Observation of Chaos in Optical Bistability

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Optical turbulence and periodic oscillations are easily seen with a hybrid optically bistable device with a delay in the feedback. The behavior of these instabilities is in good agreement with the recent work of Ikeda, Daido, and Akimoto, who predicted them for both ring-cavity and delay-line hybrid devices.

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In the last few years there has been a substantial theoretical and experimental effort on optically bistable systems (i.e., devices which exhibit two distinct states of optical transmission). Recently, Ikeda, Daido, and Akimoto[1,2] have pointed out that intrinsic bistable devices with use of a ring cavity are described by difference equations and that the stability analysis of steady states in that case is different from the usual criteria applied to differential equations. This causes instabilities to arise in steady-state solutions that have previously been described as stable.[3] Furthermore, the time dynamics in such circumstances can be chaotic. In this Letter we describe experiments on an optically bistable hybrid device which shows instabilities with periodic and chaotic dynamics that are in good agreement with the theory of Ikeda, Daido, and Akimoto.

There are several aspects of this problem that make it interesting. Intrinsic optically bistable devices can be extremely fast optical switches[4]; periodic dynamics could convert them into short-pulse generators. Optical bistability has also received substantial attention as a model problem in nonequilibrium statistical mechanics, and the appearance of chaos in this context is interesting. By comparison with turbulent flows,[5] the problem of optical turbulence (i.e., chaos) is both experimentally and theoretically very simple. Although instabilities of the type described here are not widely known in physics, they are common in ecological modeling[6] since population dynamics is realistically described by difference equations. We therefore begin by explaining how stability analysis works and how it relates to the differential-equation analysis used in most theories of optical bistability.

Let us suppose we have a difference equation $\vec{x}_{n+1}=\vec{f}(\vec{x}_n)$ where $\vec{x}_n$ is a vector, $\vec{f}$ is a nonlinear vector function, and n labels the steps. The steady states, defined by $\vec{x}_{n+1}=\vec{x}_n$, are denoted $\vec{x}^k$. The stability analysis proceeds by defining $\vec{x}_n=\vec{x}^k+\vec{\epsilon}_n$ where $\vec{\epsilon}_n$ is small. One then linearizes the problem by a Taylor's expansion about the steady state and finds the eigenvalues Λ^k of the resulting matrix. The kth state is stable if and only if $|\Lambda^k|<1$ for all eigenvalues. Let us next associate a differential equation with the difference equation. We expand x_{n+1} as $\vec{x}_{n+1}\simeq\vec{x}_n+t_R\dot{\vec{x}}_n$, where t_R is the time-step between $\vec{x}_n$ and $\vec{x}_{n+1}$. Then we drop the meaningless subscript n to obtain $t_R\dot{\vec{x}}=\vec{f}(\vec{x})-\vec{x}$, a differential equation whose steady states are the same as those of the difference equation. The eigenvalues λ^k that are obtained from the stability analysis of this differential equation are related to those of the difference equation by $\lambda^k=(\Lambda^k-1)/t_R$, and the state $\vec{x}^k$ is stable if and only if $\operatorname{Re}\lambda^k<0$. One can see that if the differential equation is unstable the difference equation is also unstable, but the *difference equation can be unstable even though the differential equation is stable.* It is in this latter regime that one finds periodic and chaotic time dynamics. We will call this the regime of "Ikeda instability," to differentiate it from other types of instabilities[7,8] that occur in optical bistability. The instability described by McCall[7] is a relaxation oscillation having a period related to the response times of the medium rather than t_R. The self-pulsing instability[8] of absorptive optical bistability can be distinguished experimentally since it has a period of t_R, while periodic pulses of the Ikeda type have period[9] nt_R where $n\geqslant 2$.

Since the ring cavity, the problem Ikeda first analyzed, is a standard model problem in optical bistability and is the motivation for introducing a delay in our hybrid, we discuss the ring first. In the limit of purely dispersive bistability, by using a cubic nonlinearity inside a ring cavity, the dynamics are given by the difference-differential equations[2,10]

$$\tau\dot{\varphi}=-\varphi+\operatorname{sgn}(n_2)|E(t-t_R)|^2, \tag{1}$$

$$E(t)=A+BE(t-t_R)\exp[i(\varphi-\varphi_0)]. \tag{2}$$

Here $E(t)$ is the normalized field in the cavity, φ is the phase lag of the field across the nonlinear medium, n_2 is the nonlinear coefficient, φ_0 is the small-signal phase lag, τ is the response time of the medium, t_R is the delay time, $A=T^{1/2}E_0$, and $B=1-T$ where T is the mirror transmission and E_0 is the incident field. In the limit $t_R \gg \tau$, these equations reduce to a continuous-time difference equation[1] which has an Ikeda instability over virtually all of what would normally be referred to as upper stable branches.[1,2] In the limit $t_R \ll \tau$, the difference equation features are largely eliminated, and normal bistable behavior is predicted.[2]

