Linear Magnetoresistance Caused by Sample Thickness Variations

G. J. C. L. Bruls, J. Bass,[a] A. P. van Gelder, H. van Kempen, and P. Wyder
Research Institute for Materials, University of Nijmegen, Toernooiveld, Nijmegen, The Netherlands
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Experiments are presented which show that surface imperfections can give rise to a larger linear magnetoresistance than previously supposed; in fact, large enough to explain many hitherto-unexplained published results. A theoretical model, which takes into account both the finite width of the sample and the Hall fields within it, is shown to describe the experimental results quantitatively, without the use of adjustable parameters.

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The appearance of linear magnetoresistance (LMR) in simple metals such as K, Al, and In has puzzled physicists for several decades. In the literature there remains the question of whether this LMR is primarily intrinsic or extrinsic. Recent interest has centered upon extrinsic causes, for instance magnetic-field nonuniformity along the sample length[1] or the presence of volume defects (voids) in the material. Theoretically[2,3] and experimentally,[4] it has been shown that macroscopic voids can give an LMR proportional to the volume fraction of the voids. However, in carefully prepared samples this volume fraction is much too small to yield the LMR which is actually observed. Macroscopic surface defects have been suggested as additional sources of LMR,[2] but, again, application of the theory of volume defects leads to an LMR too small to account for the usual observations.

In this Letter we demonstrate, both experimentally and theoretically, that there exists an additional LMR due to surface defects which is large enough to explain many of the usual observations. This LMR results from Hall voltage variations within the sample, an effect neglected in previous treatments. We present here some of the experimental data which led us to the discovery of this new effect, and briefly describe our analysis and the results it yields. More extensive data (including discussion of corrections applied for other magnetoresistive effects) and mathematical details of the analysis will be published elsewhere.[5]

Our samples were prepared from very pure polycrystalline Al plates of 2-mm thickness. Spark-erosion techniques were used to form bars of different widths L_y with regularly spaced arms for potential contacts on both sides (Fig. 1 shows a composite sample). Between some pairs of arms were machined projections (p) and between some pairs grooves (g); both crossed the whole width of the sample. In still other cases the regions between the arms were made wedge-shaped (w); these specially prepared pieces were always alternated with flat untreated ones to serve as controls. To remove mechanical damage, the samples were annealed and a variety of cross checks were made to establish that the effects described below were not due to damage introduced during sample preparation.

We made conventional four-probe measurements of transverse LMR with the magnetic field perpendicular to the plane of the arms. We always measured potentials on both sides of the sample (e.g., $V_{AA'}$ and $V_{CC'}$ in Fig. 1), and our data were calculated from the average, except for the wedge-shaped samples, where $V_{AA'}$ and $V_{CC'}$ were very different. The temperature (4.2 K), the field (up to 5 T), and the residual resistivity ratio of the samples (around 20 000) were such that $\omega_c\tau \gg 1$, where ω_c is the cyclotron frequency and τ is the relaxation time. As our measure of the LMR we use the dimensionless Kohler slope: $S=(1/\rho_s)\{\partial[\rho(B)-\rho_s]/\partial(\omega_c\tau)\} = (1/R_{\rm H})\{\partial[\rho(B)-\rho_s]/\partial B\}$. Here ρ_s is the saturation magnetoresistance which would have existed in the absence of LMR, B is the magnetic field, and $R_{\rm H}$ is the high-field Hall constant ($R_{\rm H}=1.0\times10^{-10}$ m^3/C for Al).

For a wedge-shaped sample, our model can be solved exactly. The predicted magnetoresistivity

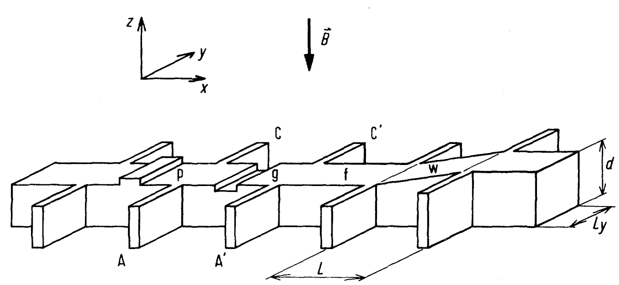

FIG. 1. Composite sample. g, groove; p, projection; f, flat portion; w, wedge-shaped portion. See text for further details.

