flow is mainly carried by a small number of electrons on the tail of the Maxwellian distribution. As pointed out by Gray and Kilkenny,[9] f_1/f_0 exceeds unity at velocities for which the energy flux is large when λ/L is of the order 0.01, and the linearized theory breaks down since f is then negative. Figure 3 plots the ratio f_1/f_0 as given by the simulation when $t=5260$ at the point immediately to the right of the heated region where the heat flow is largest. f_1/f_0 as given by the Spitzer-Harm theory is plotted for comparison and two major differences are apparent: (a) The simulation curve does not rise to values much larger than unity and (b) since f_1 is much lower at high velocities in the simulation, f_1 need not take such large negative values at low velocities to provide the return current. The limitation of f_1/f_0 to around unity reduces the large heat flow which peaks at velocities around $4(kT/m)^{1/2}$ in the Spitzer-Harm theory thus reducing the overall heat flow by an order of magnitude.

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Nonlinear Magnetic Islands and Anomalous Electron Thermal Conductivity

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It is found that nonlinear magnetic islands are formed, in an inhomogeneous magnetized plasma, in the presence of a longitudinal current. These magnetic islands may cause an enhanced electron thermal conductivity in a turbulent plasma state.

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One of the most central points in the understanding of transport processes in fusion devices is the anomalous electron thermal conductivity. Recently, it was suggested that the mechanism of this conductivity is the thermal motion of electrons along the magnetic field lines which are perturbed by a two-dimensional, extremely low-frequency electromagnetic mode.[1] Because this mode has a negligible electric field, it was called the magnetostatic mode.

The transport due to the magnetostatic mode in a homogeneous plasma was studied by several authors.[1-4] It was found that the anomalous electron diffusion is comparable to the classical in thermal equilibrium. In a turbulent plasma, however, the diffusion may exceed the classical by several orders of magnitude. In a nonuniform plasma it turns out that the magnetostatic mode obtains a real frequency which is of the order of the electron diamagnetic drift frequency ω_*.[5] As we will show, this real frequency strongly influences the diffusion. Following the procedure of Chu, Chu, and Ohkawa,[1] we can write the diffusion coefficient in the form

$$D=\frac{v_{\rm th}^2}{(2\pi)^2B_0^2}\int\langle B_k^2\rangle\frac{\gamma_k}{\omega_k^2+\gamma_k^2}d^3k\,, \qquad (1)$$

where $v_{\rm th}$ is the electron thermal velocity, B_0 is a uniform background magnetic field, B_k is the Fourier component of the transverse-magnetic-field perturbation, ω_k is the real part of the eigenfrequency and $1/\gamma_k$ is the decorrelation time. Clearly ω_k will strongly decrease the diffusion if it is larger than γ_k. For comparison we may consider two decorrelation mechanisms. First, for decorrelation due to electron-ion collisions we have[2]

$$\gamma_k = \nu_{ei}/(1+\omega_{pe}^2/k^2c^2), \tag{2}$$

where ν_{ei} is the electron-ion collision frequency, ω_{pe} is the electron plasma frequency, and c is the velocity of light. For fusion temperature we have $\nu_{ei} \lesssim \omega_*$. Since also $\omega_{pe}^2/k^2c^2 \gg 1$ we conclude that $\omega_k^2 \gg \gamma_k^2$. Second, when the decorrelation is due to anomalous diffusion, we have

$$\gamma_k = k^2 D. \tag{3}$$

Using the anomalous diffusion for a homogeneous plasma, we have[1]

$$D = (T/B_0)(2/m_e L_\parallel)\ln[(L\omega_{pe}/2\pi c)^{1/2}], \tag{4}$$

where m_e is the electron mass, T is the temperature in energy units, and $L_\parallel$ and L are the parallel and transverse scale lengths of the plasma. For fusion parameters it is easily verified that $\omega_* \gg k^2 D$.

Since the diffusion due to the linear mode is strongly reduced in an inhomogeneous plasma it is, for such a plasma, interesting to consider the diffusion due to the zero-frequency nonlinear mode.[6] In order to have a finite magnetic field associated with this mode (magnetic island), it turns out that we must include the background plasma current in our description. We thus write the parallel equation of motion of electrons for flute modes ($k_\parallel = 0$)

$$\frac{\partial v_\parallel}{\partial t} + (\vec{v}_\perp\cdot\nabla)v = -\frac{e}{m}E_\parallel - \frac{e}{mc}(\vec{v}_f\times\vec{B}_\perp)\cdot\hat{z}, \tag{5}$$

where $\vec{v}_\perp$ is the perpendicular guiding-center velocity of the electrons,

$$\vec{v}_f = \vec{v}_\perp - (c/en_eB_0)(\hat{z}\times\nabla P_e) \tag{6}$$

is the total fluid velocity, $E_\parallel$ is the parallel electric field, $\vec{B}_\perp$ is the perturbed magnetic field, and P_e is the electron pressure. We include a background current in (5) by writing

