cleus are arbitrary, the fact that the high-energy portions of both neutron and proton forward-angle spectra are fitted simultaneously by calculations with use of the *same* normalization suggests that the single nucleon-nucleon scattering mechanism accounts for most of the continuum spectrum above the evaporation peaks. The systematic variation of the ratios of neutron to proton yields with N and Z in approximate agreement with Eq. (1) provides additional evidence of the importance of the single nucleon-nucleon scattering mechanism. The combination of these results and the recent particle-particle coincidence studies of reactions induced by 100-MeV protons[5] demonstrate that the interaction of medium-energy protons with nuclei is dominated by nucleon-nucleon interactions.

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Resonance Structure of the Antisymmetrized Optical Potential

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An extension of the unsymmetrized optical-potential formalism for two-fragment elastic scattering is found which fully incorporates the Pauli principle without the use of complicated projection operators. The discrete singularities of this optical potential are shown to be correlated with the physical resonance structure of the scattering amplitude.

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A new approach to the problem of incorporating the Pauli principle (PP) into the microscopic theory of the optical potential (OP) for elastic nuclear scattering has appeared recently.[1-5] The development of Refs. 1–5 differs in several important ways from previous studies of this question.[6,7] Of particular note is the fact that projection operators which do *not* map the space, $\mathcal{H}_\Lambda$, of fully antisymmetrized states into itself are employed in Refs. 1–5. Although these projectors are simpler to work with than those defined on $\mathcal{H}_\Lambda$, it is then possible that the identification and physical interpretation of the discrete poles in the OP (as a function of the total energy) might be obscured. In the case of distinguishable particles, such poles of the OP are unambiguously correlated with the resonance structure of the scattering amplitude.[8] In this Letter we establish that this correlation also holds for the OP defined as in Ref. 1. Our major results consist in what appears to be a direct and physically transparent generalization of the *original* Feshback formalism[8] to include the PP which retains the same simplicity and practical applicability characteristic of Ref. 8.

In order to maintain a two-body description of the elastic two-fragment scattering, the antisymmetrized transition and OP operators, $T(\hat{\beta})$ and $U(\hat{\beta})$, respectively, are related by[1]

$$U(\hat{\beta}) = T(\hat{\beta}) - U(\hat{\beta}) P_\beta G_\beta T(\hat{\beta}), \quad (1a)$$

$$= T(\hat{\beta}) - T(\hat{\beta}) P_\beta G_\beta U(\hat{\beta}), \quad (1b)$$

where (1b) follows from (1a) if the solution of (1a) is unique. Here β refers to an arbitrary (but fixed) choice of the assignment of identical nucleons to the two fragments. The set of such physically equivalent choices (partitions), connected by permutations of identical nucleons, is denoted by $\hat{\beta}$. The β-channel Green's function is $G_\beta = (E - H_\beta + i0)^{-1}$ while P_β is the projector on the space spanned by the (relative motion) eigenstates $|\varphi_\beta(\vec{k})\rangle$ of the channel Hamiltonian H_β. We assume that each of the two fragments represented by $|\varphi_\beta(\vec{k})\rangle$ is in its ground state and that $\overline{\mathfrak{A}}(\beta) \times |\varphi_\beta(\vec{k})\rangle = |\varphi_\beta(\vec{k})\rangle$, where $\overline{\mathfrak{A}}(\beta)$ is the antisymmetrizer (internal to the fragments) with respect to all permutations which map β into itself. Thus, although the cluster states are individually antisymmetrized, P_β does not map $\mathcal{H}_\Lambda$ into itself. The essential features of the OP are independent of the choice of β.[1,4,9]

The operator $U(\hat{\beta})$ defined by (1) depends upon the choice of off-shell extension of the unsymmetrized transition operators which enter into $T(\hat{\beta})$. We confine ourselves here to a form[10] of these operators which yields a $U(\hat{\beta})$ free of all $\hat{\beta}$-class elastic unitarity cuts.[1,2,11] It is easy to show that, in this case,[5,12]

$$T(\hat{\beta}) = V^\beta \mathfrak{A}(\hat{\beta}) G G_\beta^{-1} + \hat{\mathfrak{A}}(\hat{\beta}) G_\beta^{-1}. \tag{2}$$

