where

$$T_H/T_0 = (\pi/e)^{1/2}.$$

Note that again if we expand the result in the coupling constant we find the same nonanalytic relation between J and the conventionally defined coupling constant g.

Now from the two universal numbers T_K/T_H and T_H/T_0 we may deduce a third one:

$$W = \frac{T_K}{T_0} = \frac{T_K}{T_H}\frac{T_H}{T_0} = 2\beta\gamma\pi^{1/2}e^{-9/4},$$

which appears in the famous relation[3]

$$\chi^i = W\pi^{-1}\mu^2/T_K,$$

and we find, substituting the values $\ln\gamma = 0.577\,216$ and $\ln\beta = 0.662\,122$, that $W/4\pi = 0.102\,676$ in agreement with Wilson's numerical answer $W/4\pi = 0.1032 \pm 0.0005$.

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Observation of a Non-Ohmic Hall Resistivity at Low Temperatures in a Two-Dimensional Electron Gas

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Hall measurements of electron inversion layers at low temperatures and electric fields are reported. For resistances of $\lesssim 10$ kΩ/□ a logarithmic dependence of the Hall coefficient on temperature and Hall voltage is observed. This indicates that the logarithmic dependences observed for the resistance of metal films and metal-oxide-semiconductor field-effect transistors are not evidence for the scaling theory of localization. These results are also difficult to resolve within the theory of interaction effects by Altshuler, Aronov, and Lee.

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Several experiments have recently observed logarithmic dependences of the resistivity on temperature and electric field which scale with sheet resistance $R_\square$ for two-dimensional (2D) systems at low temperatures.[1,2] In addition, for silicon inversion layers, Kawaguchi and Kawaji have reported a negative magnetoresistance.[3] Although these observations qualitatively agree with the basic predictions of the scaling theory of localization by Abrahams, Anderson, Licciardello, and Ramakrishnan (AALR)[4] there is now an alternative theory by Altshuler, Aronov, and Lee

(AAL)[5,6] based on the effects of Coulomb interactions in two dimensions which is equally capable of describing the experimental results. Even though both theories predict similar behavior for the logarithmic corrections to the conductance in zero field, they predict significantly different Hall-effect behavior. Therefore a measure of the Hall effect should allow one to determine which, if either, theory is correct.

In this paper we report measurements of the Hall effect in Si metal-oxide-semiconductor field-effect transistors (MOSFET's) at low temperatures and low electric fields. We have observed logarithmic dependences of the Hall coefficient R_H on temperature and Hall voltage which are in disagreement with the predictions of the scaling theory of localization. According to this theory, localization effects change the mobility of the 2D electron gas, and the Hall coefficient R_H which measures the electron density should be unaffected. Therefore the theory anticipates no logarithmic temperature dependence to the Hall resistance, in contradiction to our observations. The magnitude of the voltage and temperature dependences of $R_\square$ is the same as that predicted by the interaction theory of AAL. However, we are in a screening regime where the effects predicted by their theory should vanish. We conclude that the previously observed logarithmic rises in resistance are not evidence for the scaling theory of localization and that the interaction theory of AAL might be able to explain our results but not before a further understanding of 2D screening evolves.

Our measurements were performed on N-channel MOSFET's fabricated on (111) surfaces of p-type silicon with peak mobilities of ~ 1000 cm^2/V sec at 4.2 K. The devices were similar to those studied in Ref. 2 and the zero-field logarithmic slopes of resistance were in quantitative agreement with the results of Ref. 2. The measurements were performed in a ^3He-^4He dilution refrigerator at temperatures from 50 mK to 1 K. The conducting channels were 0.25 mm wide and 1.0 mm long, with potential probes arranged in the usual Hall geometry. The two potential probes were separated by 0.25 mm and the $R_\square$ quoted here is the resistance of a 0.25×0.25 mm^2 area at 1 K. The Hall voltage was measured as a function of Hall current using a bridge circuit described previously.[2,7] The measurements were performed in both positive and negative magnetic field configurations and carefully averaged to eliminate effects due to Hall probe misalignment and nonuniform current paths. The field was supplied from a superconducting solenoid and measurements performed to 30 kG in the low-field limit $\mu H < 0.3$.

Figure 1 shows the deviation of Hall voltage from linearity as a function of current at different temperatures with a constant resistance $R_{sub} = 445$ Ω subtracted off. Note that the curves become non-Ohmic at extremely low values of current which implies that the Hall coefficient R_H has a nonzero temperature dependence. For $R_\square \lesssim 10$ kΩ the Hall resistance is logarithmic in Hall voltage at the lowest temperatures and logarithmic in temperature at the lowest voltages as is shown in Fig. 2.

Therefore, as for the conductance, one observes that the Hall coefficient depends logarithmically on both temperature and voltage. This voltage nonlinearity is due to a temperature-dependent R_H and simple Joule heating of the electron gas. As pointed out by Anderson, Abrahams, and Ramakrishnan[8] the ratio of the coefficients of $\ln V$ and $\ln T$ should be a constant given by $2/(2+P')$, where P' is the temperature exponent of the electron-phonon scattering rate. This ratio should be independent of the physical mechanism which determines the coefficient of the $\ln T$ dependence. For our data this ratio has a value of ~ 0.45 which implies that $P' \sim 3$. This value is consistent with previous measurements in Ref. 2 and reflects the 2D nature of the electron-phonon interaction.