In our experiment, we follow the suggestion in Ref. 2 and modify a hybrid[11] optically bistable device, by introducing an electrical delay line with delay t_R in the feedback [see Fig. 1(a)]. In our hybrid[12] an optical beam passes through a PLZT (Pb-based lanthanum-doped zirconate titanate) piezoelectric crystal sandwiched between crossed polarizers. The transmitted light is monitored by a photodiode detector whose output signal is delayed and then fed back to the electrodes of the PLZT, causing changes in the refractive index and the transmission through the device. This delay-line hybrid device satisfies[2]

$$\tau\dot{\varphi} = -\varphi + A^2\{1+2B\cos[\varphi(t-t_R)-\varphi_0]\}, \qquad (3)$$

where the light transmission of the PLZT as a function of voltage V is approximately $(1-2B\times\cos\varphi)/2$, $\varphi=\pi V/V_H$, V_H is the half-wave voltage, A^2 is proportional to the input intensity, $\varphi_0=-\pi V_B/V_H$, V_B is the bias voltage, and τ is the composite detector-feedback-PLZT response time. In our experiment, B has a value close to the 0.5 for an ideal modulator. Figure 3 of Ikeda, Daido, and Akimoto[2] shows periodic oscillations and chaos obtained by solving Eq. (3) numerically with $B=0.3$. The output of our device is qualitatively indistinguishable from their solutions (and our own). Equation (1) for a ring cavity reduces to Eq. (3) in the limit $B \ll 1$. Since we see Ikeda instability in a delay-line hybrid, it almost surely exists for a ring cavity, and could have important implications.

Since optical or electronic delay lines with $t_R \gg 1$ ms are impractical, we introduce a delay by inserting a microprocessor in the feedback loop. The output from the photodectector is digitized every 160 μs. Next, since the electro-optic device gives a phase shift that goes as V^2 and we need a response proportional to V for the device to be described by Eq. (3), we take the square root of the input voltage and store it in a first-in, first-out buffer (queue). We can then adjust the delay by leaving the sampling rate constant and changing the length of the buffer. Whenever a new sample is digitized and stored in the buffer, the oldest sample is retrieved from the memory and fed back to the PLZT through a digital-to-analog converter. The response time τ was typically set at 1 ms. The output characteristics were unchanged by reasonable variations in τ and the digitizing error size, verifying that the use of a microprocessor to introduce the delay does not affect the dynamics.

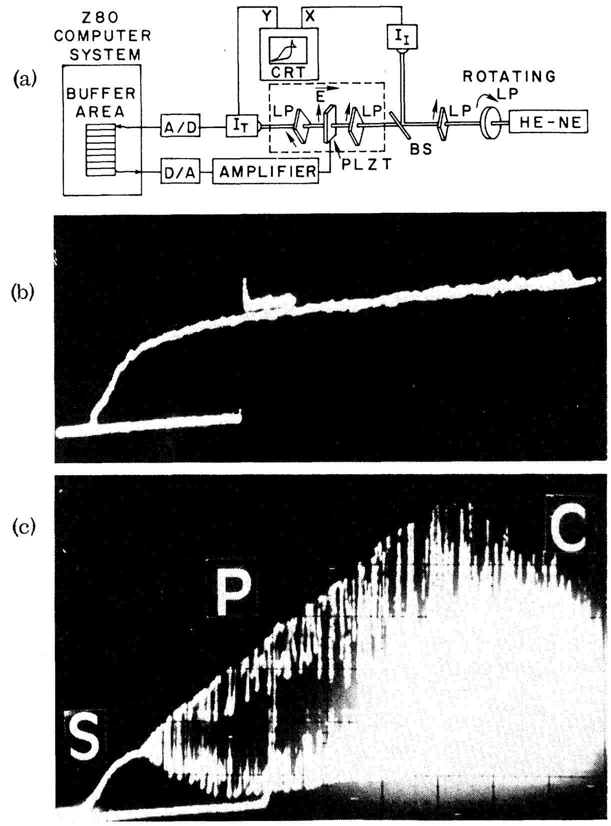

FIG. 1. (a) Block diagram of experimental apparatus. (b) Plot of output intensity (vertical axis) vs input intensity (horizontal axis) as the input intensity is cycled slowly from zero to some maximum and back over a time of 30 s. Here $t_R = 160$ μs $\ll \tau = 1$ ms. (c) Same as (b) except $t_R = 40$ ms $\gg \tau = 1$ ms. The labels, S, $\boldsymbol{P}$, and C indicate the stable, periodic, and chaotic domains of the upper branch.