The distinct identities of $\mathfrak{A}(\hat{\beta})$, $\hat{\mathfrak{A}}(\hat{\beta})$, and $\overline{\mathfrak{A}}(\beta)$ should be kept in mind in what follows.[12] We remark that $\mathfrak{A}(\hat{\beta})$ is proportional to the projector onto $\mathcal{H}_\Lambda$, so that $[H, \mathfrak{A}(\hat{\beta})] = 0$ implies

$$[\mathfrak{A}(\hat{\beta}), G_\beta^{-1}] = [\mathfrak{A}(\hat{\beta}), V^\beta], \tag{3}$$

which is useful in our subsequent analysis as well as in removing the apparent asymmetry in (2).

If we write $Q_\beta = 1 - P_\beta$, we find from (1), with the aid of (2), (3), and the resolvent identities which relate $G(z)$ and $G_\beta(z)$ for a complex parametric energy z, the Lippmann-Schwinger-type equations[13]

$$U(\hat{\beta}) = V_e^{\beta}(z) + (1+\mathfrak{N})^{-1}\hat{\mathfrak{A}}(\hat{\beta})Q_\beta G_\beta^{-1}(z) + V_e^{\beta}(z) Q_\beta G_\beta(z) U(\hat{\beta}), \tag{4a}$$

$$U(\hat{\beta}) = V_e^{\beta\dagger}(z^*) + G_\beta^{-1}(z) Q_\beta \hat{\mathfrak{A}}(\hat{\beta})(1+\mathfrak{N}^\dagger)^{-1} + U(\hat{\beta}) Q_\beta G_\beta(z) V_e^{\beta\dagger}(z^*), \tag{4b}$$

which involve the z-dependent effective interaction

$$V_e^{\beta}(z) = (1+\mathfrak{N})^{-1}[V^\beta + \mathfrak{N} G_\beta^{-1}(z)], \tag{5}$$

where $\mathfrak{N} \equiv \hat{\mathfrak{A}}(\hat{\beta}) P_\beta$.[14] The inverse, $(1+\mathfrak{N})^{-1}$, exists if P_β is constructed of correlated wave functions; the details of the proof of this assertion will be published elsewhere[15] (see also Ref. 6). We remark that the calculation of $(1+\mathfrak{N})^{-1}$ is relatively straightforward,[16] so that $V_e^{\beta}(z)$ is a comparatively uncomplicated object.

For elastic scattering we require only the P_β-projected OP, namely $\mathcal{V}_{\text{opt}} \equiv P_\beta U(\hat{\beta}) P_\beta$. Upon solving Eqs. (4), expanding the denominator, and projecting, one obtains either the closed-form expression

$$\mathcal{V}_{\text{opt}}(z) = P_\beta V_e^{\beta}(z) P_\beta + P_\beta V_e^{\beta}(z) Q_\beta [G_\beta^{-1}(z) - Q_\beta V_e^{\beta}(z) Q_\beta]^{-1} Q_\beta V_e^{\beta}(z) P_\beta, \tag{6}$$

or the alternative form for $\mathcal{V}_{\text{opt}}(z)$ given by (6) but with $V_e^{\beta}(z)$ replaced by $V_e^{\beta\dagger}(z^*)$. From this it follows that $\mathcal{V}_{\text{opt}}^\dagger(z) = \mathcal{V}_{\text{opt}}(z^*)$.[17] Equation (6) (or its alternative form) bears a striking resemblance to the original Feshbach[8] form of the OP, which follows from the neglect of nucleon identity ($\mathfrak{N} = 0$).

