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Analysis of Reactor Experiments for Neutrino Oscillations

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Data from four reactor experiments is analyzed without using any calculated $\bar{\nu}_e$ spectrum. N_{e1} and N_{e2} , for e^+ observed with $2.2 < E_{e1} < 6.7$ MeV and $4.4 < E_{e2} < 6.7$ MeV, are extracted and N_{e1}/N_{e2} is found to be 2.7 ± 0.5 , 5.6 ± 0.6 , and 8.20 ± 0.35 for the 6.5-, 8.7-, and 11.2-m experiments, respectively. In pairs, these numbers differ by 3–8 standard deviations. No distance-independent $\bar{\nu}_e$ spectrum accounts for all the data with a confidence level (C.L.) > 0.0028 . Oscillations with three (two) ν 's yield fits to all data with C.L. = 0.061 (0.033) and to the high-statistics experiments with C.L. ≈ 0.31 (0.18).

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Since the phenomena of neutrino oscillations was first discussed¹ there have been several experimental suggestions in support of that possibility.^{2,3} The recent round of discussions on this subject was intensified by the experimental findings of Reines, Sobel, and Pasierb (RSP), who measured the rates for neutral current deuteron (ncd) and charge current deuteron (ccd) reactions initiated by reactor $\bar{\nu}_e$.³ Over the years, the en-

ergy spectrum of reactor $\bar{\nu}_e$ has been experimentally measured by the inverse beta (IB) reaction $\bar{\nu}_e + p \rightarrow n + e^+$ at 6.5,⁴ 8.7,⁵ and 11.2 m (Ref. 6) from reactor sources. We shall study the e^+ energy spectra measured in those three experiments in conjunction with the deuteron experiment of RSP to examine the hypothesis of oscillations.

In the IB reaction the differential rate for e^+ with observed kinetic energy E_e at a distance L from a reactor source is given by

$$\frac{dR}{dE_e} = 0.203 \times (9.24 \times 10^{-44} \text{ cm}^2)^{-1} \times \left(\frac{P}{1 \text{ MW}} \right) \left(\frac{n_p}{10^{26}} \right) \left(\frac{L}{1 \text{ m}} \right)^{-2} \eta_s \int \sigma(E_\nu) R_e(E_e, E_e') \eta(E_e') n(E_\nu, L) dE_\nu \text{ (MeV d)}^{-1}, \quad (1)$$

where n_p is the number of protons in the target, P is the reactor power, $E_e' = E_e - 1.8$ MeV, $\sigma(E_\nu) = 9.24 \times 10^{-44} (E_\nu - 1.29) [(E_\nu - 1.29)^2 - 0.26]^{1/2} \text{ cm}^2$, $R_e(E_e, E_e')$ is the experimental energy resolution function, $\eta(E_e')$ is the energy-dependent detection efficiency, η_s is the energy-independent systematic efficiency, and $n(E_\nu, L)$ is the spectrum (number of $\bar{\nu}_e$'s per fission per megaelectronvolt) of $\bar{\nu}_e$ with

energy E_ν at distance L . If neutrinos do not oscillate then $n(E_\nu, L)$ should be independent of L , i.e., $n(E_\nu, L) = n(E_\nu, 0) = n_0(E_\nu)$ which is the spectrum of $\bar{\nu}_e$ emitted at the reactor source.

In our analysis, instead of using any one of the theoretically calculated spectra,⁷ which are different by as much as 25% to 50% and therefore cannot be reliably used for ruling in favor of or against oscillations, we shall solve for the spectra that are compatible with the data separately under the oscillation and the no-oscillation hypotheses. To that end we assume that the reactor $\bar{\nu}_e$ spectrum, $n_0(E_\nu)$, can be parametrized in the general form

$$\ln n_0(E_\nu) = \sum_{j=0}^N A_j [E_\nu / (1 \text{ MeV})]^j. \quad (2)$$

χ^2 minimization is then used to extract A_j and N from the data sets.

