Coupling of New Order-Parameter Collective Modes to Sound Waves in Superfluid ^{3}He

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A recently observed second collective-mode peak in the sound absorption in ^{3}He-B is shown to arise from weak excitation of a known order-parameter mode with weak-coupling frequency $\omega=(\sqrt{\tfrac{8}{5}})\Delta(T)$ due to particle-hole–symmetry–violating terms. The somewhat lower value of the observed frequency might be caused by strong-coupling effects or admixture of $l=3$ pair fluctuations.

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One of the more prominent manifestations of pair correlations in superfluid ^{3}He is the existence of sharp peaks in the ultrasound attenuation as a function of temperature.[1] These peaks are caused by excitation of high-frequency collective modes of the order parameter when the temperature-dependent collective-mode frequency matches the sound frequency ω. In a number of experiments in the temperature range near T_c, the existence of two such modes in the A phase and one mode in the B phase has been established.[1] A detailed theory is in good agreement with the observed sound attenuation and velocity change in this temperature regime.[2]

In recent experiments[3-5] the propagation of ultrasound in ^{3}He-B has been studied in the temperature regime $0.35<T/T_c<0.95$, for frequencies ranging from 12 to 133 MHz. In addition to the main attenuation peak, occurring approximately at $\omega=(\sqrt{\tfrac{12}{5}})\Delta(T)$, as predicted by weak-coupling theory, a second sharp peak is observed at lower temperatures. The area under this second peak is orders of magnitude smaller than the area of the main peak at a given frequency. It is the purpose of this Letter to point out that the new collective peak corresponds to a theoretically predicted mode at $\omega=(\sqrt{\tfrac{8}{5}})\Delta(T)$, which couples only weakly to sound via particle-hole–symmetry–violating terms.

The dynamics of superfluid ^{3}He appears to be reasonably well described by a time-dependent mean-field theory for diagonal and off-diagonal fields in particle-hole space,

$$\delta\epsilon_{\vec{k}}(q,t)=\sum_{k'} f_{\vec{k}\vec{k}'}\,\delta n_{\vec{k}'}(q,t)$$

and

$$\delta\Delta_{\vec{k}}(q,t)=\sum_{k'} g_{\vec{k}\vec{k}'}\,\delta g_{\vec{k}'}(q,t),$$

respectively. Here $\delta n_{\vec{k}}(q,t)=\delta\langle c_{\vec{k}_+}^{\dagger}(t)c_{\vec{k}_-}(t)\rangle$ and $\delta g_{\vec{k}}(q,t)=\delta\langle c_{\vec{k}_-}(t)c_{\vec{k}_+}(t)\rangle$ are the diagonal and off-diagonal distribution functions ($\vec{k}_\pm=\vec{k}\pm\tfrac{1}{2}\vec{q}$) and $f_{\vec{k}\vec{k}'}$ and $g_{\vec{k}\vec{k}'}$ are the Fermi-liquid and pair interactions, respectively. Solving the equations of motion for $\delta n_{\vec{k}}$ and $\delta g_{\vec{k}}$ one finds the sound dispersion relation[2]

$$\omega^2=c_1^2q^2\{1-2[(c_0-c_1)/c_1]\,\xi\}\,. \tag{1}$$

Here $c_1^2=\tfrac{1}{3}(1+F_0^s)(1+\tfrac{1}{3}F_1^s)$, $(c_0-c_1)/c_1=\tfrac{2}{5}(1+\tfrac{1}{5}F_2^s)/(1+F_0^s)$, and F_l^s are the spin-symmetric Landau parameters. The function ξ is given by $\xi=\tfrac{5}{2}\sum_k P_2(\hat{k}\cdot\hat{q})\delta n_k/\delta n$, where P_2 is the $l=2$ Legendre polynomial. Explicitly,