Equation (6) contains only partially antisymmetrized projectors, P_β and Q_β, which refer to a particular partition β. This simplifies some matters compared to the treatment of Ref. 6 but it also appears to introduce a new problem. In particular, we expect[1,2] $\mathcal{V}_{\text{opt}}(z)$ to be free of *all* $\hat{\beta}$-class elastic unitarity cuts but since, e.g., $P_\beta Q_{\beta'} \neq 0$ for $\beta \neq \beta' \in \hat{\beta}$, it is not manifest that (6) possesses this property. Evidently, however, the right-hand side of (6) has no elastic unitarity cut in the β channel. Then, because the matrix elements $\langle \varphi_\beta(\vec{k}')|\mathcal{V}_{\text{opt}}|\varphi_\beta(\vec{k})\rangle$ are independent of the choice of β,[1,4] it follows that these momentum-space elements of the OP possess no $\hat{\beta}$-class elastic unitarity cuts.[15] This result holds for any truncation which preserves the label-transforming character of Q_β.[4,15]

The standard structural analysis of the OP employs a spectral decomposition of the Green's-function term in (6).[6,8] However, in contrast to the $\mathfrak{N} = 0$ case, the effective Hamiltonian, $H_e \equiv H_\beta + Q_\beta V_e^{\beta} Q_\beta$, which enters into that Green's function in (6), is not Hermitian. Thus in order to effect such a decomposition one must introduce biorthogonal states.[6] In particular, from (6) we see that $\mathcal{V}_{\text{opt}}$ has discrete pole singularities for real energies when there are nontrivial, normalizable solutions, $Q_\beta|\chi_R\rangle$ and $Q_\beta|\chi_R^\dagger\rangle$, of the biorthogonal partner *resonance equations*[14]

$$(E_R - H_e)Q_\beta|\chi_R\rangle = 0, \quad (7a)$$

$$(E_R - H_e^\dagger)Q_\beta|\chi_R^\dagger\rangle = 0, \quad (7b)$$

at the energy E_R, which we suppose to be nondegenerate. We then introduce a biorthogonal representation of Q_β:

$$Q_\beta = \mathbf{Q}_\beta^{(0)} + \sum_R Q_R, \quad (8)$$

where $Q_R = Q_\beta|\chi_R\rangle\langle\chi_R^\dagger|Q_\beta$, $Q_R Q_{R'} = Q_R\delta_{R,R'}$, and $\mathbf{Q}_\beta^{(0)}Q_R = Q_R Q_\beta^{(0)} = 0$.

If we employ (8) in (6) for $z = E + i0$, we obtain[14]

$$\mathcal{V}_{\text{opt}}(E) = \mathcal{V}_{\text{opt}}^{(0)}(E) + \sum_R \frac{P_\beta V_e^\beta Q_\beta|\chi_R\rangle\langle\chi_R^\dagger|Q_\beta V_e^\beta(E)P_\beta}{E - E_R}. \quad (9)$$

The quantity $\mathcal{V}_{\text{opt}}^{(0)}(E)$ is given by (6) but with Q_β replaced by $Q_\beta^{(0)}$ throughout. The analysis of Ref. 8 when applied to (9) implies that, for E above the threshold for elastic scattering, the poles of $\mathcal{V}_{\text{opt}}(E)$ at $E = E_R$ correspond to physical resonances only if their residues are positive semidefinite and therefore Hermitian operators. This condition is needed to define nonnegative resonance widths. Since the numerators of the poles in (9) are *not* Hermitian it is possible that the poles of $\mathcal{V}_{\text{opt}}(E)$ do not necessarily represent resonances. We next show that this possibility is never realized.

The identity (3) can be rewritten as[14]

$$V_e^\beta(z) + (1+\mathfrak{N})^{-1}\hat{\mathfrak{a}}(\hat{\beta})Q_\beta[G_\beta^{-1}(z) - V_e^{\beta\dagger}] = V_e^{\beta\dagger}(z^*) + [G_\beta^{-1}(z) - V_e^\beta]Q_\beta\mathfrak{a}(\hat{\beta})(1+\mathfrak{N}^\dagger)^{-1}. \quad (10)$$

