Stochastic Plasma Heating by a Large-Amplitude Standing Wave

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Strong heating of a laboratory plasma is experimentally observed in the presence of a standing wave when the wave amplitude exceeds a certain threshold. This threshold amplitude is absolutely measured from observations of the ponderomotive force effects and compared to the theoretically expected value for the onset of large-scale stochasticity.

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The motion of a particle in the field of two electrostatic waves is subject to the so-called stochastic instability. The onset of such an instability has been extensively studied numerically.[1-3] This situation applies to the case of a standing wave. Then particles with zero velocity may execute random motion when the stochastic instability sets in. Such an irreversible process has been predicted to lead to strong particle heating in a plasma.[4] Numerical simulations confirm this prediction. This Letter reports measurements of increased electron temperature in a laboratory plasma in the presence of a large-amplitude standing wave. To compare the observed results with theoretical predictions, the absolute amplitude of the wave in the plasma is needed. This is provided through the ponderomotive force which produces a modulation of the plasma density, the relative amplitude of which is related to the absolute amplitude of the wave.[5]

The equation of motion of a particle in a standing wave of frequency $\omega_0/2\pi$ and wavelength $\lambda_0 = 2\pi/k_0$ can be simply written as

$$d^2X/dT^2 = \psi(X,T), \tag{1}$$

with

$$\psi(X,T) = (p/2)[\rho^2 \cos(X-T) - \cos(X+T)] \tag{2a}$$

$$= p \sin X \sin T + (\rho^2 - 1)(p/2)\cos(X-T), \tag{2b}$$

where, in dimensionless units, $X = k_0 x$ and $T = \omega_0 t$ are the spatial and temporal coordinates, $p = ek_0E_-/m\omega_0^2$ is the electric field strength of the backward traveling component of the wave and $\rho^2 = E_+/E_-$ is the amplitude ratio of the forward to backward traveling components of the wave; ρ^2 is related to the standing-wave ratio (SWR) S by $S = (\rho^2-1)/(\rho^2+1)$. When $\rho = 1$, one recovers the case of a pure standing wave; if one considers the particle motion in each of the wave components, one can distinguish two types of particles: (i) the ones which are trapped in the potential troughs of the wave and constitute a resonance and (ii) the circulating ones; these two classes being separated by a frontier curve called the separatrix. This separatrix is expected to disappear above an amplitude p_0 of the wave. This p_0 is estimated to be 0.5 from the "overlapping resonances" criterion,[6] in which the mutual perturbations of the two waves are neglected. In fact, it has been shown (i) by numerical simulations[7] that large scale stochasticity (i.e., a particle trapped in a potential trough of one wave may wander in a potential trough of the other wave) sets on for $p_0 \sim 0.27$, and (ii) by theoretical calculations[8] that the mutual influence of resonances is to repel each other, challenging the overlapping resonance criterion. An approximate renormalization theory[7] was recently built that gives an estimation of the instability threshold for any value of ρ in agreement within 5% to 10% with the numerical simulation results; for $\rho = 1$, it gives $p_0 = 0.25$, in agreement with Greene's result[9] on the standard mapping. This theory exhibits a renormalization transformation that allows one to consider the system at a smaller and smaller scale as it is iterated and relates the instability to the destruction of KAM tori[10] between the two resonances.

The irreversibility introduced by the stochastic instability should lead to the heating of particles. A self-similar treatment, confirmed by a one-dimensional simulation, has shown[4] that, for $p \gg 1$, the particle energy gain is proportional to p.

The experiments were performed in a collisionless argon plasma,[11] 1.50 m long, 2 cm in diameter, drifting from a discharge source along a uniform magnetic field (1.3–2.5 kG). The plasma density of a few times 10^8 cm^{-3} is measured by a resonant cavity located at the source output. All electronic collision processes have mean free paths larger than 10^4 cm. The electron distribution is measured by using a multigrid electrostatic analyzer located at the end of the machine. In order to excite the standing wave, one can use at the source end of the machine, one of two fixed Langmuir probes and, at the cold end, either a fixed Langmuir probe or a plate perpendicular to the plasma column. The choice of the appropriate emitters is made according to the excited wavelength in order to obtain a well-defined standing-wave pattern. This pattern can be observed from the wave amplitude received on an axially movable probe. This probe is also used to detect the ion saturation current, giving longitudinal density profiles. The frequency $\omega_0/2\pi = 20-90$ MHz is smaller than the electron plasma frequency and the waves are Gould-Trivelpiece modes.[12]

The field of the standing wave will exert a ponderomotive force on an electron:

$$F(X) = -(\partial/\partial X)\langle\psi^2(X,T)\rangle = (\rho^2 p^2 \sin 2X)/2, \quad (3)$$

where the brackets denote time averaging and the associated potential is normalized to $m\omega_0^2/2k_0^2$. In the (X,V) phase space, this force corresponds to the presence of a secondary resonance with wave number equal to 2, exactly centered at $v=0$ when $\rho=1$. It will produce a density modulation[5] given by

$$n/n_0 = 1 + [\rho^2 p^2/4(T_e + T_i/Z)]\cos 2X, \quad (4)$$

where T_e and T_i are also normalized to $m\omega_0^2/2k_0^2$ and Z is the ion charge. Equation (4) shows that the density is a maximum at the nodes of the standing electric field. Since $T_i \ll T_e$, the density modulation reads

$$\delta n/n = \rho^2 p^2/4T_e. \quad (5)$$

This simple relation shows how the absolute amplitude of the field can be deduced from the density modulation measurements.

