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Glueball Spectrum in Extended Quantum Chromodynamics

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Extended quantum chromodynamics in which the magnetic structure of the color gauge symmetry plays the important role in the dynamics is proposed as a phenomenological theory of the strong interaction. In the one-loop approximation the masses of the scalar and the axial-vector magnetic glueballs are estimated to be around 2.2 and 1.5 GeV, and the leading linear trajectory of the 2^{++} electric glueball is estimated to be $\alpha_x(s) \simeq 0.48s\,(\mathrm{GeV}^{-2}) + 1.01$. The trajectory is proposed as the Pomeron.

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One of the most challenging problems in contemporary physics is to clarify the physical meaning of quantum chromodynamics (QCD), in particular to obtain the physical spectrum of the theory. To resolve this problem we have recently constructed *the extended gauge theory*, or *the gauge theory in the large*,[1] in which the magnetic structure of the underlying gauge symmetry plays the important role in the dynamics. Based on the group SU(2), the extended theory has been shown to exhibit a manifest confinement of the color. More importantly the theory could tell us how to construct the physical states, in particular the glueball states, and to calculate its mass spectrum. The purpose of this Letter is to discuss some characteristic features of the extended QCD based on the group SU(3), and to present its glueball spectrum. Because of the limitation of the space, the presentation in this Letter will be short and brief. The detailed discussions on the subject will be published separately.[2]

One of the virtues of the extended QCD is its manifest confinement of the color. The confinement is obtained through the dual Meissner effect, as has been speculated by Nambu[3] and Mandelstam,[4] among others. Specifically, the confinement mechanism may be summed up by the following three characteristic features[1,2]: First, it allows *color reflection invariance*, the 24-element subgroup of SU(3), as the residual symmetry which remains unbroken after the magnetic condensation of the vacuum and thus plays the crucial role for us to construct the actual physical states of the theory. Secondly, there must exist two *magnetic glueballs*, the scalar and the axial-vector collective modes of the vacuum, whose masses determine the scale at which the confinement sets in. Thirdly, any physical state made of the colored constituents should reproduce *the string picture*[5] of hadrons in the high-energy limit when one of the constituents is hit with a large momentum.

Another virtue of the extended theory is the factorization of the gauge potential into two parts,[1,2] the color-confining *binding gluons* and the gauge-covariant *valence gluons*. The factorization is achieved by the explicit introduction of the topological degrees of the gauge symmetry into the potential. As an immediate consequence one obtains *the electric glueballs*, the color-singlet bound states of the valence gluons, in the physical spectrum of the theory. Unlike the magnetic glueballs, however, the electric ones form a very rich spectrum which consists of families of linear trajectories. With all these attractive features, the real advantage of the extended theory is that it may allow us to calculate the mass spectrum of the theory. With some simplifying technical assumptions[1,2] the masses of the scalar and the axial-vector magnetic glueballs are estimated to be around 2.2 and 1.5 GeV, respectively, in the one-loop approximation. For the electric glueballs, the universal slope α_X' is estimated to be $\alpha_X' \simeq 0.54\alpha'$ ($\alpha' \simeq 0.9$ GeV^{-2} is the Regge slope of the $q\bar{q}$ meson trajectories). As for the ground states the mass of the 2^{++} ground state of the leading $^5S_2$ trajectory made of a valence gluon pair could be estimated to be around 1.4 GeV in a crude nonrelativistic approximation. Granting that, the leading 2^{++} trajectory $\alpha_X(s)$ of the gluon pair may be described by $\alpha_X(s) \simeq 0.48s(\mathrm{GeV}^{-2}) + 1.01$, which we propose as the Pomeron of the theory.

The extended QCD is different from the conventional one in that the gauge potential is chosen to include not only the local degrees but also the topological (i.e., the magnetic) degrees of the underlying gauge symmetry *explicitly* into the dynamics. The magnetic structure of the color SU(3) may be described by two internal Killing vectors[5] (of unit norm), a λ_3-like octet $\hat{m}$ and its symmetric product (the d product) $\hat{m}' = \sqrt{3}\,\hat{m} * \hat{m}$ which is λ_8-like. With the explicit introduction of $\hat{m}$ and $\hat{m}'$ into the potential one may write the extended potential $\vec{B}_\mu$ as[2]