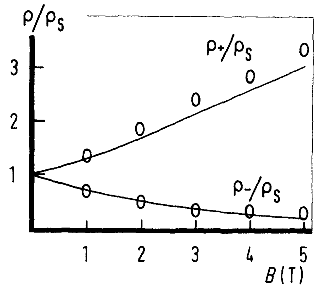

FIG. 2. Asymmetric behavior of the magnetoresistivity of wedge-shaped sample. ρ_+ and ρ_- refer to resistivities from measurements on opposite sides of the sample (see text). Drawn lines are computed values; symbols are data points corrected for residual magnetoresistivity of flat control pieces.

is highly asymmetric and not strictly linear in B. These features are illustrated in Fig. 2. For a sample with grooves or projections, the model predicts a magnetoresistivity which *is* linear in B, and this LMR is predicted to be proportional to (1) the relative depth Δ of the groove (or height of the projection); and (2) the width L_y of the sample. The first feature is illustrated in Fig. 3 for grooves in a sample of width 3.6 mm. Similar results were found for two additional widths and for projections as well as grooves. The second feature is illustrated in Fig. 4 for three different sample widths and three groove depths.

We explain these observations by considering a sample of a homogeneous, uncompensated metal with no open orbits. The width L_y of the sample is constant, but its thickness $d(x)$ parallel to the magnetic field $B=B_z$ varies along the sample length. For $\beta=\omega_c\tau \gg 1$, the transport equations for this metal are (see, e.g., Ref. 6)

$$E_x=\rho_s(J_x+\beta J_y), \quad E_y=\rho_s(-\beta J_x+J_y). \tag{1}$$

Here E_i and J_i $(i=x,y)$ are functions of x, y, and z. We do not use the equivalent equation relating E_z and J_z.

These equations are solved by reducing the number of variables from three to two. This was previously done[2,3] by averaging them over the y coordinate, i.e., the direction perpendicular to both the macroscopic current and the magnetic field. This procedure yields the usual LMR for volume or surface defects mentioned above. We, in contrast, average over the z direction, a procedure which preserves effects which are otherwise suppressed. We define

$$\langle E_i\rangle=(1/d)\int_0^d E_i\,dz, \qquad \langle J_i\rangle=(1/d)\int_0^d J_i\,dz=K_i/d. \tag{2}$$

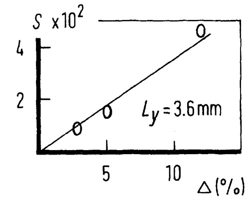

FIG. 3. Kohler slope S as a function of relative thickness variation Δ for grooves in a sample of width L_y =3.6 mm. The straight line is calculated; the symbols are data corrected for residual magnetoresistivity of flat control pieces and LMR due to "volume defect" theory (see text and Ref. 5).

Since we have continuity in two dimensions, $(\partial K_x/\partial x)+(\partial K_y/\partial y)=0$, we can find a function F such that $K_x=(\partial F/\partial y)$ and $K_y=-(\partial F/\partial x)$. The transport equations become

$$E_x=(\rho_s/d)[\partial F/\partial y-\beta(\partial F/\partial x)], \qquad E_y=(\rho_s/d)[-\beta(\partial F/\partial y)-\partial F/\partial x], \tag{3}$$

where E_i, $(\partial F/\partial i)$, and d depend upon x and y.