$$\xi=\tfrac{5}{2}\sum_k P_2(\hat{k}\cdot\hat{q})\Big\{\frac{\vec{v}\cdot\vec{q}}{\omega-\vec{v}\cdot\vec{q}}(1-\lambda_k)\delta\epsilon_k-\lambda_k\delta\epsilon_k+\frac{\omega}{4|\Delta_k|^2}\lambda_k\tfrac{1}{2}\mathrm{Tr}_\sigma[(\delta\Delta_k)\Delta_k^{+}-\Delta_k(\delta\Delta_k^{+})]$$
$$+\frac{\xi_k}{|\Delta_k|^2}\lambda_k\tfrac{1}{2}\mathrm{Tr}_\sigma[(\delta\Delta_k)\Delta_k^{+}+\Delta_k(\delta\Delta_k^{+})]\Big\}\frac{1}{\delta n}, \tag{2}$$

where

$$\lambda_k=\frac{|\Delta_k|^2}{E_k^2-\omega^2/4}\,\frac{1}{2E_k}\tanh\frac{E_k}{2T}, \tag{3}$$

$E_k=(\xi_k^2+|\Delta_k|^2)^{1/2}$, $\xi_k=k^2/2m^*-\mu$, and $\vec{v}=\vec{k}/m^*$. For later reference we define $\lambda(\omega,T)=\Delta^2\bar{\lambda}=\int_{-\infty}^{\infty}d\epsilon\,\lambda_k$.

We parametrize the nonequilibrium gap parameter by

$$\delta\Delta_k=\Delta\sum_{j=1}^{3} i\sigma_j\sigma_2 d_j(\hat{k}),$$

where the σ_j are the Pauli matrices. For $l=1$ pairing, $d_j(\hat{k}) = d_{j\alpha}\hat{k}_\alpha$. The equilibrium gap parameter for ^{3}He-B may be taken as $d_{j\alpha}{}^0 = \delta_{j\alpha}$ (in the present context a relative spin-orbit rotation is irrelevant). Performing the spin traces and angular integrations in (2), one finds that only the combination of $d_{j\alpha}$'s given by $d'' = \frac{1}{2}(2d_{zz}'' - d_{xx}'' - d_{yy}'')$, where $d_{j\alpha}'' = \frac{1}{2}[d_{j\alpha}(\vec{q},\omega) - d_{j\alpha}{}^*(-\vec{q},-\omega)]$ couples strongly to the sound $(\vec{q}\,||\,\hat{z})$. The last term in (2) couples to the order-parameter variable $d' = \frac{1}{2}(2d_{zz}' - d_{xx}' - d_{yy}')$, where $d_{j\alpha}' = \frac{1}{2}[d_{j\alpha}(\vec{q},\omega) + d_{j\alpha}{}^*(-\vec{q},\omega)]$, but because of the extra factor ξ_k the energy integral would vanish in the case of exact particle-hole symmetry. This term has been neglected in previous theories. The relative importance of the last term in (2) may be expressed by the following dimensionless parameter characterizing the degree of particle-hole-symmetry violation,

$$\eta = \frac{1}{\Delta}\frac{N'(\epsilon_F)}{N(\epsilon_F)}\frac{1}{\lambda}\int_0^{\epsilon_c} d\xi\, \xi^2\lambda(\xi) \simeq \frac{\Delta}{2\epsilon_F}\ln\left(\frac{\epsilon_c}{\Delta}\right) \text{ as } T\to 0. \tag{4}$$

Here $N(\epsilon)$ is the density of states and $\epsilon_c \simeq 0.1\epsilon_F$ is a cutoff energy simulating the energy dependence of the interaction function $g_{kk'}$. Since very little is known about the energy dependence of $N(\epsilon)$ and $g_{kk'}$, we restrict ourselves to the order-of-magnitude estimate (4), with the result $\eta \sim 3\times10^{-3}$ to 5×10^{-3} for pressures from 0 to 12 bars.

The order-parameter components $d_{j\alpha}$ may be calculated from the time-dependent gap equation[2]

$$d_j(\hat{p}) + (4\pi)^{-1}\int d\Omega_k\, G(\hat{p}\cdot\hat{k})\{\tfrac{1}{2}\lambda[\omega^2 + 2\eta\omega\Delta - (\vec{v}\cdot\vec{q})^2(1+2\eta'\Delta/\omega) - 2\Delta^2]d_j(\hat{k}) - G_1{}^{-1}d_j(\hat{k}) + 2\lambda[d_j{}^*(\hat{k}) - 2\hat{k}_i d_i{}^*(\hat{k})\hat{k}_j]\}$$
$$= (4\pi)^{-1}\int d\Omega_k\, G(\hat{p}\cdot\hat{k})\hat{k}_j[\omega + 2\eta\Delta + \vec{v}\cdot\vec{q}(1+2\eta'\Delta/\omega)]\,\delta\epsilon(\hat{k}). \tag{5}$$

