to refer to, I must first introduce the kinematic arena where the action is to take place. Fix a three-manifold $\mathscr{I}$, equipped with a collection of pairs $(\underline{g}_{ab}, n^a)$ of nowhere vanishing fields so that (i) $\underline{g}_{ab} V^b = 0 \leftrightarrow V$ is proportional to n; (ii) $\mathcal{L}_n \underline{g}_{ab} = 0$; (iii) $(\underline{g}, n)$ and $(\underline{g}', n')$ are both in the collection if and only if $\underline{g}' = \omega^2 \underline{g}$ and $n' = \omega^{-1} n$ for some ω satisfying $\mathcal{L}_n \omega = 0$; and (iv) n^a is a complete vector field and its manifold of orbits is diffeomorphic to S^2. This is the required arena.

Fix a pair $(\underline{g}, n)$ on $\mathscr{I}$. Denote by $\mathcal{C}$ the collection of connections D on $\mathscr{I}$ satisfying Eq. (1). D and $\tilde{D}$ in $\mathcal{C}$ will be said to be equivalent if they satisy Eq. (3) for some f. Denote the space of equivalence classes by Γ. This affine space Γ is the phase space of radiative modes.[10]

Let us first "count" the degrees of freedom. It is easy to show that, for any two connections D and $\tilde{D}$ in $\mathcal{C}$, $(D_a - \tilde{D}_a) K_b = \Sigma_{ab} K_c n^c$ for some symmetric tensor field Σ_{ab} satisfying $\Sigma_{ab} n^b = 0$. It then follows from Eq. (3) that the difference between any two elements of Γ can be completely characterized by γ_{ab}, the trace-free part of Σ_{ab}. Thus, by fixing an arbitrary element as origin, Γ can be coordinatized by these γ_{ab}. How many components does γ_{ab} have? Precisely two.

Next, I introduce a symplectic structure Ω on Γ. Since Ω is to be a tensor field on Γ, and since tangent vectors at any point of Γ represent linearized radiative modes, one needs, in effect, the expression of the symplectic tensor of linearized gravity off arbitrary asymptotically flat space-times. A simple extension of the known results[11] gives

$$\Omega_{\{D\}}(\gamma, \tilde{\gamma}) := \int_{\mathscr{I}} (\gamma_{ab} \mathcal{L}_n \tilde{\gamma}_{cd} - \tilde{\gamma}_{ab} \mathcal{L}_n \gamma_{cd}) \underline{g}^{ac} \underline{g}^{bd} d\mathscr{I}, \quad (7)$$

where γ and $\tilde{\gamma}$ are arbitrary tangent vectors at the point $\{D\}$ of Γ, and $\underline{g}^{ac}$ any "inverse"[7] of $\underline{g}_{ac}$ on $\mathscr{I}$. Equation (2) implies that the expression is conformally invariant. Also, as one might expect, Ω has the dimensions of action and is constant with respect to the affine structure of Γ.

The action of the Bondi-Metzner-Sachs (BMS) group[4] B on $\mathscr{I}$ provides a powerful check that (Γ, Ω) is the "correct" phase space. Not only does it preserve the collection $\mathcal{C}$ of preferred connections on $\mathscr{I}$, but it also respects the equivalence relation in Eq. (3). Furthermore, the resulting action of B on Γ leaves Ω invariant.[10] One can compute the Hamiltonians generating the corresponding canonical transformations. For the special case of BMS translations, αn^a, I obtain

$$H(\{D\}) = -\tfrac{1}{2} \int_{\mathscr{I}} \alpha N_{ab} N_{cd} \underline{g}^{ac} \underline{g}^{bd} d\mathscr{I}. \quad (8)$$

This is precisely the Bondi-Sachs formula for flux of four-momentum.[12]

This phase-space formulation is very reminiscent of gauge theories: The basic variable in both cases is a connection. One may therefore regard $\{D\}$ as the "potential" and N_{ab}—the nontrivial part of the curvature of $\{D\}$—as the "field."