In the scaling theory of localization by AALR, the conductance at $T = 0$ K of a disordered system depends in a universal manner on its length scale L. They argued that the logarithmic slopes ver-

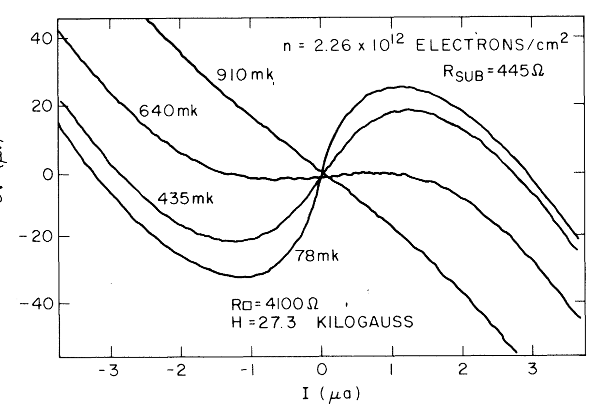

FIG. 1. Hall voltage shown as a function of Hall current at different temperatures. The data shown have a constant resistance $R_{sub} = 445$ Ω subtracted off.

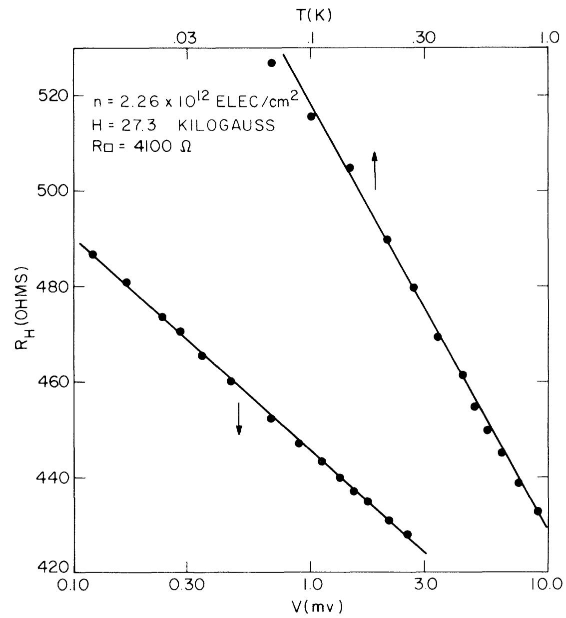

FIG. 2. The logarithmic dependences of the Hall resistance from Fig. 1 shown as a function of temperature (upper scale) and voltage (lower scale).

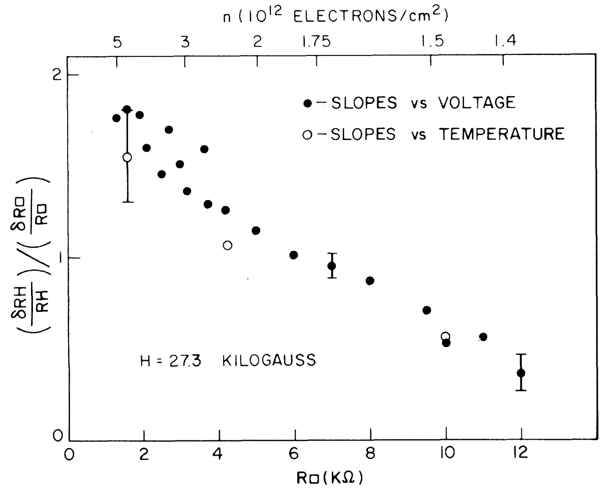

FIG. 3. The ratio $(\delta R_H/R_H)/(\delta R_\square/R_\square)$ shown as a function of $R_\square$ for a field of 27.3 kG.

sus temperature should be given by $(e^2/\pi^2\hbar)(\alpha P/2)R_\square$. AALR expect $\alpha = 1$ for two noncommunicating gases of spins up and down and $\alpha = \frac{1}{2}$ if the elastic spin-flip scattering length is short in comparison with the other lengths in the problem. P is the exponent for the inelastic scattering rate, $\tau_{in} \sim T^{-P}$. For electron-electron scattering in the presence of impurities one expects $P \sim 1$.[9] Therefore the theory of AALR is consistent with the value $\alpha P \sim 1$ as determined by the experiments.[1,2]

However, it has been shown by AAL that Coulomb-interaction effects in disordered Fermi systems in two dimensions also give logarithmic corrections to the resistance whose slopes versus temperature are given by $(e^2/\pi^2\hbar)(\frac{1}{2})R_\square$, which is similar to the scaling theory and the results of the experiments. We have a situation where two distinct theories describe quantitatively the same behavior. The scaling theory ignores the effects of particle interactions while the Coulomb-interaction theory of AAL takes no account of localization effects.