Here $G(\hat{p}\cdot\hat{k}) = N(\epsilon_F)g_{\vec{p}\vec{k}} = -\sum_l (2l+1)G_l P_l(\hat{p}\cdot\hat{k})$ is the dimensionless pair interaction and η' is another particle-hole-asymmetry parameter, defined by (5) with $\xi^2\lambda_k$ in the integrand replaced by $\{\xi^2\lambda_k + \Delta^2\xi(d/d\xi)[(1/2E)\tanh(E/2T)]\}$. For $T \ll T_c$, one finds $\eta' \cong (\omega^2/4\Delta^2)(\Delta/2\epsilon_F) \ll \eta$. We neglect η' in the following. Solving (5) for d'' and d', keeping only the $l=1$ component of the pair interaction, expanding powers of $(v_F q/\omega)$ and η, and substituting the result into (2), one finds the quantity ξ in the sound dispersion relation (3) as

$$\xi = 1 - \lambda + \frac{2}{5}\frac{c_1{}^2q^2}{\omega^2 - \frac{12}{5}\Delta^2 - \frac{7}{15}(v_Fq)^2 + 2i\omega\gamma} + 8\eta^2\lambda\frac{c_1{}^2q^2}{\omega^2 - \frac{8}{5}\Delta^2 - \frac{7}{15}(v_Fq)^2 + 2i\omega\gamma}. \tag{6}$$

Equations (6) and (1) imply that the sound attenuation $\alpha = -q[(c_0 - c_1)/c_1]\mathrm{Im}(\xi)$ (when $\alpha \ll g$ is assumed) has two resonance peaks as a function of temperature, corresponding to oscillations of $d_{j\alpha}''$ $(\omega^2 = \frac{12}{5}\Delta^2)$ and $d_{j\alpha}'$ $(\omega^2 = \frac{8}{5}\Delta^2)$ of quadrupolar symmetry, $d_{j\alpha} \propto (\delta_{j\alpha} - 3\hat{q}_j\hat{q}_\alpha)$. The motion may be visualized as a squashing of the order-parameter structure in the direction $\hat{q}$. The mode involving d'' has been called "squashing" mode (sq). We propose the term "real squashing" mode (rsq) for the new mode with frequency $(\sqrt{\frac{8}{5}})\Delta$, because it involves the real part of d.

In (6) we have also included the principal effect of quasiparticle collision processes by adding an imaginary damping term $2i\omega\gamma$ to the denominators in (6). γ is related to the energy-dependent quasiparticle lifetime $\tau(E)$ by[2]

$$\gamma = \frac{1}{\lambda}\int_{-\infty}^{\infty} d\xi\,\frac{1}{\tau(E)}\left(-\frac{df}{dE}\right)\frac{\Delta^2}{E^2 - \omega^2/4}, \tag{7}$$

for $\omega\tau \gg 1$. Here $f(E)$ is the Fermi function. γ is essentially the quasiparticle relaxation rate times the relative density of thermal excitations.

In Fig. 1, γ is plotted as a function of temperature, with $\omega^2 = a\Delta^2(T)$, for $a = \frac{8}{5}$ (γ is rather insensitive to the value of a). The quasiparticle lifetime $\tau(E)$ has been taken from Einzel and Wölfle.[6] It is apparent from Fig. 1 that the damping of the collective modes decreases rather rapidly for $T\to 0$, thus giving rise to enormously sharp and high acoustic attenuation peaks. The structure described by (6) is observed experimentally.[3-5] However, the collective-mode frequencies at $T \ll T_c$, as determined from the peak positions, are found to be 10% (sq) and 20% (rsq) lower than the ideal values $(\sqrt{\frac{12}{5}})\Delta(T)$ and $(\sqrt{\frac{8}{5}})\Delta(T)$, respectively, even when gap renormalization is taken into account. In the following, we discuss three effects which can cause such a frequency shift.