Quantization.—I begin by introducing an operator-valued distribution $\underline{N}_{ab}(x)$—the news—on $\mathcal{G}$, satisfying the canonical commutation relation (CCR):

$$[\underline{N}(f),\underline{N}(\tilde{f})]=(\hbar/i)\Omega(f,\tilde{f})\mathcal{G}, \tag{9}$$

with $\underline{N}(f)=\int \underline{N}_{ab}f_{cd}\underline{g}^{ac}\underline{g}^{bd}d\mathcal{G}$, where f_{ab}, satisfying $f_{ab}=f_{(ab)}$, $f_{ab}n^b=0$, and $f_{ab}\underline{g}^{ab}=0$, is a test field in $\mathcal{S}(\mathcal{G})$.[13] [*Classical* observables, $N(f):=N_{ab}f_{cd}\underline{g}^{ac}\times\underline{g}^{bd}d\mathcal{G}$, generate *affine-structure-preserving* canonical transformations on (Γ,Ω) with Poisson brackets given by $\{N(f),N(f')\}=\Omega(f,f')$; hence the introduction of $\underline{N}_{ab}(x)$, and Eq. (9).] The Fock representation can be constructed as usual. Let $\mathcal{H}$ denote the complex Hilbert space of positive-frequency[14] fields $f_{ab}{}^{+}$ on $\mathcal{G}$, with the inner-product $\langle f^+,f'^+\rangle=(i/\hbar)\Omega(f^+,f'^+)$. This $\mathcal{H}$ is the one-graviton space: One can represent $\underline{N}(f)$ by sums of creation and annihilation operators on the symmetric Fock space $\mathcal{F}$ based on $\mathcal{H}$. This representation of the CCR is, of course, irreducible. It provides a (reducible) unitary representation of the BMS group B. The induced representation of *any* Poincaré subgroup of B can be decomposed in to two irreducible parts: They have the same mass ($m=0$) and spin ($|s|=2$), but opposite helicities. [Graviton state $f_{ab}{}^{+}$ is right (left) handed if $\epsilon^{mnp}l_p\underline{g}_{nb}=\pm i f_{ab}{}^{+}$, where l_p is any covector on $\mathcal{G}$ with $l\cdot n=1$.] The generators of various BMS transformations provide us the quantum momentum-angular-momentum operators.

There exist, in addition, other representations of the CCR of physical interest. Consider, to begin with, the classical observable $Q_{ab}(\theta,\varphi)=\int_{-\infty}^{+\infty}N_{ab}(u,\theta,\varphi)\,du$ on Γ, where we have used a Bondi system on $\mathcal{G}$. $\{N(f),Q_{ab}\}=0$ for all f. Consequently, one might expect, in quantum theory, unitarily inequivalent representations of the CCR labeled by values of the "internal charge" $Q_{ab}(\theta,\varphi)$. This is indeed the case. [The situation is completely analogous to the two-dimensional model,[15] $\Box\Phi=0$, with N_{ab} playing the role of the "field" $d\Phi/dx$, and, Q_{ab} of the "topological charge" $\Phi(x=\infty)-\Phi(x=-\infty)$.] In classical general relativity Q_{ab} has a natural geometrical interpretation—it represents the amount by which the shear-free cross sections of $\mathcal{G}$ in the distant future are supertranslated relative to those in the distant past—whence Q_{ab} takes values in the quotient ST/T of the supertranslation subgroup ST of B by the translation subgroup T. Thus, the emergence of new quantum sectors is intertwined with the enlargement of the translation group to the supertranslation group. The enlargement itself has been a puzzling (but essential) feature of gravitation radiation in the *exact* theory. From a purely quantum theoretic viewpoint, on the other hand, one can show that the new representations are the natural analogs of the infrared sectors in electrodynamics. Thus, one now has an "explanation" of the supertranslation ambiguities: They arise as a combined effect of the infrared behavior and the geometrical nature of the gravitational field. One's experience with Maxwell and Yang-Mills theories indicates that the contributions of the new sectors to asymptotic states would *have* to be included if one is to obtain a consistent scattering—or, superscattering—matrix description.

To apply this framework, one would consider the following situation: asymptotic states come in from $\mathcal{G}^-$, interact—perhaps form a black hole which subsequently evaporates—and scatter off to $\mathcal{G}^+$. One now has a new kinematic basis for describing such processes. Since the topology in the "interior" is unconstrained, one might hope that the formalism would capture interesting phenomena which escape perturbative treatments off a fixed background. Finally, because the framework has many features in common with the covariant, canonical, H space and twistor approaches to quantum gravity, even as it stands, it provides a common platform to compare and contrast ideas and formulas that have emerged from these approaches.