First, however, we solve for the $\bar{\nu}_e$ spectrum $n(E_\nu, L)$ "seen" at the distance of each IB experiment using a parametrized form as in (2). The

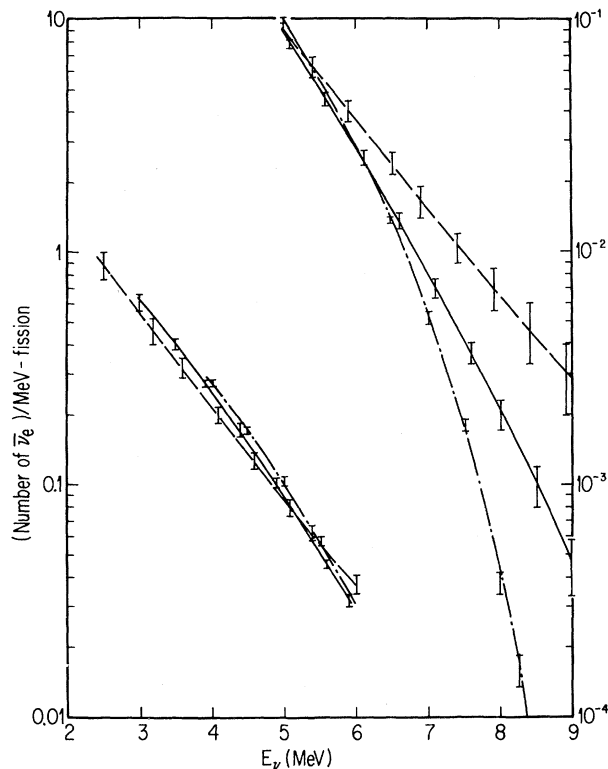


FIG. 1. The $\bar{\nu}_e$ spectra fitted to produce the e^+ spectra observed at 6.5 (dashed line), 8.7 (solid line), and 11.2 m (dash-dotted line). The vertical scale on left is for $E_\nu < 6$ MeV, and the one on the right is for $E_\nu > 5$ MeV.

resulting spectra, shown in Fig. 1, exhibit an interesting trend.⁸ For $E_\nu \geq 6$ MeV, the 6.5-m spectrum is the highest and the 11.2-m one is the lowest, with the 8.7-m one lying between those two. For $E_\nu \leq 6$ MeV, that ordering is reversed. To analyze this trend we divide the overlapping energy range of the three experiments into two halves and integrate each of these spectra for the intervals $4.0 < E_\nu < 8.5$ MeV and $6.2 < E_\nu < 8.5$ MeV. The statistical errors on these integrated rates will be appreciably less than on individual data points. To remove the normalization uncertainties we take the ratio $R_\nu(\text{expt})$ of those two integrals for each experiment. We find (see Fig. 2) that $R_\nu(\text{expt}) = 7.9 \pm 0.9$, 14.3 ± 1.2 , and 21.7 ± 1.0 , respectively, for the 6.5-, 8.7-, and the 11.2-m experiments differing from each other, taken in pairs, by 4.3, 4.7, and 10.3 standard deviations. With use of the weighted mean $\bar{R}_\nu(\text{expt}) = 14.1 \pm 0.6$ these measurements disagree by $\chi^2/\text{d.f.} \approx 105/2$, i.e., confidence level (CL) $< 10^{-5}$.