(i) *Fermi-liquid corrections.*—It has been shown previously[7] that the squashing-mode frequency is shifted by the Landau parameter $F_2{}^s$ in particular at lower T: $\omega_{sq}{}^2 = \frac{12}{5}\Delta^2[1 + \frac{2}{25}\lambda F_2{}^s]$. $\lambda(\omega_{sq})$ is plotted in Fig. 1. A corresponding shift

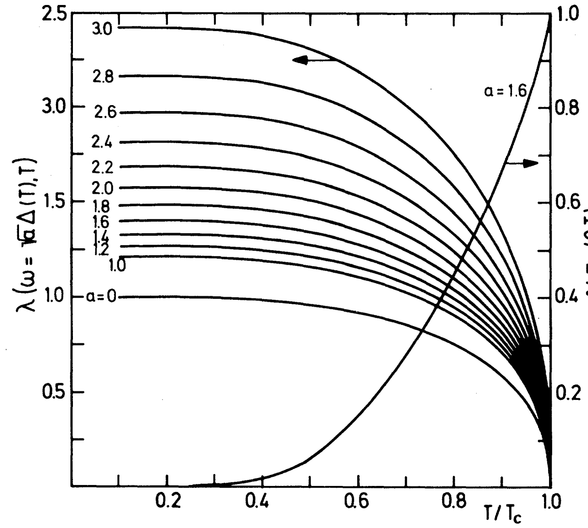

FIG. 1. Coupling constant $\lambda(\omega = a^{1/2}\Delta(T), T)$ for various values of a (left scale) and linewidth parameter γ normalized to the quasiparticle relaxation rate in the normal state $1/\tau_N(0,T)$ (right scale) as a function of reduced temperature.

of the rsq mode is negligible because of the smallness of the coupling parameter η^2. F_2^s may be inferred from the measured values of $(c_0 - c_1)/c_1$. Unfortunately, the evaluation depends crucially on the effective mass parameter m^*/m. Taking m^*/m from Wheatley's tables[1] one finds a small positive F_2^s (~ 0.5), whereas employing the recent m^*/m values of Alvesalo *et al.*,[8] which are considerably lower, one finds $F_2^s \cong -1$, at pressures from 0 to 10 bars.

(ii) Strong-coupling effects.—Both the squashing and real squashing modes are associated with minima in the static free energy with respect to order-parameter distortions of the appropriate symmetry. One may estimate[9] strong-coupling corrections of the collective-mode frequencies from the strong-coupling corrections of the parameters β_i in the Ginzburg-Landau free energy as $(\delta\omega_{sq}/\omega_{sq}) \sim \frac{1}{2}\delta\beta_1/\beta_1 \sim -2\%$ and $\delta\omega_{rsq}/\omega_{rsq} \sim \frac{1}{2}(\delta\beta_3 + \delta\beta_4 + \delta\beta_5)/(\beta_3 + \beta_4 + \beta_5) \sim -5\%$, with insertion of values for β_i appropriate for low pressure from the literature.[10]

(iii) f-wave pair fluctuations.—The collective-mode frequencies are also influenced by higher-angular-momentum fluctuations of the order parameter.[11-13] Solving the gap equation (5), and keeping only G_1 and G_3, one finds

$$\omega_i^2 = \Delta^2\{(2 - b_i) - [2a_i b_i + (2 - b_i)^2]^{1/2}\},$$

for $i = \text{sq, rsq}$ where $b_i = (G_1^{-1} - G_3^{-1})/\lambda(\omega_i)$, $a_{sq} = \frac{12}{5}$, and $a_{rsq} = \frac{8}{5}$. The ω_i approach the weak-coupling values $a_i^{1/2}\Delta$ for $b_i \to -\infty$ as $(-b_i)^{-1}$, such that even relatively small G_3 can cause sizable shifts of ω_i. With an estimated value of $G_1 \simeq 0.25$ (0.35) at low (high) pressure (when a cutoff energy $\epsilon_c \sim 50$ mK is assumed in the formula for T_c) a value of $G_3 \sim 0.1$ would give rise to a decrease of ω_i by 20%. Sauls and Serene[12] have discussed the possibility of new collective modes with frequencies less than 2Δ by coupling to $l=3$ and $l=5$ fluctuations. They have found a total-angular-momentum $J=4$ mode for values of G_3 and G_5 of the order of G_1. There exist many more overdamped modes in the pair-breaking continuum ($\omega > 2\Delta$). Unfortunately we have little information on the value of G_3 at present.

