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Proton Decay Rate

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The pionic two-body decay amplitude of the proton in the SU(5) grand unified gauge theory is computed by using the soft-pion method in the reference frame of the pion at rest. The normalization condition of the relativistic three-body Bethe-Salpeter wave function of the proton is used in the computation. It is shown that the partial decay rate of the process, proton $-e^{+}\pi^0$, is $(0.86 \times 10^{30} \text{ yr})^{-1}$ and $(1.4 \times 10^{31} \text{ yr})^{-1}$ for $m_X = 4.0 \times 10^{14}$ GeV and 8.0×10^{14} GeV, respectively. These values are on the boundary of the present experimental limit.

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One of the most remarkable consequences of grand unified gauge theories is that the proton decays. Several authors have made estimates of the proton decay rate in the SU(5) and SO(10) mod-
els^{1,2} and have found the lifetime to be in the range of $10^{31} - 10^{33} \text{ yr}^3$, which is close to the present experimental lower bound⁵ ($\sim 10^{30}$ yr). Renewed attempts to observe such decays are under way and are expected to give some results soon if the proton lifetime is in the above-mentioned range.

Among various decay modes, the mode $p-e^+$ $+\pi^0$ seems to be the most appropriate one for detection in most experiments in progress if its branching ratio is significantly large. However, there is some uncertainty in the theoretical estimates for this process since the estimates are made based on either the SU(6) wave functions or the Massachusetts Institute of Technology bag model wave functions, which are either nonrelativistic or noncovariant. The proton, consisting of three light quarks, is likely to be a relativistic system and the relativistic corrections may not be negligible. In this article, I present a relativistie calculation of the pionie two-body decay of protons based on the partially conserved axialvector current hypothesis (PCAC) and current algebra.

In order to use the soft-pion method, I shall calculate the decay amplitude in the rest frame of the pion. If the soft-pion limit is taken literally, the momentum of the incident proton becomes infinite:

$$
(\rho^2 + M^2)^{1/2} = \frac{M^2 + m^2 + \mu_{\pi}^2}{2\mu_{\pi}} \to \infty, \text{ as } \mu_{\pi} \to 0,
$$
 (1)

where M, m , and μ_{π} are the masses of the proton, the positron, and the pion, respectively. However, it is important to note that we can use the soft-pion method either in the ordinary reference frame of the pion at rest or in the infinitemomentum reference frame of the incident proton.⁶ The two situations are identical in the limit μ_{π} + 0. In fact, this method has been used in the analysis of nonleptonic hyperon weak decays and has led to successful sum rules for the p -wave amplitudes as well as for the s -wave amplitudes.⁷

The interaction Lagrangian in the SU(5) gauge model, which is relevant to the proton decay, is given by

$$
\mathcal{L} = (4G\sqrt{\sqrt{2}})\epsilon_{ijk}[\bar{u}_{kL}{}^c\gamma_{\mu} u_{jL}(2\bar{e}_L{}^{\dagger}\gamma_{\mu}d_{iL} + \bar{e}_R{}^{\dagger}\gamma_{\mu}d_{iR} + \bar{\mu}_L{}^{\dagger}\gamma_{\mu} s_{iL} + \bar{\mu}_R{}^{\dagger}\gamma_{\mu} s_{iR})
$$

$$
+ \bar{u}_{kL}{}^c\gamma_{\mu}d_{jL}(\bar{\nu}_{eR}{}^c\gamma_{\mu}d_{iR} + \bar{\nu}_{\mu}{}^c\gamma_{\mu} s_{iR})],
$$
 (2)

where $(u_k, d_k, s_k)_{L(R)}$ and $(e^-, \nu_e, \mu^-, \nu_\mu)_{L(R)}$ are the left- (right-) handed quarks of color index k and the leptons, respectively. The coupling constant G is given in terms of the grand unified coupling constant g_{GUT} and the heavy gauge field mass m_x , by

$$
G = (g_{\text{GUT}}^2/4\sqrt{2m_X}^2),\tag{3}
$$

while λ is the quantum-chromodynamic enhancement factor^{1,3} (λ = 3.7).