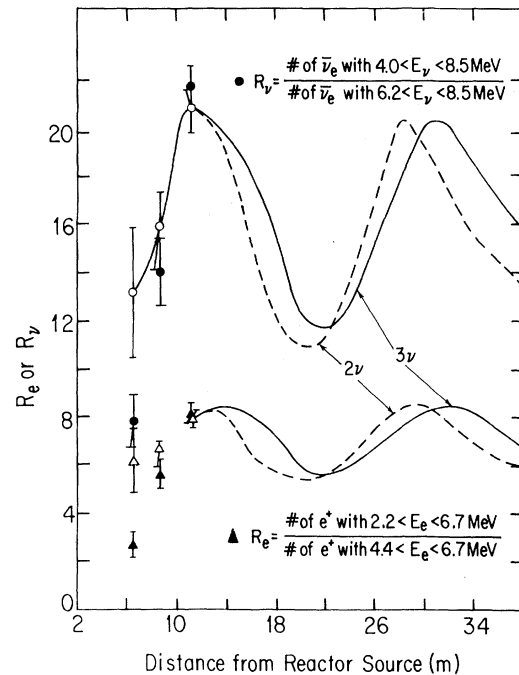


FIG. 2. Shown as a function of distance are measured values of R_e (solid triangles) and deduced R_ν (solid circles), and predicted values of R_e (open triangles) and R_ν (open circles) from our joint oscillation (2ν or 3ν) solutions. Predicted values of R_e and R_ν for longer distances are shown by solid (3ν solution) and by dashed (2ν , $\delta m^2 = 0.8 \text{ eV}^2$ solution) lines. Also illustrated is the distance dependence of R_e with, for example, the characteristics of the 6.5-m detector.

Indeed such a distance dependence is exhibited by the e^+ histograms themselves. We extract the numbers N_{e1} and N_{e2} of e^+ observed in the intervals $2.2 \leq E_e \leq 6.7$ MeV and $4.4 \leq E_e \leq 6.7$ MeV. We find that the ratio $R_e(\text{expt})$ of those two numbers (see Fig. 2) equals 2.7 ± 0.5 , 5.6 ± 0.6 , and 8.20 ± 0.35 , respectively, for the IB experiments performed at 6.5, 8.7, and 11.2 m. Again these numbers taken in pairs differ by 3.6, 3.6, and 8.7 standard deviations. Once again, using the weighted mean $\bar{R}_e(\text{expt}) = 6.33 \pm 0.26$, these measurements disagree by $\chi^2/\text{d.f.} = 79/2$, i.e., $\text{CL} < 10^{-5}$. Note that the distance dependence originates predominantly from the difference in the three spectra for $E_e \geq 4$ MeV.

We now search to see if there is any distance-independent $\bar{\nu}_e$ spectrum irrespective of its shape that can account for the observed differential e^+ spectra.⁹ We first disregard both the 6.5-m experiment and the deuteron experiment and use only the 26 points lying in the overlapping interval ($2.2 \leq E_e \leq 6.7$ MeV) for the 8.7- and the 11.2-m experiments.¹⁰ By varying the degree N in the general form [Eq. (2)] for the reactor spectrum, we find that the maximum attainable CL for such a solution is 0.025, i.e., $\chi^2/\text{d.f.} \approx 31.5/18$.

We next use all the four experiments and search for a no-oscillation solution to all the 54 data points.¹⁰ For the deuteron experiment³ of RSP, performed at 11.2 m from the reactor, we use the measured rates $[(165 \pm 25)/\text{d}]$ for the ncd reaction: $\bar{\nu} + d \rightarrow n + p + \bar{\nu}$ and $[(28 \pm 12)/\text{d}]$ for the ccd reaction: $\bar{\nu}_e + d \rightarrow n + n + e^+$ as two data points in our analysis. Once again we vary the degree N in Eq. (2) and find that the maximum attainable CL

for such a fit is $\text{CL} \approx 0.0028$ with $\chi^2/\text{d.f.} = 76.8/46$. The resulting no-oscillation reactor $\bar{\nu}_e$ spectrum has the following parameters (A_0, \dots, A_4):

$$0.91, 0.528, -0.5432, 0.09239, -0.006209; \\ \chi^2/\text{d.f.} = 76.8/46. \quad (3)$$