$$\vec{B}_\mu = A_\mu \hat{m} + A_\mu' \hat{m}' - (1/g)\hat{m}\times\partial_\mu\hat{m} - (1/g)\hat{m}'\times\partial_\mu\hat{m}' + \vec{X}_\mu, \tag{1}$$

where $A_\mu = \hat{m}\cdot\vec{B}_\mu$ and $A_\mu' = \hat{m}'\cdot\vec{B}_\mu$ are the λ_3-like and the λ_8-like Abelian components of the potential. In the magnetic gauge where $\hat{m}$ and $\hat{m}'$ become the space-time independent $\hat{\xi}_3$ and $\hat{\xi}_8$, one obtains[2]

$$\vec{B}_\mu \to (A_\mu + \tilde{C}_\mu)\hat{\xi}_3 + (A_\mu' + \tilde{C}_\mu')\hat{\xi}_8 + X_\mu{}'\hat{\xi}_i \tag{2}$$

where ($i = 1, 2, 4, 5, 6,$ and 7), and where the singular "magnetic potentials" $\tilde{C}_\mu$ and $\tilde{C}_\mu'$ will now describe the magnetic fields of the colored monopoles,[5] i.e., the topological singularities of the Killing vectors $\hat{m}$ and $\hat{m}'$ which violate the Bianchi identity. From the potential (1) one finds

$$\vec{X}_\mu = (1/g)(\hat{m}\times D_\mu\hat{m} + \hat{m}'\times D_\mu\hat{m}'), \tag{3}$$

which shows that $\vec{X}_\mu$ *is the gauge-covariant piece of the potential*. For this reason we will call $\vec{X}_\mu$ *the valence gluons*. There are three of them (red, blue, and yellow) which may be written as

$$R_\mu = (X_\mu{}^1 + iX_\mu{}^2)/\sqrt{2}, \quad B_\mu = (X_\mu{}^6 + iX_\mu{}^7)/\sqrt{2}, \quad Y_\mu = (X_\mu{}^4 - iX_\mu{}^5)/\sqrt{2}. \tag{4}$$

The rest of the potential will be called *the binding gluons* since they provide, together with the condensed vacuum, the color confining force as we will see in a moment.

The Lagrangian for the extended QCD may be obtained from the conventional one by replacing its potential with the extended one (1). Introducing the monopole field operators[6] φ and φ' which create the topological singularities of $\hat{m}$ and $\hat{m}'$, and its *regular* magnetic potentials C_μ and C_μ', one may obtain the following Lagrangian[2] for the extended QCD in the magnetic gauge

$$\begin{aligned}\mathcal{L}_E = & -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{4}F_{\mu\nu}'^2 - \tfrac{1}{4}H_{\mu\nu}^{*2} - \tfrac{1}{4}H_{\mu\nu}'^{*2} + \bar{r}\gamma^\mu[i\partial_\mu + \tfrac{1}{2}g(A_\mu + \tilde{C}_\mu) + (g/2\sqrt{3})(A_\mu' + \tilde{C}_\mu')]r \\ & + \bar{b}\gamma^\mu[i\partial_\mu - \tfrac{1}{2}g(A_\mu + \tilde{C}_\mu) + (g/2\sqrt{3})(A_\mu' + \tilde{C}_\mu')]b + \bar{y}\gamma^\mu[i\partial_\mu - (g/\sqrt{3})(A_\mu' + \tilde{C}_\mu')]y \\ & - \tfrac{1}{2}|[\partial_\mu + ig(A_\mu + \tilde{C}_\mu)]R_\nu - [\partial_\nu + ig(A_\nu + \tilde{C}_\nu)]R_\mu|^2 \\ & - \tfrac{1}{2}|[\partial_\mu - i\tfrac{1}{2}g(A_\mu + \tilde{C}_\mu) + i(\sqrt{3}/2)g(A_\mu' + \tilde{C}_\mu')]B_\nu - [\partial_\nu - i\tfrac{1}{2}g(A_\nu + \tilde{C}_\nu) + i(\sqrt{3}/2)g(A_\nu' + \tilde{C}_\nu')]B_\mu|^2 \\ & - \tfrac{1}{2}|[\partial_\mu - i\tfrac{1}{2}g(A_\mu + \tilde{C}_\mu)] - i(\sqrt{3}/2)g(A_\mu' + \tilde{C}_\mu')]Y_\nu - [\partial_\nu - i\tfrac{1}{2}g(A_\nu + \tilde{C}_\nu) - i(\sqrt{3}/2)g(A_\nu' + \tilde{C}_\nu)]Y_\mu|^2 \\ & + |[\partial_\mu + i(4\pi/g)(\tilde{A}_\mu + C_\mu)]\varphi|^2 + |[\partial_\mu + i(4\pi/g)\sqrt{3}(\tilde{A}_\mu' + C_\mu')]\varphi'|^2 \\ & + \text{other interaction terms},\end{aligned} \tag{5}$$