Using the PCAC relation for pions,

$$
\varphi^{\alpha} = (F_{\pi}\mu_{\pi}^{2})^{-1}\partial A_{\mu}^{\alpha}/\partial x_{\mu}, \quad \alpha = 1, 2, 3,
$$
\n⁽⁴⁾

where $\sqrt{2}F_{\pi}$ is the decay constant of $\pi^+\to\mu^+\nu_{\mu}$ ($\sqrt{2}F_{\pi}=0.945\mu_{\pi}$) and where $A_{\mu}{}^{\alpha}$ is the axial-vector current, and applying the standard technique of the Lehmann-Symanzik-Zimmermann reduction formalism in the rest frame of the pion, I obtain the following expression for the decay amplitude for $p \rightarrow e^+ \pi^0$.

$$
(2q_0)^{1/2} \langle e(\vec{p}), \pi^0(\vec{0}) | \mathcal{L} | p(\vec{p}) \rangle
$$

= $-i \frac{\mu_{\pi}^2 - q_0^2}{F_{\pi} \mu_{\pi}^2} \langle e | [Q_5^3, \mathcal{L}] | p \rangle - q_0 \sum_{l} \left[\frac{\langle e | A_0^3(0) | l \rangle \langle l | \mathcal{L} | p \rangle}{q_0 + p_0^2 - p_0^2 + i \epsilon} - \frac{\langle e | \mathcal{L} | l \rangle \langle l | A_0^3(0) | p \rangle}{q_0 + p_0^2 - p_0^2 - i \epsilon} \right]_{\vec{p}} l_{\vec{p}} l_{\vec{p}} \rangle.$ (5)

The limit $q_0 \rightarrow 0$ selects the intermediate state l in the summation of Eq. (5) to be equal to either the initial or the final particle state. Thus, I obtain

$$
f = (2q_0)^{1/2} \langle e\pi^0 | \mathcal{L} | p \rangle = \frac{-i}{F_{\pi}} \{ \langle e | [Q_5^3, \mathcal{L}] | p \rangle + \sum_{\substack{\text{intermediate} \\ \text{spin sum}}} \langle e | \mathcal{L} | p \rangle \langle p | A_0^3(0) | p \rangle \}.
$$
 (6)

Here I have used the identity $\langle e|A_0^3(0)|e\rangle = 0$. The first term of Eq. (6) is computed by using

$$
Q_5^3 = \frac{1}{2} \int \left(u_i \, {}^{\dagger} \gamma_5 u_i - d_i \, {}^{\dagger} \gamma_5 d_i \right) d^3 x \,, \tag{7}
$$

giving, '

$$
[Q_{5}^{3}, \mathcal{L}] = (4G\lambda/\sqrt{2})\epsilon_{ijk}\frac{1}{2}[\bar{u}_{k}c\gamma_{\mu}u_{jL}(2\bar{e}_{L}^{\dagger}\gamma_{\mu}d_{iL} - \bar{e}_{R}^{\dagger}\gamma_{\mu}d_{iR}) + \bar{u}_{k}c\gamma_{\mu}d_{jL}\bar{\nu}_{eR}c\gamma_{\mu}d_{iR}].
$$
\n(8)

Then Eq. (6) becomes

$$
f = \frac{-i}{F_{\pi}} \frac{4G\lambda}{\sqrt{2}} \epsilon_{ijk} \left(\frac{m}{g_p}\right)^{1/2} \overline{u}_e^{(e)}(p) \langle 0|u_{ka} d_{ic} u_{jb}|p \rangle \left(c^{-1}\gamma_{\mu} \frac{1+\gamma_5}{2}\right)_{ab}
$$

$$
\times \left[\frac{1}{2} \left(\gamma_{\mu} \frac{1+3\gamma_5}{2}\right)_{ec} + \left(\gamma_{\mu} \frac{3+\gamma_5}{2}\right)_{ec} \frac{M}{2E_p} g_A \overline{u}(p) \gamma_4 \gamma_5 u(p)\right],
$$
(9)

where \mathcal{E}_p , $u^{(e)}(p)$ and E_p , $u(p)$ are the energy and the Dirac spinor for the positron and the proton, respectively, and g_A is the axial-vector coupling constant of the neutron β decay.