We now study how well neutrino oscillations¹¹ can account for all the four reactor experiments.¹⁰ We find a three-neutrino fit with $N=5$ and mixing matrix elements $U_{e1}=0.92$, $U_{e2}=0.32$, $U_{e3}=0.23$ with $m_2^2 - m_1^2 = 0.8$ eV², $m_3^2 - m_1^2 = 2.4$ eV², and $m_3^2 - m_2^2 = 1.6$ eV² having $\text{CL} \approx 0.061$, i.e., $\chi^2/\text{d.f.} \approx 57.0/42$. The resulting oscillation (3ν) reactor spectrum has the parameters (A_0, \dots, A_5):

$$0.22, 0.745, -0.4345, 0.04341, 0.001336, \\ -0.0004028; \chi^2/\text{d.f.} = 57/42. \quad (4)$$

For this fit $\eta_s = 1.04, 1.08$, and 1.005 for the 6.5-, 8.7-, and 11.2-m experiments.

When any one of the three neutrinos is effectively decoupled a 2ν oscillation fit to the data¹⁰ is obtained with values $\sin\theta \approx 0.35$ (0.34, 0.25), i.e., $\sin^2 2\theta \approx 0.43$ (0.41, 0.23) and $\delta m^2 \approx 0.8$ (3.7, 2.2) eV² having $\text{CL} \approx 0.033$ (0.024, 0.011), i.e., $\chi^2/\text{d.f.} \approx 62.7/44$ (64.4/44, 68.4/44).

The reactor $\bar{\nu}_e$ spectrum given by Eq. (4) obtained under the oscillation hypothesis agrees fairly well with the range of theoretically calculated spectra⁷ for $E_\nu \lesssim 7.0$ MeV and falls below both of them for $E_\nu \gtrsim 7.0$ MeV.

The no-oscillation joint fit (3) and the oscillation joint fit (4) each imply ncd and ccd rates within 1–1.4 standard deviations of the measured rates.

TABLE I. Comparison of confidence levels for hypotheses.

No.	Input data	Hypothesis	$\chi^2/\text{d.f.}$ (CL)
(1)	8.7+11.2 m (only overlapping data points)	No oscillations	31.5/18 (0.025)
(2)	Data from all four reactor experiments, i.e., 6.5, 8.7, 11.2 m +ncd +ccd	No oscillations	76.8/46 (0.0028)
(3)	Same as No. 2	Oscillations	57/42 (0.061)
(4)	8.7+11.2 m +ncd +ccd, i.e., disregard 6.5 m experiment	Oscillations	(same solution as for No. 3)
(5)	Same as No. 1	Oscillations	< 28/25 (> 0.31)
		(same solution as for No. 3)	< 18.5/14 (> 0.18)

The CL for the various solutions are presented in Table I. The no-oscillation hypothesis is not supported with or without the 6.5-m experiment. The oscillation fit to all the reactor data, although better (by a factor of 20) than the no-oscillation solution, has $\chi^2/\text{d.f.} = 57/42$, i.e., CL = 0.061 only. Most of the χ^2 (29 for 16 data points) in that solution originates from the 6.5-m experiment. In that experiment the reactor off background was measured for only $\frac{1}{7}$ th of the reactor on time and was therefore poorly determined. We thus examine the agreement of the other three experiments with our best fit. For that purpose we add up the χ^2 in that fit from those three experiments and find that the joint oscillation solution is in very good agreement (see No. 4 in the Table) having $\chi^2/\text{d.f.} < 28/25$, i.e., CL > 0.31. In particular, we notice (from No. 4 in the Table) that in that oscillation solution the overlapping 26 data points of the 8.7- and the 11.2-m experiments contribute $\chi^2/\text{d.f.} < 18.5/14$, i.e., CL > 0.18 to be compared to the no-oscillation best fit $\chi^2/\text{d.f.} = 31.5/18$, i.e., CL = 0.025 given in No. 1 in the Table.