Figure 1 shows a typical standing-wave pattern for $\omega_0/2\pi = 40$ MHz. The electric field amplitude oscillates with a spatial period of $\lambda_0/2$. Its SWR corresponds to $\rho = 1.5$. The axial density profile also shows a stationary density modulation, in opposite phase with the wave amplitude, in agreement with Eq. (4). The natural decreasing plasma density along the axis is also observed, producing a wavelength reduction toward the column end. The density modulation was observed to increase linearly with the injected power, as expected from Eq. (5), with a constant SWR. In the following, this density modulation is taken as the measure of wave amplitude.

The electron temperature is deduced from the dependence of the electron current collected on the electrostatic analyzer as a function of grid bias. As the wave amplitude increases one observes strong heating of the bulk plasma electrons. In contrast with previous experiments,[13] one does not look at the plasma distribution deformation in the neighborhood of the wave phase velocity, as this velocity is much greater than the thermal velocity. The results of temperature measurements are reported in Fig. 2 as a function of the measured wave amplitude p. The dashed line corresponds to experimental best fit. A sharp increase of the temperature at a value p_c is observed. This value is compared with the

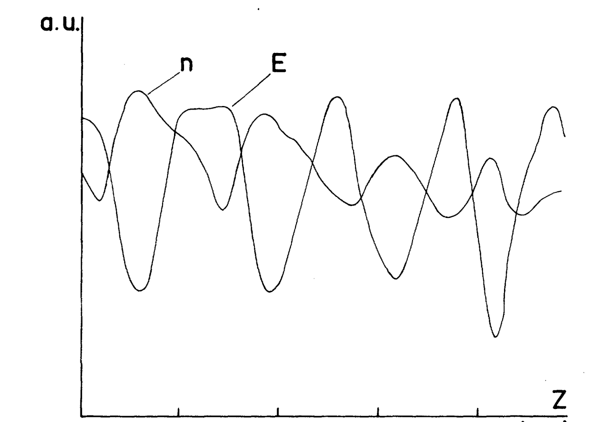

FIG. 1. Axial profiles of the amplitude of the standing wave and the associated ion saturation current for $\omega_0/2\pi = 40$ MHz.

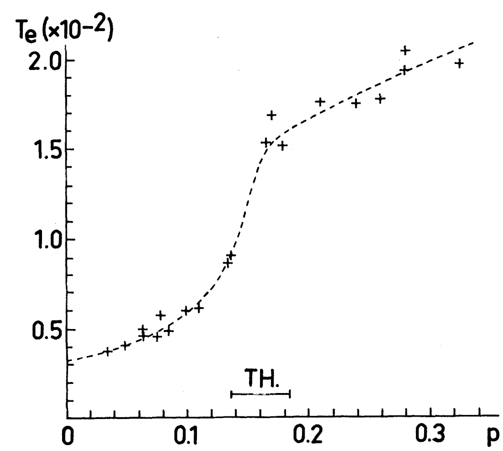

FIG. 2. Measured electron temperature (normalized to $m\omega_0^2/2k_0^2 = 75$ eV) vs p. The horizontal bar is the theoretical prediction for the stochasticity threshold weighted by the experimental error on the SWR ($1.4 < \rho < 2.2$). $\omega_0/2\pi = 30$ MHz.

large-scale stochasticity threshold given by the renormalization theory[7]; this theoretical threshold is shown as a horizontal bar, where the error is due to the precision in the SWR measurement that affects the abscissa. Good agreement is found between predicted and measured threshold.

Other nonlinear mechanisms could be invoked to explain the observed heating. Wave breaking was shown to lead to a randomization of the particle orbits[14] when the amplitude of a purely traveling wave is increased. The predicted threshold[15] for the occurrence of such a phenomenon is given by $p = (\omega_p/\omega_0)^2/\rho^2$ where ω_p is the plasma frequency; in our case, it should correspond to $p \sim 12$, a much higher value than our threshold p_c. Frequency spectrum measurements have not shown any parametric decay or sidebands. This last point is not surprising since sideband growth requires the existence of long-lived bunches of electrons bouncing in the wave's potential troughs[1]; this is excluded by the presence of the mutually perturbing waves with same-order amplitudes constituting the standing wave.

To point out the particular role of the standing wave in the heating mechanism, an attempt was made to eliminate one of its components by decreasing the reflection coefficient on the end plate. This could be done by biasing it at a fixed positive voltage in order to increase electron absorption as was shown in previous experiments.[16] Unfortunately, at the considered large wave amplitudes, no net variation of the SWR was observed; nevertheless, for a given value of the injected power, the heating was seen to decrease when ρ increases, as theoretically predicted.

In conclusion, this experiment shows good agreement with theoretical predictions based on the occurrence of large scale stochasticity to explain the observed heating and evidences consequences of stochasticity on a rather macroscopic level.

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