where $F_{\mu\nu}, F_{\mu\nu}', H_{\mu\nu}^*, H_{\mu\nu}'^*$ are the Abelian field strengths corresponding to the potentials A_μ, A_μ', C_μ, C_μ', and $\tilde{A}_\mu$, $\tilde{A}_\mu'$, $\tilde{C}_\mu$, $\tilde{C}_\mu'$ are the singular "dual potentials" of the dual fields $F_{\mu\nu}^*$, $F_{\mu\nu}'^*$, $H_{\mu\nu}$, $H_{\mu\nu}'$, respectively,[7] and the quark triplet is denoted by r, b, and y. In the absence of any colored source the Lagrangian (5) is reduced to

$$\mathcal{L}(m) = -\tfrac{1}{4}H_{\mu\nu}^{*2} - \tfrac{1}{4}H_{\mu\nu}'^{*2} + |[\partial_\mu + i(4\pi/g)C_\mu]\varphi|^2 + |[\partial_\mu + i(4\pi/g)\sqrt{3}C_\mu']\varphi'|^2. \tag{6}$$

Then assuming that the Lagrangian is both ultraviolet finite and infrared unstable, one may obtain in the one-loop approximation the following Coleman-Weinberg-type effective potential[8]

$$V_{\text{eff}} = (24\pi^2/g^4)\{\varphi_0^4 + (\varphi^*\varphi)^2[2\ln(\varphi^*\varphi/\varphi_0^2) - 1]\} + (216\pi^2/g^4)\{\varphi_0'^4 + (\varphi'^*\varphi')^2[2\ln(\varphi'^*\varphi'/\varphi_0'^2) - 1]\}, \tag{7}$$

where φ_0 and φ_0' are the vacuum expectation values of the fields φ and φ'. This implies that the theory will create a dynamical condensation of the monopoles to cure its infrared divergences. Once the condensation is achieved the dual dynamics of the Lagrangian will ensure the confinement of any colored flux in a finite region of space.[1,2]

The condensation of the vacuum creates the magnetic glueballs, the massive collective modes of the vacuum. At first glance one would expect four such modes, two scalars μ, μ', and two axial vectors m, m'. The potential (7) fixes their mass ratios as ($\alpha_s = g^2/4\pi$)

$$\kappa^2 = \mu^2/m^2 = 3/2\pi\alpha_s, \quad \kappa'^2 = \mu'^2/m'^2 = 9/2\pi\alpha_s = 3\kappa^2. \tag{8}$$

On the other hand once the condensation is assured, one may take a more phenomenological point of view and try the familiar quartic potential[2]

$$V_{\text{eff}} = (48\pi^2/g^4)\lambda(\varphi^*\varphi - \varphi_0^2)^2 + (432\pi^2/g^4)\lambda'(\varphi'^*\varphi' - \varphi_0'^2)^2, \tag{9}$$

from which one obtains

$$\begin{aligned}\kappa^2 &= \frac{\mu^2}{m^2} = \frac{3}{2\pi\alpha_s}\lambda \\ \kappa'^2 &= \frac{\mu'^2}{m'^2} = \frac{9}{2\pi\alpha_s}\lambda' = 3\frac{\lambda'}{\lambda}\kappa^2.\end{aligned} \tag{10}$$

Since the mass ratio (8) may be reliable only in the strong-coupling limit,[2,8] in the following we will use the potential (9) in our calculation of the mass spectrum. Now the masses of the magnetic glueballs can be estimated by evaluating the string tension k_q of the $q\bar{q}$ mesons of the theory

$$k_q = \frac{1}{2\pi\alpha'} = \gamma_q\varphi_0^2 = \frac{\alpha_s}{8\pi}\gamma_q m^2, \tag{11}$$

where γ_q is a dimensionless parameter[2] which can be calculated from the string solution of the quark pair. Before one calculates the masses, however, it is important to realize that[2] within the framework of the extended QCD the color electric charge can uniquely be defined only up to the 24-element color-reflection degrees of free-

TABLE I. An estimate of the masses of the magnetic glueballs.