Finally, we indicate how well the measured values of R_ν and R_e (Fig. 2) can be accounted for by oscillations. Since R_e is directly related to R_ν , we discuss only R_ν , although both are shown in Fig. 2. The oscillation solutions (2ν or 3ν) yield $R_\nu = 13.2 \pm 2.7$, 15.8 ± 1.6 , and 20.7 ± 1.0 for the 6.5-, 8.7-, and 11.2-m experiments to be compared to the measured values $R_\nu(\text{expt}) = 7.9 \pm 0.9$, 14.3 ± 1.2 , and 21.7 ± 1.0 , respectively. Thus the oscillation solutions are in very good agreement with the 8.7- and the 11.2-m experiments, but are about 1.9σ away from the 6.5-m experiment. In comparison, relevant to the no-oscillation hypothesis, the weighted average is $\bar{R}_\nu(3) = 14.1 \pm 0.6$ and it is 6.9σ away from the 6.5 m experiment. Once again, if we disregard the 6.5 m experiment then the weighted average for the 8.7- and 11.2-m experiments is $\bar{R}_\nu(2) = 18.7 \pm 0.8$ and it still disagrees from the measurements at the two distances by $\chi^2/\text{d.f.} = 22/1$, i.e., CL < 10^{-5} . In comparison the joint oscillation solution yields $\chi^2 = 2.6$ for the two data points, i.e., CL = 0.27.

Relevant to the oscillation experiments being currently planned we also show in Fig. 2 the values of R_ν and R_e as a function of distance implied by our oscillation solutions to the existing reactor data. We emphasize that this method of treating the integrated data is more accurate than dealing with the differential spectra and we note the dramatic distance dependence it exhibits.

In conclusion, taking the efficiencies, stated uncertainties, and other aspects of the experimental data at its face value, this analysis shows that the four reactor experiments, or for that matter the 11.2- and the 8.7-m IB decay experiments alone, cannot be satisfactorily accounted for by any distance-independent reactor $\bar{\nu}_e$ spectrum. Neutrino oscillations with mass differences ranging from ≈ 0.6 to ≈ 4 eV² are preferred by the data. Favored values of the oscillation parameters for our 3ν fit are $U_{e1} \approx 0.92$, $U_{e2} \approx 0.32$, $U_{e3} \approx 0.23$, $m_2^2 - m_1^2 \approx 0.8$ eV², $m_3^2 - m_1^2 \approx 2.4$ eV², and $m_3^2 - m_2^2 \approx 1.6$ eV². The possibility that one of the three neutrinos is effectively decoupled from $\bar{\nu}_e$ cannot be ruled out. For such effective 2ν solutions the favored values of the parameters are $\sin\theta \approx 0.35$ (0.34, 0.25), i.e., $\sin^2 2\theta \approx 0.43$ (0.41, 0.23), and $\delta m^2 \approx 0.8$ (3.7, 2.2) eV², respectively.

A detailed account of this work will be published elsewhere.

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⁸Errors shown in Fig. 1 are obtained by using the covariance matrix for the fits given.

⁹The reactor $\bar{\nu}_e$ spectra at the Savannah River Plant (Refs. 3 and 6) (0.88²³⁵U, 0.04²³⁸U, 0.08²³⁹Pu fissions) are estimated to differ from those at the reactor used

in Ref. 5 (only ²³⁵U fissions) by no more than 2.4% for 1 MeV < E_ν < 7 MeV using the calculations of Ref. 7.

¹⁰In this count the systematic efficiencies (10%, 8%, and 13.8% for the 6.5-, 8.7-, and 11.2-m IB decay experiments, respectively) are included as separate data points.

¹¹Standard theoretical expressions for the $\bar{\nu}_e$ oscillations emerging from neutrino flavor mixing and given in papers of Ref. 2 are used. However, experimental energy resolution and reactor size effects are fully taken into account.