$$v_\parallel(x,y,t) = v_z(x,y,t) + v_0(x). \tag{7}$$

The parallel electric field is a pure induction field, i.e.,

$$E_\parallel = -c^{-1}\partial A/\partial t, \tag{8}$$

where A is the parallel component of the vector potential $\vec{A} = A(x,y,t)\hat{z}$. For the perpendicular electron velocity we have

$$\vec{v}_e = (c/B_0)(\hat{z}\times\nabla\varphi) + (\vec{B}_\perp/B_0)v_\parallel, \tag{9}$$

where $-\nabla\varphi$ is the perpendicular electric field. In order to determine this field we use the equation

$$\nabla\cdot\vec{j} = e\nabla\cdot(n_i\vec{v}_{\perp i} - n_e\vec{v}_{\perp e}) = 0, \tag{10}$$

where

$$\vec{v}_{\perp i} = \frac{c}{B_0}\hat{z}\times\nabla\varphi - \frac{c}{B_0\Omega_{ci}}\left(\frac{\partial}{\partial t} + v_{0y}\frac{\partial}{\partial y}\right)\nabla\varphi \tag{11}$$

and v_{0y} is the zeroth-order ion-fluid drift. In order to close this system of equations we now need the electron continuity equation with the electron velocity given by (9), the quasineutrality condition and the relations

$$\vec{B}_\perp = \nabla A\times\hat{z} \tag{12}$$

and

$$j_\parallel = -(c/4\pi)\Delta A, \tag{13}$$

where we consider the parallel current $j_\parallel$ to be due to electrons only.

In order to find the excitation level of the zero-frequency nonlinear mode we apply the reductive perturbation method[7] to the system (5)–(13), using the ordering relations[8]

$$\varphi(x,y,t) = \varphi^{(0)}(x) + \sum_{l,\alpha}\epsilon^\alpha\varphi_l^{(\alpha)}(x,\xi,\tau)\exp[il(k_y y - \omega t)] \tag{14a}$$

$$\begin{pmatrix} A \\ n \end{pmatrix} = \sum_\alpha \epsilon^\alpha \begin{pmatrix} A \\ n \end{pmatrix}_l^{(\alpha)}(x,\xi,\tau)\exp[il(k_y y - \omega t)], \tag{14b}$$

where

$$\xi = \epsilon(y - \lambda t), \quad \tau = \epsilon^2 t, \quad x = x,$$

and ϵ is a small parameter. In (14a) we also introduced a background potential $\varphi^{(0)}(x)$ which we will assume to have a linear x dependence. In the x direction we introduce periodic boundary conditions with period L. To first order of ϵ we obtain an eigenvalue equation in x giving standing-wave solutions of the form $\sin k_m x$ where $k_m = 2m\pi/L$. Corresponding to this solution we have the linear dispersion relation

$$\omega_1 = D\left[\omega_* + k_y v_0\left(\frac{(\kappa+2\eta)v_0}{\Omega_{ce}} - \frac{m_e}{m_i}\frac{k_y v_0(\kappa+\eta)^2}{k^2\omega_2}\right)\right], \tag{15}$$

where $\omega_1 = \omega - (k_y c/B_0)d\varphi/dx^{(0)}$, $D=(1+k^2c^2/\omega_{pe}^2)^{-1}$, $k^2 = k_y^2 + k_m^2$, $\omega_2 = \omega_1 - k_y v_{0y}$, $\Omega_{ce} = eB_0/m_e c$, $\eta = -d\ln v_0/dx$, $\kappa = -d\ln n_0/dx$, and n_0 is the background density. In obtaining (15) we used the relation

$$\varphi_1^{(1)} = [k_y v_0(\kappa+\eta)\Omega_{ci}/k^2\omega_2]A_1^{(1)}. \tag{16}$$

We notice that the electric field is introduced by the background current velocity v_0. The dispersion relation (15) contains the possibility for a linear instability, i.e., excitation of the linear mode by the current. For typical fusion plasma parameters, however, the mode is stable. We also find from (16) that for such parameters we have $|\varphi_1^{(1)}| \ll |A_1^{(1)}|$.

To the second order in ϵ we obtain the condition $\lambda = \partial\omega/\partial k_y$.

To third order we obtain a nonlinear Schrödinger equation (NSE) in ξ and τ for $A_1^{(1)}$ and a zero-frequency component $A_0^{(2)}$. In order to simplify the coefficients of the NSE we make use of the facts that $k^2c^2 \ll \omega_{pe}^2$ and $\partial\omega/\partial k_y \approx \omega/k_y$. Several possibilities for modulational instability exist. As an example, if the condition

$$\frac{\omega_1\omega_2}{k_y^2} > \frac{1}{3}\frac{(\kappa+\eta)^2}{k^2}\frac{m_e}{m_i}\frac{\omega_{pe}^2}{k^2c^2}v_0^2$$

is fulfilled, we have modulational instability when $k_y^2 > 3k_m^2$. Because of this instability we obtain a zero-frequency mode which is localized in the y direction. Localization in the x direction is due to the imposed boundary conditions. The zero-frequency mode is driven by the ponderomotive force and can be expressed as

$$A_0^{(2)} = 8\frac{k_y^3 v_0(\kappa+\eta)\Omega_{ci}}{B_0 K}\frac{k_m^2\lambda_2}{k^2\omega_2}\left(1 - k_y\kappa\frac{v_0^2+v_{te}^2}{\omega_1\Omega_{ce}}\right)\frac{\partial}{\partial x}|A_1^{(1)}|^2, \tag{17a}$$

where $\lambda_2 = \partial\omega_2/\partial k_y$ and

$$K = 12\frac{k_m^2k^2c^2}{k_y^2\omega_{pe}^2}\omega_1\omega_2(k_m^2 - k_y^2) - \frac{m_e}{m_i}\frac{(\kappa+\eta)^2}{k^2}v_0^2[4k_m^2(k_m^2-k_y^2) - k^4]. \tag{17b}$$