Details will appear elsewhere.

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[7]For details, see Geroch, Ref. 5.

[8]E. T. Newman and R. Penrose, Proc. Roy. Soc. London, Ser. A 305, 175 (1968).

[9]Under $g_{ab} \to \omega^2 g_{ab}$, the transformation of $\{D\}$ is well defined although that of an individual D is not. We have, as in Ref. 5, $N_{ab} \to N_{ab}$ and $*K^{ab} \to \omega^5 *K^{ab}$.

[10]Actually, Γ admits the structure of an affine Fréchet manifold. Ω is then continuous and weakly nondegenerate. (Γ, Ω) provides a new representation of the BMS group, perhaps of mathematical interest in its own right.

[11]T. N. Palmer, J. Math. Phys. 19, 2324 (1978); R. Geroch and B. C. Xanthopoulos, J. Math. Phys. 19, 714 (1978).

[12]The flux expressions for general BMS vector fields are derived in a joint paper with M. Strubel, to be published.

[13]$\mathcal{S}(\mathscr{I})$ is the space of C^∞ fields such that they and all their derivatives fall off faster than any power of u as $u \to \pm\infty$ on $\mathscr{I}$ where u is an affine parameter along n^a.

[14]f^+ is obtained via Fourier decomposition of f with respect to u and is unchanged under permissible rescalings of n^a.

[15]See, e.g., R. F. Streater and I. F. Wilde, Nucl. Phys. B24, 561 (1970).

Masses and Widths of Odd-Parity N and Δ Resonances

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The masses and widths of $S = 0$ resonances in the $(\underline{70}, 1^-)$ and $(\underline{56}, 1^-)$ multiplets have been fitted with use of a quark model with hyperfine interactions. Three models for the decay of these and other states have been examined, and it is concluded that the usual spectator model for the decays must be modified. No positive evidence is found for a tensor force, while conflicting evidence is found for a three-body spin-orbit force, and the $(\underline{56}, 1^-)$ mass is lower than expected.

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The quark-shell model of baryons explains most qualitative features of the baryon spectrum. There is a gross level structure which can be identified with the harmonic-oscillator spectrum and which is split into SU(6)⊗O(3) multiplets.[1] Splittings within SU(6)⊗O(3) multiplets have been ascribed to quark masses and quantum-chromodynamics- (QCD) motivated flavor-independent hyperfine interactions.[2] Koniuk and Isgur, using a simple meson-emission model, have provided a qualitative fit to many partial widths.[3] Here we reexamine more quantitatively, using newer data, properties of the $M < 2$ GeV odd-parity $S = 0$ baryons along with the elastic widths of several other leading-trajectory baryons.

In agreement with the earlier work, we conclude that the QCD-perturbed quark-shell model is qualitatively successful. In detail, however, our results are significantly different in several respects: (1) The $(\underline{56}, 1^-)$ multiplet is lower than predicted. (2) We find no evidence for a tensor force. (3) There is conflicting information about a possible three-body spin-orbit force. (4) Spectator-independent decay models fail to describe the elastic widths.

We use as input data results from the Carnegie-Mellon University–Lawrence Berkeley Laboratory partial-wave analysis[4] (see Table I). The main difference from previous work is that we include components of the $(\underline{56}, 1^-)$ multiplet. The observed $(\underline{56}, 1^-)$ is so low that its octet components mix strongly with the $(\underline{70}, 1^-)$, profoundly altering the pattern of perturbations involving the $(\underline{70}, 1^-)$. It is therefore important to establish whether any simple extension of current ideas is able to accommodate the extra states. The masses of the one-star $D_{13}(1880)$ and an unseen S_{11} are unconstrained in our fits, but we include upper bounds on their elastic widths. In most previous models, the D_{33}-S_{31} mass difference has not been considered.[5] This difference may indicate a need for three-body spin-orbit forces.[6]

The S_{11} partial widths in Table I are quite different from those adopted in previous work or from Particle Data Group averages.[7] There are smaller differences in other widths. Significant