In order to find the expression for the three-quark wave function of the proton, $\langle 0|u_{ka}(0)d_{ic}(0)u_{jb}(0)|p\rangle$, we define the three-body Bethe-Salpeter amplitude for the octet baryons, ${}^{9}B$,

$$
\langle 0|T\psi_{i\alpha}{}^{\alpha}(\mathbf{x}_{1})\psi_{j\beta}{}^{\beta}(\mathbf{x}_{2})\psi_{k\alpha}{}^{\delta}(\mathbf{x}_{3})|B\rangle = (M/E_{p})^{1/2}\epsilon^{ijk\frac{1}{2}}(\chi_{abc}{}^{(\xi)}U_{\alpha\beta\gamma}{}^{(\xi)} + \chi_{abc}{}^{(\eta)}U_{\alpha\beta\gamma}{}^{(\eta)})\varphi(\xi,\eta,\rho)e^{ipX},\tag{10}
$$

where

$$
X = \frac{1}{3}(x_1 + x_2 + x_3), \quad p = p_1 + p_2 + p_3, \quad \xi = x_1 - x_2,
$$

\n
$$
p_{\xi} = \frac{1}{2}(p_1 - p_2), \quad \eta = \frac{1}{2}(x_1 + x_2 - 2x_3), \quad p_{\eta} = \frac{1}{3}(p_1 + p_2 - 2p_3),
$$
\n(11)

$$
\chi_{abc}^{(\xi)} = \left(\frac{-i\gamma p + M}{2M} \gamma_5 C\right)_{ab} u_c(p), \quad \chi_{abc}^{(\eta)} = (1/\sqrt{3})(\chi_{bca}^{(\xi)} - \chi_{cab}^{(\xi)}), \tag{12}
$$

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and

$$
U_{\alpha\beta\gamma}^{(\xi)} = \epsilon_{\alpha\beta\gamma} B_{\gamma}^{\delta}, \quad U_{\alpha\beta\gamma}^{(\eta)} = (1/\sqrt{3})(U_{\beta\gamma\alpha}^{(\xi)} - U_{\gamma\alpha\beta}^{(\xi)}).
$$
\n(13)

The Greek index in Eq. (10) represents the ordinary SU(3) index of the strong interactions (not the color index) and B stands for the 3×3 matrix of the baryon octet.

The normalization of the above amplitude can be written as

$$
-iM\int \frac{d^4p_{\xi}d^4p_{\eta}d^4p_{\xi}d^4p_{\eta}}{(2\pi)^{16}}\bar{\chi}_{\rho}(p_{\xi}p_{\eta})\frac{\partial}{\partial p_{0}}[I(p_{\xi},p_{\eta}',p_{\xi},p_{\eta};p)+\overline{G}(p_{\xi}',p_{\eta}',p_{\xi},p_{\eta};p)]\chi_{\rho}(p_{\xi},p_{\eta})=p_{0},\quad(14)
$$

where $\chi_{p}(p_{\xi}, p_{\eta})$ is the Fourier transform of

$$
\chi_{\rho}(\xi,\eta) = \frac{1}{2} \epsilon^{ijk} (\chi_{abc}^{(\xi)} U_{\alpha\beta\gamma}^{(\xi)} + \chi_{abc}^{(\eta)} U_{\alpha\beta\gamma}^{(\eta)}) \varphi(\xi,\eta,\rho).
$$
\n(15)

 \overline{G} is the kernel for the six-point Green's function and

$$
I(p_{\xi},p_{\eta},p_{\xi},p_{\eta};p) = (2\pi)^{8}\delta(p_{\xi} - p_{\xi})\delta(p_{\eta} - p_{\eta})[S_{\mathbf{F}}^{A(\frac{1}{3}p + p_{\xi} + \frac{1}{2}p_{\eta})S_{\mathbf{F}}^{B(\frac{1}{3}p - p_{\xi} + \frac{1}{2}p_{\eta})S_{\mathbf{F}}^{C(\frac{1}{3}p - p_{\eta})}]^{-1}
$$
 (16) with

$$
[S_{\mathrm{F}}^{\ A}(k)]^{-1} = i(i\gamma k + m_A). \tag{17}
$$

Equation (14) is a generalization of the normalization condition for the two-body Bethe-Salpeter ampli
tude.¹⁰ The explicit computation of Eq. (10) gives tude.¹⁰ The explicit computation of Eq. (10) gives

