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Propagation of Sound in a Spin-Glass

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Sound propagation and damping are studied near the spin-glass transition for a time-dependent Landau-Ginzburg model coupled to phonons. The sound speed decreases linearly with $T-T_g$, and the sound damping diverges as $(T-T_g)^{-1}$, when the spin-glass transition, T_g, is approached from high temperatures. At the spin-glass transition, the sound speed has a term proportional to $\omega^{1/2}$, and the sound damping is proportional to $\omega^{-1/2}$.

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The spin-glass (SG) phase was proposed by Edwards and Anderson (E-A)[1] to explain the low-temperature properties of some amorphous magnetic alloys. According to E-A, the phase is characterized by a nonvanishing value of the different time correlation of a spin at a given site so that the spin may be visualized as being frozen in time. In view of this picture, the dynamics of spins are of considerable interest in understanding the phase transition. Several authors[2-5] have studied spin relaxation and damping near the spin-glass freezing temperature, T_g. In this Letter, we study propagation of sound with a view to investigating spin dynamics in and just above the spin-glass phase, and we report results on the propagation velocity and damping of sound for a simple model. Our result for the temperature dependence when the SG transition is approached from above T_g agrees with the observed decrease in the sound speed.[6,7] There are no systematic data for the frequency dependence of the speed, nor on the sound damping; we encourage work to test our prediction.

We study a time-dependent Ginzburg-Landau (TDGL)[2,8,9] model. If $\sigma(x)$ is the local spin density and $\vec{\psi}(x)$ a longitudinal phonon field, then the model is defined by the Hamiltonian

$$H=\int[d^dx/(2\pi)^d]\tfrac{1}{2}\{[r_0+\varphi(x)]\sigma^2(x,t)+|\nabla\sigma(x,t)|^2+\tfrac{1}{4}u[\sigma(x,t)^2]^2+C_0{}^2\sum_{\mu=1}^{d}[\partial_\mu\psi^\mu(x,t)]^2+|\vec{\Pi}(x,t)|^2$$
$$+g_0\sigma^2(x,t)\nabla\cdot\vec{\psi}(x,t)\}+\int d^dx[h(x,t)\sigma(x,t)+\vec{f}(x,t)\cdot\vec{\psi}(x,t)],\qquad(1)$$

where $\vec{\Pi}(x)=\partial\vec{\psi}/\partial t$ is the phonon momentum density. $\varphi(x)$ is a time-independent Gaussian random variable

$$\langle\varphi(x)\varphi(x')\rangle_c=\Delta\delta(x-x').$$

[We denote configuration average by angular brackets with a subscript c.] When g_0 is zero, the phonon field, $\vec{\psi}(x)$, has the usual form of a free massless field. For this case, the spin dynamics near the spin-glass transition are known.[2] When g_0 is not zero, the coupling term may be looked upon as producing a local change in r_0. The change depends on the sign of the gradient which, in a lattice picture, says that the exchange interaction is enhanced when neighboring spins move towards each other.

The spin dynamics are described by the Langevin equation of motion,

$$\partial\sigma(x,t)/\partial t=-\Gamma_0\delta H/\delta\sigma(x,t)+\zeta(x,t).\qquad(2a)$$

For phonons, we take an equation of motion similar to that of a randomly driven, damped harmonic oscillator,

$$\frac{\partial\vec{\Pi}(x,t)}{\partial t}=-\frac{\delta H}{\delta\vec{\psi}(x)}+\gamma_0\nabla^2\frac{\delta H}{\delta\Pi(x,t)}+\xi(x,t).\qquad(2b)$$

$\zeta(x,t)$ and $\xi(x,t)$ are statistically independent noise sources for the spins and phonons, respectively, and their statistics are related to the respective kinetic coefficients Γ_0 and γ_0 by the fluctuation-dissipation theorem:

$$\langle\zeta_i(x,t)\zeta_j(x',t')\rangle=2\Gamma_0\delta(x-x')\delta(t-t')\delta_{ij},\qquad(3a)$$

$$\langle\xi_\mu(x,t)\xi_\nu(x',t')\rangle=-2\gamma_0\nabla^2\delta(x-x')\delta(t-t')\delta_{\mu\nu}.\qquad(3b)$$