We note that K has a zero close to $k_y = k_m$. This corresponds to a resonance where our assumption $A_0^{(2)} \sim \epsilon A_1^{(1)}$ breaks down and we should use another ordering relation where $A_0 \sim A_1^{(1)}$.

In the region $k_y^2 > 3k_m^2$, the first term in (17b) dominates. From (17a) we can see that $A_0^{(2)}$ is proportional to the background current velocity v_0. This means that the background current is necessary for the formation of the nonlinear magnetostatic mode which in the following will be referred to as a nonlinear magnetic island.

Let us now consider the diffusion due to the nonlinear magnetic islands. For this purpose we use Eq. (1), where we put $\omega_k = 0$. Using (12) and keeping only the first part of (17b), we obtain, from (1),

$$D = \frac{4v_{\rm th}^2(\kappa+\eta)^2v_0^2\Omega_{ci}^2\omega_{pe}^4a^2}{9(2\pi)^2B_0^4v_{De}^4c^4}\int\frac{1}{\gamma_k}\frac{k_y^4k_m^2}{k^6(k_m^2-k_y^2)^2}|A_1^{(1)}|^4L^2\,d^3k, \tag{18}$$

where $a = 1 - \kappa(v_0^2 + v_{te}^2)/v_{De}\Omega_{ce}$ and $v_{De} = \kappa T_e/eB_0$. In order to obtain the diffusion coefficient from D we need to know the spectrum of the magnetic fluctuations. Introducing an effective temperature $T_{\rm eff}$ corresponding to a turbulent level of excitation into the thermal equilibrium spectrum,[1] we have

$$\left|\frac{eA_1^{(1)}}{T}\right|^2 = \frac{4\pi e^2T_{\rm eff}}{L_\parallel L^2T^2k^2}\frac{\omega_p^2}{\omega_p^2+k^2c^2}. \tag{19}$$

For a low level of excitation, i.e., $T_{\rm eff} \gtrsim T_1$, the appropriate decorrelation time γ^{-1} is given by (2). We then obtain the diffusion coefficient

$$D \approx a^2\left(\frac{T_{\rm eff}}{T}\right)^2 D_{\rm B}\left(\frac{v_{\rm th}v_0}{c^2}\right)^2\left(\frac{\omega_{pe}}{k_0c}\right)^4\frac{v_{\rm th}^2}{(k_0\lambda_{\rm D})^4v_{De}^2}\frac{(\kappa+\eta)^2}{\kappa^2(L_\parallel L)^2}\frac{e}{n_0c}\frac{\Omega_{ci}^2}{B_0\nu_{ei}}, \tag{20}$$

where k_0 is the lower limit of the integration in (18) and D_B is the Bohm diffusion coefficient. In obtaining (20) we estimated (18) by putting $k_y^2 = 4k_m^2$ and by using the surface element $d^2k = 2\pi k \times dk$, thus avoiding the singularity. For a higher level of excitation, i.e., $T_{eff} \gg T$, we should use γ given by (3). We then have

$$D \approx \frac{a}{4\pi}\frac{T_{eff}}{T}\frac{v_0\Omega_{ci}}{\kappa^2c^2}\frac{\omega_{pe}^2}{k_0^2d^2}\frac{v_{th}(\kappa+\eta)}{n_0LL_{\parallel}k_0^2\lambda_{De}^2}. \qquad (21)$$

For typical parameter values, (21) can be estimated as

$$D \sim 10^{-8}(T_{eff}/T)k_0^{-4}. \qquad (22)$$

At a turbulence level of $B_\perp/B_0 \sim 10^{-4}$ and with k_0 determined by the system size, the diffusion given by (22) and the corresponding thermal conductivity κ_t given by[9]

$$\kappa_t = nD$$

are several orders of magnitude above the classical.

We note that the description in terms of an effective temperature does not take into account the details of the k spectrum. To the authors knowledge, however, no calculation or measurement of the electromagnetic turbulent spectrum in fusion devices yet exists. We point out also that if k_0 is determined by some mechanism other than the limitation due to the system size, magnetic shear, for example, this may change the scaling properties of the diffusion coefficient. Another mechanism that may interact with the studied diffusion is the convective cell diffusion (2)–(4). Because of this diffusion, γ_k is modified.

Finally, we stress that the magnetic islands give rise to only electron diffusion. Since the particle diffusion must be ambipolar, the main effect of the nonlinear magnetic islands will be to increase the electron thermal conductivity in a turbulent plasma state.

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