The sharpness of the collective resonances at low temperatures gives rise to enormous sound attenuation peaks, which are difficult to measure experimentally. It may be more feasible, then, to determine the group velocity of sound, which varies dramatically near resonance.[3-5] From the dispersion relation (1) and (6), one finds

$$v_g = \frac{d\omega}{dq} = c_1^2\frac{q}{\omega}\left[1 + 2\left(\frac{c_0 - c_1}{c_1}\right)(1-\lambda) + 4\sum_i B_i R_i\right]\left[1 + 2\sum_i B_i R_i^2\left(1 - \frac{4\omega_i^2\gamma^2}{(\omega^2 - \omega_i^2)^2}\right)\right]^{-1}. \tag{8}$$

Here $i = \text{rs, rsq}$ and $B_{sq} = \frac{2}{5}\lambda(c_0 - c_1/c_1)$, $B_{rsq} = 20\eta^2 B_{sq}$, and $R_i = c_1^2 q^2(\omega^2 - \omega_i^2)[(\omega^2 - \omega_i^2)^2 + 4\omega_i^2\gamma^2]^{-1}$. For simplicity we have assumed $\text{Im}(q)/\text{Re}(q) \ll 1$.

We now briefly compare the theoretical strength and width of the new mode with experiment. From the area under the sound attenuation peak measured in Refs. 5, 3, and 4 at pressures of 3, 9, and 13 bars and sound frequencies of 45, 60, and 84 MHz, respectively, one determines coupling strengths $10^6 B_{rsq}^{exp} = 2.7$, 4.5, and 1.25, from which the particle-hole–asymmetry parameter η is inferred as $10^3\eta = 4.1$, 4.7, and 3.5, in fair agreement with the estimate (4). The width of the attenuation peaks ($\Delta T/T_c$) is found from the first two experiments as 3×10^{-3} and 4×10^{-3}, which compare reasonably well with the collision-induced theoretical widths 3×10^{-3} and 6×10^{-3}. Here we have used a normal-state relaxation time $\tau_N(0)T^2 = (0.6 - 0.01P)$ μsec (mK)^2, with P in bars, and the BCS gap function $\Delta(T)$.

At low temperatures, quasiparticle collision damping becomes very small. Another resonance broadening mechanism is introduced by dispersion. Determining this broadening self-consistently with the sound attenuation, one finds a linewidth $\Delta\omega_i \sim \omega_i(v_F/c_1)B_i^{1/2}$, which should dominate the width of the rsq peak below $0.5T_c$.

In conclusion, we believe to have shown that the new collective mode may be identified as a $J=2$, $J_z=0$ mode of the real part of the order parameter, which couples weakly to sound via particle-hole–asymmetric terms. Tewordt and Schopohl[14] have predicted a linear splitting of this $J=2$ multiplet in a magnetic field H. For nonparallel orientations of H and $\vec{q}$, the $J_z \neq 0$ modes couple to the sound as well, giving rise to a fivefold splitting of the sound attenuation peak. This has been observed recently by Avenel, Varoquaux, and Ebisawa.[5] The strong direct evidence in favor of a $J=2$ mode makes any alternative explanation[12] employing $J=4$ or $J=6$ modes unlikely.

In ^{3}He-A the order-parameter collective modes are damped considerably by pair breaking, even at $T \ll T_c$. The two well-defined p-wave collective modes[2] (the clapping and flapping modes) couple to sound waves already in the case of exact particle-hole symmetry. There is indeed a new mode with frequency $\omega \simeq 1.74\Delta_0(T)$, where Δ_0 is the maximum of the anisotropic gap (essentially oscillations of the magnitude of the equilibrium gap), which couples to sound only via particle-hole–asymmetric terms. However, this contributes to the sound attenuation $\alpha(T)$ only a broad peak of magnitude about 10^{-3} of the pair-breaking contribution and angular symmetry $P_2(\hat{q}\cdot\hat{l})$ ($\hat{l}$ is the axis of the gap).

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