Despite the similarity of the predictions for the conductance the two theories have radically different expectations for the Hall effect. In a result first obtained by Fukuyama[10] the localization theory predicts $\delta R_H/R_H = 0$, where δR_H is the logarithmic decrement per decade for the Hall coefficient. In other words the localization theory predicts *no* logarithmic corrections to the Hall coefficient. This comes about because localization effects manifest themselves in the mobility and not the density of the 2D electron gas. However, it is clear qualitatively from Fig. 1 that this is not the case as nonlinearities are observed, i.e., there is a temperature dependence to R_H.

The Coulomb-interaction picture predicts a much different result. AAL[4] have shown that their theory predicts a logarithmic dependence of the Hall coefficient on temperature. These workers are able to calculate these dependences for their model in the limit of weak screening, small magnetic fields, and small $R_\square$. In these limits AAL predict that the ratio $(\delta R_H/R_H)/(\delta R_\square/R_\square)$ should be 2. This factor of 2 comes about because corrections to the Hall current are found to vanish and the Hall constant R_H then goes like the square of the resistance. The calculation is perturbative and is not easily extended to the general case of finite H and $R_\square$.

From data such as shown in Fig. 2 we can determine this ratio as a function of H and $R_\square$. In Fig. 3 this ratio is shown as a function of $R_\square$ for $H = 27.3$ kG. Note that in the limit of $R_\square \to 0$, the data are clearly consistent with $\lim_{R_\square \to 0}(\text{ratio}) = 2$. Shown in Fig. 4 are data at $R_\square \sim 4100\ \Omega$ as a function of magnetic field. These data show that the limiting behavior of the ratio as $H \to 0$ is also consistent with 2 despite the large error bars which are due to the difficulty of measuring the Hall effect with high resolution at low magnetic fields. Therefore our results are consistent with

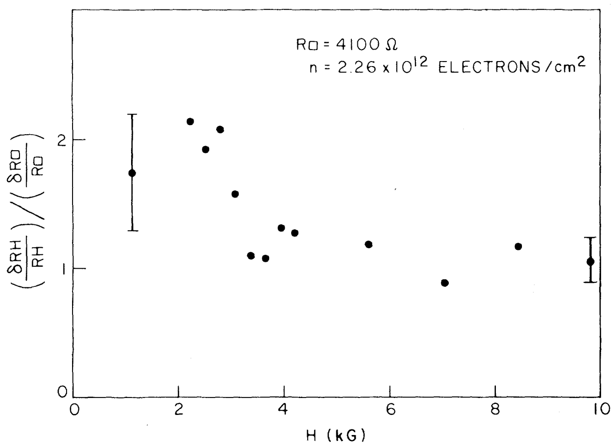

FIG. 4. The ratio $(\delta R_H/R_H)/(\delta R_\square/R_\square)$ shown as a function of magnetic field for a channel resistance of 4100 Ω/□.

the behavior predicted by AAL and the ratio shows the same limiting values as expected by their theory.

There still, however, remains a problem with this interpretation. This theory predicts these effects in the limit of a long screening length, i.e., $2k_f/\kappa \gg 1$ (κ is the screening constant). In the other limit $2k_f/\kappa < 1$, the Hartree term is opposite in sign to the exchange term responsible for these corrections and exactly cancels it. This implies that in this limit, these logarithmic terms should not be observed. Our estimates indicate that for these devices we are in the range $2k_f/\kappa \sim 0.1-0.3$. In other words screening should be very effective. This agreement with the theory of AAL in a regime where the effect predicted by their theory should diminish is disturbing.

There are several possible explanations. One is that we simply do not understand screening in this limit. From previous measurements, it is established that exponential localization does indeed occur in the range $R_\square > 10$ kΩ. In that region we would expect the screening length to diverge in analogy to the situation in three dimensions,[11] because of a reduced ability of the electrons to respond to a point charge. The details of the nature of this divergence in two dimensions, the relationship of this to the Coulomb effects, and the localization transition have not yet been determined. Another possibility is that neither theory is complete and correct and that the agreement with respect to these ratios with the theory of AAL is fortuitous. A more complete theory would then be needed taking into account both interaction and localization effects in order to explain the observations of these logarithmic dependences.

In conclusion, we have measured the Hall effect in silicon inversion layers at low temperatures and low electric fields. In the regime where logarithmic behavior is seen in the conductance ($R_\square \lesssim 10$ kΩ) we have observed logarithmic dependences of the Hall resistance on temperature and Hall voltage. Our observations conflict with the expectations of the scaling theory of localization which in its present form predicts no logarithmic dependences. In the limits of $R_\square \to 0$ and $H \to 0$ we obtain the value for the ratio $(\delta R_H/R_H)/(\delta R_\square/R_\square) = 2$ predicted by AAL. However we estimate that we are in a regime of $2k_f/\kappa$ where the effect predicted by AAL should vanish. It is possible that in order to explain our observations a more complete theory is needed which includes both Coulomb interaction and localization effects. We conclude that the previously observed logarithmic dependences for the resistance of thin metal films and Si MOSFET's are not evidence for the scaling theory of localization in its present form.

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