We assume that the averaged field is rotation-free (a detailed discussion of this assumption will be given elsewhere[5]), and call $(\partial d/\partial x)/d=1/a$. This gives the equation

$$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}-\frac{1}{a}\left(\frac{\partial F}{\partial x}+\beta\,\frac{\partial F}{\partial y}\right)=0. \tag{4}$$

This equation can be solved exactly in closed

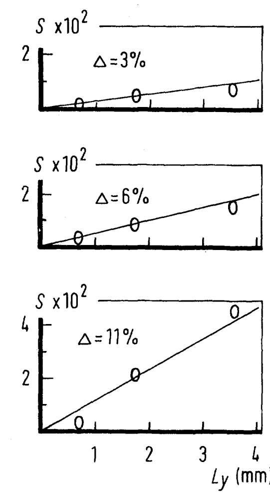

FIG. 4. Kohler slope S as a function of sample width L_y for grooves with fixed values of thickness variation Δ. The straight lines are calculated; the symbols are data corrected for residual magnetoresistivity of flat control pieces and LMR due to "volume defect" theory (see text and Ref. 5).

form if we take a to be constant, which corresponds to a wedge-shaped sample of thickness $d = d_0 \exp(x/a)$. The solutions are

$$K_y = -\partial F/\partial x = 0,$$
$$K_x = \frac{\partial F}{\partial y} = I(\beta/2a)\frac{\exp(\beta y/a)}{\sinh(\beta L_y/2a)}, \quad (5)$$

where I is the total current.

If we choose axes down the middle of the sample, then what we measure are the voltages $V_x(\pm L_y/2)$ at the sides of the sample $(y = \pm L_y/2)$ between the positions $x = 0$ and $x = L$ (see Fig. 1). Calculating E_x and integrating from 0 to L gives

$$V_x(\pm L_y/2) = \frac{I\rho_s}{2d_0}\frac{\exp(\mp\beta L_y/2a)}{\sinh(\beta L_y/2a)} \times [1 - \exp(-L/a)], \quad (6)$$

where $[1 - \exp(-L/a)] = (d_1 - d_0)/d_1 = \Delta$ is the relative thickness variation of the wedge-shaped sample (for our sample, 10%). The difference between these two voltages, $V_x(-L_y/2) - V_x(+L_y/2) = (I\rho_s\beta\Delta)/d_0$, is just equal to the difference in Hall voltage between the place on the sample having thickness d_1 and the place having thickness d_0. That means that the path integral of the electric field, and hence the rotation, is zero. Our mechanism thus works as follows: The requirement that the (averaged) field is rotation-free forces differences in Hall voltage caused by thickness variations to appear along the length of the sample, thereby producing a large LMR.

Finally, we define $\rho_\pm = V_x(\mp L_y/2)(d_0 L_y/LI)$, and obtain

$$\rho_\pm = (\rho_s \beta L_y \Delta/2L)\frac{\exp(\pm\beta L_y/2a)}{\sinh(\beta L_y/2a)}. \quad (7)$$

Figure 2 shows that this equation fits the asymmetric data for our wedge-shaped sample with no adjustable parameters. Furthermore, these solutions satisfy the consistency condition that their average, $\rho = (\rho_+ + \rho_-)/2$, is just equal to the true resistivity defined via the power dissipation in the sample.[5]

Since it is clear from Fig. 2 that we can accurately describe the magnetoresistivity for the wedge-shaped geometry, we apply the same solutions to other geometries. A sample with a groove or a projection is approximated by two wedges of opposite slope bounded by flat pieces. Placing two such wedges in series has the consequence that, seen over the whole sample, the asymmetry vanishes. Adding the resistivities of the flat pieces and of the two wedges, and going to the limit of steep steps (i.e., making a smaller while keeping Δ constant), we get for a single groove or projection, an LMR of

$$\rho(\beta) = \rho_s(\beta L_y \Delta/L + 1) \quad \text{or} \quad S = L_y\Delta/L. \quad (8)$$

We see that S is linearly dependent on Δ, the relative depth (height) of the groove (projection), and also on the width of the sample. For a sample with $L/L_y = 1$, and just a single groove of relative depth $\Delta = 1\%$, we find $S = 10^{-2}$, which is already larger than many reported values in the literature[7,8] ($S = 10^{-3} - 10^{-1}$). For a given sample length, we have checked that two and three grooves give values of S about two and three times larger, respectively, than one groove alone. We thus see that even a single small groove can produce an S of the usually observed magnitude, while at the same time the mechanism can account for a wide variation in values of S, since the width of the sample and the form, number, and orientation of the defects all exert a large influence.

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[a]Permanent address: Physics Department, Michigan State University, East Lansing, Mich. 48824.

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