$$
\int \frac{d^4 p \, \xi d^4 p \, \eta}{(2\pi)^8} |\tilde{\varphi}|^2 \left[\frac{1}{2} \frac{(p p \, \eta)^2}{M^2} + \frac{2}{3} \frac{(p p \, \xi)^2}{M^2} - 2 \left(\frac{1}{3} M - m_q \right)^2 \right] = 1, \tag{18}
$$

where $\tilde{\varphi}(p_{\varepsilon}, p_{\varepsilon}, p)$ is the Fourier transform of $\varphi(\xi, \eta; p)$. Then, for the wave function for the relativiswhere $\varphi(p_{\xi}, p_{\eta}, p)$ is the rourier to the network

harmonic-oscillator potential,
\n
$$
\varphi(\xi,\eta;\rho) = N \exp\left\{-\frac{\alpha}{6} \left[2\left(\frac{\rho\hat{\eta}}{M}\right)^2 + \hat{\eta} + 2\left(\frac{\rho\xi}{M}\right)^2 + \hat{\xi}^2\right]\right\},
$$
\n(19)

where

$$
\hat{\xi} = \xi/\sqrt{2}, \quad \hat{\eta} = (2/3)^{1/2}\eta, \tag{20}
$$

the normalization condition gives

$$
N = (\alpha / 3\pi)^2 [\alpha - 2(M - 3m_q)^2]^{-1/2}.
$$
 (21)

Assuming that $M \approx 3m_a$, I obtain

$$
\varphi(0,0,p) \equiv N = (3\pi)^{-1/2} (\alpha/3\pi)^{3/2}.
$$
 (22)

Using Eqs. (10), (12), (13), and (19), I can compute the matrix element of Eq. (9), giving

$$
f = \frac{-i}{F_{\pi}} \frac{4G\lambda}{\sqrt{2}} 6N \left(\frac{mM}{g_{\rho}E_{\rho}}\right)^{1/2} \frac{1}{2} \left[\overline{u}^{(e)}(p)\Gamma_{1}u(p) + \overline{u}^{(e)}(p)\Gamma_{2}u(p)\frac{M}{2E_{\rho}}g_{A}\overline{u}(p)\gamma_{4}\gamma_{5}u(p)\right],
$$
\n(23)

where

$$
\Gamma_{i} = -\gamma_{\mu} \frac{a_{i} + b_{i} \gamma_{5}}{2} C \gamma_{\mu}^{T} \frac{-i \gamma^{T} p + M}{2M} C^{-1} \gamma_{\mu} \frac{1 + \gamma_{5}}{2} + \frac{1}{3} \gamma_{\mu} \frac{a_{i} + b_{i} \gamma_{5}}{2} \frac{-i \gamma p + M}{2M} \gamma_{5} C \frac{1 + \gamma_{5}^{T}}{2} \gamma_{\mu}^{T} C^{-1} + \frac{1}{3} \gamma_{\mu} \frac{a_{i} + b_{i} \gamma_{5}}{2} Tr \left(\frac{-i \gamma p + M}{2M} \gamma_{5} \gamma_{\mu} \frac{1 + \gamma_{5}}{2} \right)
$$

= $-b_{i} + a_{i} \gamma_{5} + \frac{a_{i} + b_{i}}{3} (1 - \gamma_{5}) \frac{i \gamma p + M}{M} \rightarrow -b_{i} + a_{i} \gamma_{5}, \quad i = 1, 2.$ (24)

Using the values

$$
a_1 = \frac{1}{2}, \quad b_1 = \frac{3}{2}, \quad a_2 = 3, \quad b_2 = 1,
$$
 (25)

I obtain

$$
\Gamma_1 = -\frac{3}{2} + \frac{1}{2}\gamma_5, \quad \Gamma_2 = -1 + 3\gamma_5. \tag{26}
$$

The bilinear Dirac spinors in the rest frame of the pion which appear in Eq. (23) can be computed by