In Fourier space Eqs. (2a) and (2b) have the explicit form

$$\left(-\frac{i\omega}{\Gamma_0}+r_0+q^2\right)\sigma_i(q,\omega)=-\frac{u}{2}\sum_j\int\frac{d^dq'}{(2\pi)^d}\int\frac{d^dq''}{(2\pi)^d}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}\sigma_j(q',\omega')\sigma_j(q'',\omega'')\sigma_i(q-q'-q'',\omega-\omega'-\omega'')$$
$$-g_0\sum_\mu\int\frac{d^dq'}{(2\pi)^d}\int\frac{d\omega'}{2\pi}\sigma_i(q-q',\omega-\omega')\{iq_\mu{}'\psi^\mu(q',\omega')\}$$
$$-\int\frac{d^dq'}{(2\pi)^d}\varphi(q-q')\sigma_i(q',\omega)+(\zeta/\Gamma_0)(q,\omega)+h_i(q,\omega),\qquad(4a)$$

$$(-i\omega\gamma_0q^2-\omega^2+C_0{}^2q^2)\psi_\mu(q,\omega)=\tfrac{1}{2}i(g_0q_\mu)\sum_i\int[d^dq'/(2\pi)^d]\int(d\omega'/2\pi)\sigma_i(q',\omega')\sigma_i(q-q',\omega-\omega')$$
$$+\xi(q,\omega)+f(q,\omega).\qquad(4b)$$

[Latin subscripts stand for the components of spin, σ, and Greek subscripts for the components of displacement, $\vec{\psi}(x,t)$.] In zeroth order, the response functions $G=\delta\langle\sigma\rangle/\delta h$ and $D=\delta\langle\psi\rangle/\delta f$ are

$$G^0(q,\omega)=(-i\omega/\Gamma_0+r_0+q^2)^{-1},\qquad(5a)$$

$$D^0(q,\omega)=(-\omega^2+C_0{}^2q^2-i\omega\gamma_0q^2)^{-1}.\qquad(5b)$$

We are interested in sound propagation and damping. To this end, we solve Eqs. (4a) and (4b) perturbatively for $\psi(q,\omega)$ and obtain the phonon self-energy. The diagrammatic representation of the perturbation series follows naturally from the similar treatment of TDGL models.[8,9] In Fig. 1, a double wavy line stands for the full phonon response function, D; a single wavy line stands for D_0; a solid line stands for the spin response function, C; and a dotted line stands for the random field φ. To zeroth order in the spin-phonon coupling, g_0, and at the mean field level in the spins,[2,3] the frequency dependence of the spin response function G is obtained from the self-energy diagram which is of first order in Δ, i.e.,

$$G(q,\omega)=[r_0+q^2-\Sigma(\omega)-i\omega/\Gamma_0]^{-1}$$
$$\equiv[-i\omega/\Gamma(\omega)+G^{-1}(q,0)]^{-1},$$

where

$$\Sigma(\omega)=\Delta\int[d^dk/(2\pi)^d][-i\omega/\Gamma(\omega)+G^{-1}(k,0)]^{-1}.$$

We can solve these self-consistent equations for

$\Gamma(\omega)$ and, to lowest nontrivial order, we have[2,3]

$$\frac{\Gamma(\omega)}{\Gamma_0}=\frac{1-\lambda}{2}\left\{1+\left(1-\frac{4i\omega\tau_1}{(1-\lambda)^2}\right)^{1/2}\right\}, \tag{6}$$

where $\lambda=\Delta\Pi(0,0)$, $\Pi(0,0)=\int[d^dk/(2\pi)^d]G^2(k,0)$, and $\tau_1\Gamma_0\equiv\Delta\int[d^dk/(2\pi)^d]G^3(k,0)$. (The characteristic time τ_0 is only a weak function of temperature.)

For large $\omega[\omega\tau_1\gg(1-\lambda)^2]$, $\Gamma(\omega)$ shows the $\omega^{1/2}$ behavior typical of the spin-glass transition.[2-5,10] The transition to the SG phase takes place[2,3] when

$$1=\Delta\int[d^dk/(2\pi)^d]G^2(k,0),$$

so that

$$1-\lambda\propto t\equiv T-T_g.$$

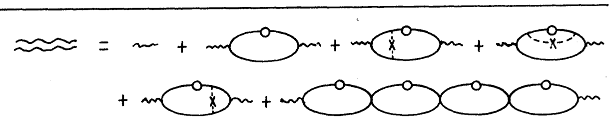

FIG. 1. Expansion for the phonon response function. Dotted lines with a cross stand for configuration average of random φ fields.