In Fig. 1(b) we show the *usual* hysteresis curve for the case $t_R = 160$ μs $\ll \tau$ obtained by slowly cycling the incident laser intensity from zero to a maximum and back to zero. In Fig. 1(c) the only change was to make $t_R = 40$ms $\gg \tau$. While the lower branch of the curve is the same as in Fig. 1(b) the upper branch is very different. One can distinguish three qualitatively different regimes in the upper branch, which are labeled S, P, and C in the figure. When the incident power is low, the upper branch is stable (S) and is identical to the upper branch in Fig. 1(b). For higher powers, the upper branch is unstable, and appears as a "wash" rather than as a well-defined line. This comes from the variation of the output intensity with time. Within this unstable regime, two characteristic patterns can be seen. When the wash is continuous (C), as is the case in the right portion of the branch, one has a chaotic output that appears to wander randomly in time between an upper and lower bound as in Fig. 2(c). In the central portion of the upper branch (P) the wash has a dark area in the center, because the output is periodic with period[9] $2t_R$ and close to a square wave [Fig. 2(b)]. The curves in Fig. 2 are in close agreement with Fig. 3 of Ref. 3 which gives a theoretical result for the hybrid with use of Eq. (3).

We observe only periods of $2t_R$ and $4t_R$, and never encounter periods of t_R, which rules out self-pulsing.[8] By varying t_R and τ, we eliminate any relationship between τ and the observed period of $2t_R$, which rules out pulses of the McCall[7] variety. The periodic part of the upper branch is periodic throughout its domain, but the chaotic portion is observed to have very small domains within it which are periodic. In these domains one observes traces with fundamental periods of either $2t_R$ or $4t_R$ with roughly equal probability. At some settings we are able to see a stable domain at the high-intensity end of the upper branch. In the unstable regime that borders this stable domain, the intensity again varies periodically with period $2t_R$, but the modulation depth is much smaller than in the periodic domain shown in Fig. 1. These results are in excellent qualitative agreement with the computations in Refs. 2 and 3 and agree also with our own calculations.

In summary, we report the first experimental observation of Ikeda optical instabilities, resulting in periodic and chaotic outputs. Ikeda instability may be of value in constructing new optical devices. If this instability is not desired, it may be avoided by using round-trip times shorter than the response time of the nonlinear medium. This was most likely the case for previous experiments on short semiconductor etalons.[4] Intrinsic devices have been studied experimentally[13] in the regime $\tau \ll t_R$, but no instability was reported. Two points are worth noting, however. First, the detector response was several t_R in Ref. 13, so that the instability could not have been seen because of time averaging. Secondly, a Fabry-Perot cavity was used rather than a ring, and it is an open question as to how this instability affects that case. Finally, we note that several recent theoretical studies, such as the proposed phase-switching of dispersive systems,[14] have postulated conditions under which the Ikeda instability exists. These proposals will have to be reanalyzed.

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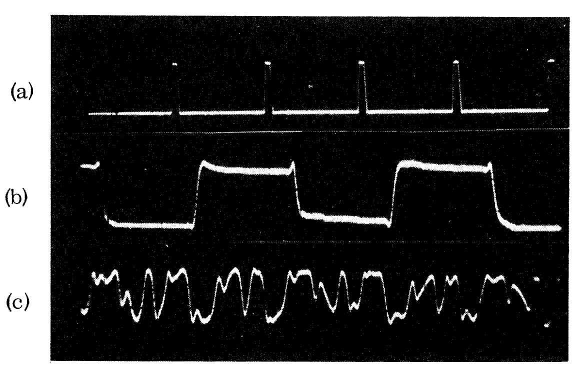

FIG. 2. (a) Time calibration; one pulse every delay time $t_R = 40$ ms. (b) Intensity vs time in the periodic domain. (c) Intensity vs time in the chaotic domain.

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[9]The period is always slightly larger than nt_R by an amount related to the medium decay time. This effect is analogous to a similar effect in a passively mode-locked laser. We have verified that our observed periods converge on nt_R as $\tau \to 0$, and we prefer to simplify the discussion by using the limiting value in the text.