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using the formulas

$$
\overline{u}(p)_{\gamma_4\gamma_5 u}(p) = -\frac{|p|}{M} \xi^{\dagger} \overline{\dot{\sigma}} \cdot \hat{\dot{p}} \xi, \quad \overline{u}^{(e)}(p)u(p) = \frac{1}{2}(Mm)^{-1/2} [(M+m)^2 - \mu_{\pi}^2]^{1/2} \xi^{(e)\dagger} \xi,
$$
\n
$$
\overline{u}^{(e)}(p)_{\gamma_5 u}(p) = \frac{1}{2}(Mm)^{-1/2} [(M-m)^2 - \mu_{\pi}^2]^{1/2} \xi^{(e)\dagger} \overline{\dot{\sigma}} \cdot \hat{\dot{p}} \xi,
$$
\n(27)

where ξ and $\xi^{(e)}$ are the two-component spinor of proton and positron, respectively, and $\mathbf{\hat{\tilde{p}}}$ is the uni vector in the direction of \bar{p} . Alternatively, one can use Table III of Ref. 7 for the infinite-momentum reference frame of the incident proton. Both methods give the same result upon neglecting the pion mass. I get the final form for the decay amplitude

$$
\frac{f}{(2q_0)^{1/2}} = \frac{-i}{F_\pi} \frac{1}{(2q_0)^{1/2}} \frac{12G\lambda N}{\sqrt{2}} \left(\frac{mM}{\mathcal{E}_p E_p}\right)^{1/2} \overline{u}^{(e)}(p) (A + B_{\gamma_5}) u(p), \tag{28}
$$

where

$$
A = -\frac{3}{2} \{ 1 + [(M - m)/(M + m)]_{\mathcal{G}_A} \} \approx -\frac{3}{2} (1 + g_A) = -3.39,
$$

\n
$$
B = \frac{1}{2} \{ 1 + [(M + m)/(M - m)]_{\mathcal{G}_A} \} \approx \frac{1}{2} (1 + g_A) = 1.13.
$$
\n(29)

(Here, use is made of the empirical value, $g_A = 1.260 \pm 0.007$.)

The decay rate of $p-e^{\dagger}\pi^{0}$, then, can be computed from Eq. (28), giving

$$
\Gamma(p \to e^{\pm} \pi^0) = \frac{M}{16\pi} \left(\frac{12G\lambda N}{\sqrt{2}F_{\pi}}\right)^2 (|A|^2 + |B|^2) = \frac{M}{16\pi} \left(\frac{3g_{\text{ GUT}}^2 \lambda N}{2F_{\pi}m_{X}^2}\right)^2 \frac{5}{2} (1 + g_A)^2.
$$
\n(30)

For typical values of the parameters^{4, 9, 11}

$$
g_{\text{GUT}}^2/4\pi = 0.024
$$
, $\lambda = 3.7$, $\alpha = 0.4 \text{ GeV}^2$, (31)

I obtain the partial decay time

 $\tau (p -$

$$
e^{+}\pi^{0}) = \left[\Gamma(p + e^{+}\pi^{0})\right]^{-1} = \begin{cases} 0.86 \times 10^{30} \text{ yr for } m_{X} = 4.0 \times 10^{14} \text{ GeV} \\ 1.4 \times 10^{31} \text{ yr for } m_{X} = 8.0 \times 10^{14} \text{ GeV}. \end{cases}
$$
(32)

These values are on the boundary of the experimental limit,⁵ and are reasonably close to those
obtained by Gavela *et al.*¹² $\{[\Gamma(\phi + e^+\pi^0)]^{-1} = 0.5\}$ obtained by Gavela *et al.*¹² $\{[\Gamma(\rho + e^{\dagger}\pi^0)]^{-1} = 0.53\}$ \times 10³⁰ y for $m_x = 4 \times 10^{14}$ GeV. The results obtained by other groups may be compared with Eq. (32) by assuming a typical branching ratio^{4, 12-14} (32) by assuming a typical branching ratio^{4, 12-14} $\Gamma(p \rightarrow e^{\dagger} \pi^0)/\Gamma(p \rightarrow two bodies) = \frac{1}{3}$; Goldman and Ross (Ref. 3), Din, Girardi, and Sorba (Ref. 13), Buras $et al.$ (Ref. 1), and Donoghue (Ref. 4) give, respectively $10^{-30} [\Gamma(\rho + e^{\mu \tau})]^{-1} = 1.8, 2.7, 3.7,$ and 30 yr for $m_X = 4 \times 10^{14}$ GeV. For a comprehensive review and many references on this subject, see Ref. 14. Note also that if we take the value of the parameter α of the proton wave function to be 0.5 GeV², which is an alternative choice suggested in Ref. 9, we would have obtained $[\Gamma(\rho + e^+ \pi^0)]^{-1} = 0.44 \times 10^{30}$ yr for $m_x = 4.0 \times 10^1$ GeV. This is very close indeed to the value ob-
tained by Gavela *et al*.¹² tained by Gavela et al.¹²