Figure 1 shows some diagrams contributing to the phonon self-energy in lowest order in the spin-phonon coupling. Here we sum all the ladder vertex corrections inside the bubbles at both ends and across the noise vertex. To order q^2 in external momentum, this sum is

$$B(\omega)=\frac{2}{\Gamma_0}\int\frac{d\nu}{2\pi}\frac{\int[d^dk/(2\pi)^d]G(k,\nu)G(-k,-\nu)G(k,\omega-\nu)}{[1-\Delta\Pi(\nu,\omega-\nu)][1-\Delta\Pi(\nu,-\nu)][1-\Delta\Pi(-\nu,\omega-\nu)]}, \tag{7}$$

where

$$\Pi(\nu,\nu')=\int[d^dk/(2\pi)^d]G(k,\nu)G(-k,\nu').$$

Then by summing the chain of bubbles arising from the four-spin vertex, u, we have

$$\Lambda(\omega)=-g_0^2nB(\omega)/[1+u(n+2)B(\omega)] \tag{8}$$

for the phonon self-energy. Thus the full phonon response function is

$$D^{-1}(\omega)=-\omega^2+q^2[C_0^2+\mathrm{Re}\Lambda(\omega)]-i[\omega\gamma_0+\mathrm{Im}\Lambda(\omega)]q^2\equiv-\omega^2+[C^2(\omega)-i\omega\gamma(\omega)]q^2. \tag{9}$$

Our results for sound propagation and damping are as follows:

(i) At zero-external frequency and $T>T_g$, the dominant frequency dependence in the integrand of Eq. (7) comes from factors of $[1-\Delta\Pi(\nu',-\nu)]^{-1}$. These factors are independent of ν in the region $\nu\ll t^2/\tau_1$, and $\sim\nu^{-1/2}$ in the region $\nu\gg t^2/\tau_1$. Thus a simple dimensional analysis gives

$$B(0)\propto t^{-1} \tag{10}$$

for the leading contribution of a bubble. From Eqs. (8)–(10), we find that the sound speed decreases as t, as we approach the SG transition from above T_g. Experimental data also suggest a similar behavior.

(ii) For a finite external frequency, ω, we examine Eq. (7) at $t=0$. Using (6), we obtain

$$B(\omega)\propto(-i\omega)^{-1/2}. \tag{11}$$

The real part of (11) says that the sound speed has a $\omega^{1/2}$ dependence on frequency and the imaginary part gives anomalous damping, i.e., in Eq. (9), $\gamma(\omega)\propto\omega^{-1/2}$.

(iii) Crossover: The behavior of the sound speed changes from linear in t [as in (i)] to temperature-independent $\omega^{1/2}$ behavior [as in (ii)], with the crossover frequency $=t^2/\tau_1$. This is sketched in Fig. 2. The sound damping, γ, grows like $1/t$ for $\omega<t^2/\tau_1$, and for $\omega>t^2/\tau_1$, it shows the anomalous $\omega^{-1/2}$ behavior discussed in (ii). The anomalous sound damping and the $\omega^{+1/2}$ frequency dependence of sound speed for $\omega>t^2/\tau_1$ may be traced back to the $\omega^{-1/2}$ tail[2,3] in the correlation function.

(iv) If we accept that

$$1=\Delta\Pi(0,0) \tag{12}$$

throughout the SG phase,[11,12] then the sound speed and damping in the SG phase have the same value as at the transition temperature T_g.

These results, obtained from a calculation in lowest order in g_0^2, agree well with the observed decrease in the sound speed at high temperatures. There are no data for the frequency dependence of the sound speed and it will be interesting to see, for the validity of our high-temperature calculation, if our prediction of a $\omega^{1/2}$ behavior holds. It will also be interesting to see the anomalous damping near the transition temperature.

We are investigating the higher-order terms in g_0^2. These terms are divergent ($\sim 1/t$) at (and be-

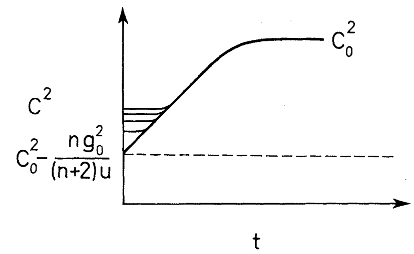

FIG. 2. Temperature dependence of the sound speed for different values of external frequency.

low) T_g. Therefore, the results presented here hold only down to some temperature $T^*(g_0) = T_g + \text{const}\, g_0^2$. The low-temperature problem is under study.

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