[10]We follow, for the most part, the notation of Ikeda, Daido, and Akimoto, Ref. 2 for easy comparison. The equations can be derived from the Maxwell-Bloch equations in the limit that T_2 is very short, the system is very far off resonance (i.e., no absorption) and one can expand the nonlinearity to third order. Then $\tau = T_1$.

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Nonlinear Stationary Interchange Modes in a Plasma

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Nonlinear two-dimensional interchange modes, excited in a nonuniform magnetized plasma, are found as analytical stationary solutions of the ion vortex equation. The effects of deviation from charge neutrality and nonuniform gravity have been included in these solutions. For an exponential type of density and enstrophy functions it is possible to reduce the equation governing the electric potential to the Liouville equation ($\Delta\Phi = \exp\lambda\Phi$).

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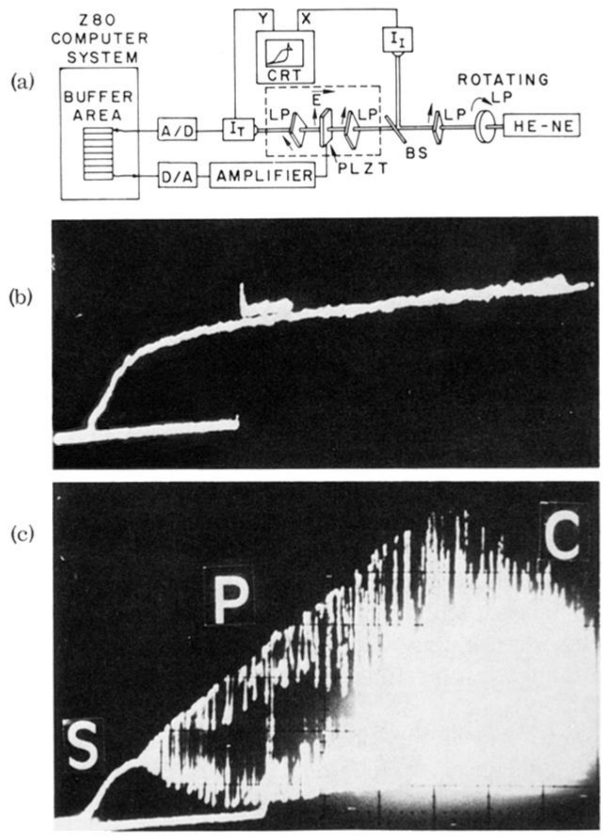
Low-frequency vortex modes play a fundamental role both for the confinement of laboratory plasmas and in the understanding of astrophysical and ionospheric processes.[1-4] In many of these applications, because of the very low frequency of such modes, a nonlinear description of the motion becomes necessary for the study of the long-time behavior of the system. The dynamics of these processes can often be analyzed in terms of two space coordinates in the plane perpendicular to the magnetic field. This formulation, nevertheless, allows for the possibility of thermalization of electrons along the magnetic field lines.[3] Moreover, for long-wavelength perturbations the quasineutrality condition is usually used. This assumption is, as we will show, justified only when the ion plasma frequency is much larger than the ion cyclotron frequency.

Recently some interesting nonlinear properties (such as cascade processes) of drift and interchange modes were found by expanding the well-known ion vortex equation in the limit where the vorticity is much smaller than the ion cyclotron frequency.[5] For the analysis of interchange modes the additional assumption of weak nonlinearity was used in the investigation of spectral cascade processes, while a reductive perturbation method, assuming a small wave amplitude, was applied for the description of the nonlinear convective cell formation.[6]



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(a)

(b)

(c)

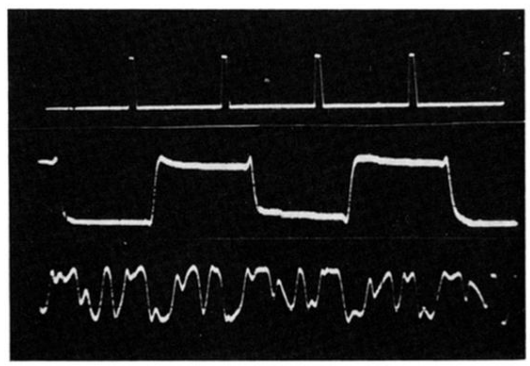

FIG. 2. (a) Time calibration; one pulse every delay time $t_K = 40$ ms. (b) Intensity vs time in the periodic domain. (c) Intensity vs time in the chaotic domain.