From Eqs. (28) and (29), we can deduce that the decay amplitude of $p - e^{+}\pi^{0}$ is predominantly s wave and the asymmetry parameter is given by

$$
\alpha = 2 \text{ Re}(A^*B) / (|A|^2 + |B|^2) = -0.60. \tag{33}
$$

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= \overline{u}^T , $\overline{u}^c = -u^T C^{-1}$, $CC^{\dagger} = 1$, $C^T = -C$, and $C^{-1} \gamma_{\mu} C = \gamma_{\mu}$

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Analysis of Reactor Experiments for Neutrino Qscillations

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Data from four reactor experiments is analyzed without using any calculated $\bar{\nu}_e$ spectrum. $N_{e,1}$ and $N_{e,2}$, for e^+ observed with $2.2 \times E_{e,1} \times 6.7$ MeV and $4.4 \times E_{e,2} \times 6.7$ MeV, are extracted and N_{e1}/N_{e2} is found to be 2.7 ± 0.5, 5.6 ± 0.6, and 8.20 ± 0.35 for the 6.5-, 8.7-, and 11.2-m experiments, respectively. In pairs, these numbers differ by 3-8 standard deviations. No distance-independent $\overline{\nu}_e$ spectrum accounts for all the data with a confidence level (C.L.) > 0.0028 . Oscillations with three (two) ν 's yield fits to all data with C.L.=0.061 (0.033) and to the high-statistics experiments with C.L. \approx 0.31 (0.18).

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Since the phenomena of neutrino oscillations was first discussed' there have been several experimental suggestions in support of that possiwas first discussed¹ there have been several experimental suggestions in support of that possi-
bility.^{2,3} The recent round of discussions on this subject was intensified by the experimental findings of Reines, Sobel, and Pasierb (RSP), who measured the rates for neutral current deuteron (ncd) and charge current deuteron (ccd) reaction initiated by reactor $\bar{\nu}_e$,³ Over the years, the en-

ergy spectrum of reactor $\bar{\nu}_e$ has been experimentally measured by the inverse beta (IB) reaction taily measured by the inverse beta (b) react.
 $\bar{\nu}_e + p \rightarrow n + e^+$ at 6.5,⁴ 8.7,⁵ and 11.2 m (Ref. 6) from reactor sources. We shall study the e^+ energy spectra measured in those three experiments in conjunction with the deuteron experiment of RSP to examine the hypothesis of oscillations.

In the IB reaction the differential rate for e^+ with observed kinetic energy E_e at a distance L from a reactor source is given by

$$
\frac{dR}{dE_e} = 0.203 \times (9.24 \times 10^{-44} \text{ cm}^2)^{-1} \times \left(\frac{P}{1 \text{ MW}}\right) \left(\frac{n_b}{10^{26}}\right) \left(\frac{L}{1 \text{ m}}\right)^{-2} \eta_s \int \sigma(E_\nu) R_e(E_e, E_e') \eta(E_e') n(E_\nu, L) dE_\nu \text{ (MeV d)}^{-1}, \tag{1}
$$

where n_{p} is the number of protons in the target, P is the reactor power, $E_{e}^{'}=E_{v}-1.8$ MeV, $\sigma(E_{v})$ = 9.24 \times 10⁻⁴⁴(E_v – 1.29)[$(E_v$ – 1.29)² – 0.26]^{1/2} cm², $R_e(E_e, E_e')$ is the experimental energy resolution function, $\eta(E_e')$ is the energy-dependent detection efficiency, η_s is the energy-independent systematic efficiency, and $n(E_y, L)$ is the spectrum (number of $\overline{\nu}_e$'s per fission per megaelectronvolt) of $\overline{\nu}_e$ with