λ	$\bar{\mu}$ (GeV)	$\bar{m}$ (GeV)
1/4	1.20	1.75
1/2	1.69	1.62
1	2.17	1.52
2	2.90	1.41

TABLE II. An estimate of the slope α_X' and the intercept $\alpha_X(0)$ of the leading 2^{++} glueball trajectory.

λ	α_X' (GeV^{-2})	$\alpha_X(0)$
1/4	0.518	1.07
1/2	0.501	1.04
1	0.482	1.01
2	0.470	0.98

dom, even after one has chosen one's own magnetic gauge. The color reflection invariance which is obtained from the fact[1,2] that the non-Abelian magnetic charge is well defined only up to the Weyl reflection tells us that the two modes μ and μ' (and also m and m') should actually describe one and the same mode $\bar{\mu}$ (and $\bar{m}$, respectively). This reflection invariance may be incorporated by insisting[2] $\bar{\mu}=\mu=\mu'$ and $\bar{m}=m=m'$, or equivalently

$$\varphi_0'=(1/\sqrt{3})\,\varphi_0, \quad \lambda'=\tfrac{1}{3}\lambda. \tag{12}$$

With this one can calculate γ_q (and consequently $\bar{\mu}$ and $\bar{m}$) as a function of α_s for a fixed λ. To estimate the masses uniquely, of course, it is necessary to know the magnitude[9] of α_s. If we assume that α_s may be fixed by the running coupling $\bar{\alpha}$ at $s=\bar{\mu}\bar{m}$ we obtain[10] Table I for the masses $\bar{\mu}$ and $\bar{m}$.

The physical spectrum of the extended theory must also contain the electric glueballs, the color-singlet bound states of the valence gluons. There must exist (at least) three types of electric glueballs,[11] one made of the valence gluon pair and two others made of the symmetric product (the d product) as well as the antisymmetric product (the f product) of three valence gluons, each of which forms a family of linear trajectories of its own. All the three types, moreover, will have one and the same universal slope α_X'. The string tension k_X of these glueballs may be written as

$$k_X=\frac{1}{2\pi\alpha_X'}=\gamma_X\,\varphi_0^{\,2}=\frac{\alpha_s}{8\pi}\gamma_X m^2. \tag{13}$$

Then with the assumption (12) the ratio between the two string tensions $R=k_q/k_X$ may be calculated[2] as a function of α_s for a fixed λ, which is shown in Fig. 1. As for the ground states, the mass of the 2^{++} ground state of the ${}^5S_2$-glueball trajectory made of the gluon pair may be estimated to be 1.43 GeV (for $\lambda=1$) in a simple-minded nonrelativistic approximation.[1] If we take the naive approximation seriously, the leading 2^{++} glueball trajectory $\alpha_X(s)$ may be described by

$$\alpha_X(s)=\alpha_X's+\alpha_X(0), \tag{14}$$

where α_X' and $\alpha_X(0)$ are given by[2] Table II.

The result strongly suggests that the ${}^5S_2$ trajectory could be identified as the Pomeron trajectory of the theory. If so we may indeed have found a solid dynamical basis for the two-component duality[12] of strong interaction in the extended QCD.

Admittedly the extended QCD in its present form is far from complete. In fact it has several unsettled problems of its own.[1,2] Nonetheless the theory could be viewed a step forward in our effort to understand the dynamical basis of strong interaction.

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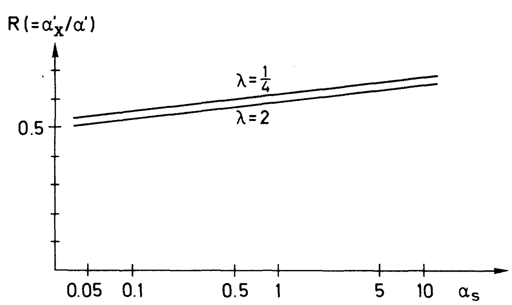

FIG. 1. Ratio of string tensions, $R=k_q/k_x$, for fixed λ as a function of α_s.

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