If we multiply (10) on both sides by Q_β we obtain

$$M(z)(z - H_e^\dagger) = (z - H_e)M^\dagger(z^*), \quad (11)$$

where $M(z) = Q_\beta(1+\mathfrak{N})^{-1}[V^\beta G_\beta + \hat{\mathfrak{a}}(\hat{\beta})]Q_\beta$. Then we infer from (7) and (11) that $Q_\beta|\chi_R\rangle$ is proportional to $|\mathfrak{M}_R\rangle \equiv Q_\beta\mathfrak{M}^\dagger(E_R)Q_\beta|\chi_R^\dagger\rangle$, unless $|\mathfrak{M}_R\rangle = 0$, viz.,

$$Q_\beta|\chi_R\rangle = C_R Q_\beta\mathfrak{a}(\hat{\beta})(1+\mathfrak{N}^\dagger)^{-1}Q_\beta|\chi_R^\dagger\rangle. \quad (12)$$

An expression for C_R follows by performing the inner product of (12) with $\langle\chi_R^\dagger|Q_\beta(1+\mathfrak{N})^{-1}(1+\mathfrak{N})$. Then, if we use the fact that $\mathfrak{a}(\hat{\beta})$ is proportional to the projection on $\mathcal{K}_\Lambda$, and the identity (i) $P_\beta\mathfrak{a}(\hat{\beta})(1+\mathfrak{N}^\dagger)^{-1} = P_\beta$, the normalization condition $\langle\chi_R^\dagger|Q_\beta|\chi_R\rangle = 1$ implies that $C_R \geq 0$.

Let us suppose that $|\mathfrak{M}_R\rangle \neq 0$ so that we can use (12) to simplify the numerators of the pole terms in (9). One finds with the aid of (10) that

$$\mathcal{V}_{\text{opt}}(E) = \mathcal{V}_{\text{opt}}^B(E) + \sum_R [C_R N_R(E_R)/(E - E_R)], \quad (13)$$

which is our central result. The background potential is Hermitian analytic,[17] contains no resonance poles, and is given by

$$\mathcal{V}_{\text{opt}}^B(E) = \mathcal{V}_{\text{opt}}^{(0)}(E) + \sum_R C_R\{[N_R(E) - N_R(E_R)]/(E - E_R) - P_\beta(1+\mathfrak{N})^{-1}\hat{\mathfrak{a}}(\hat{\beta})Q_\beta|\chi_R^\dagger\rangle\langle\chi_R^\dagger|Q_\beta V_e^\beta(E)P_\beta\}. \quad (14)$$

The operator $N_R(z)$, defined in (15), is positive semidefinite for real z:

$$N_R(z) \equiv P_\beta V_e^{\beta\dagger}(z^*)Q_\beta|\chi_R^\dagger\rangle\langle\chi_R^\dagger|Q_\beta V_e^\beta(z)P_\beta. \quad (15)$$

Thus the diagonal matrix elements of the residues, $C_R N_R(E_R)$, of the poles of $\mathcal{V}_{\text{opt}}(E)$ are nonnegative. It follows then that the expression (13) for the OP embodies all of the mathematical properties and hence the *same physical interpretation* as the original Feshbach representation[8] for the OP without the PP. We have therefore obtained what appears to be a consistent and simple generalization of the formalism of Ref. 8 to include the PP, provided that (12) holds.

We next investigate the case for which (12) breaks down. This occurs if and only if $|\mathfrak{M}_R\rangle = 0$, which is equivalent to the condition that there is a vector $|p_R\rangle = P_\beta|p_R\rangle$ such that

$$\mathfrak{a}(\hat{\beta})[Q_\beta|\chi_R^\dagger\rangle + |p_R\rangle] = 0, \quad (16)$$

where $|p_R\rangle \equiv 0$ is permitted. Equation (16) follows from (12) with the aid of the identity $Q_\beta[\mathfrak{A}(\hat{\beta})(1+\mathfrak{N}^\dagger)^{-1}-(1+\mathfrak{N})^{-1}\mathfrak{A}(\hat{\beta})]Q_\beta=0$. We use (16) to explicate the E dependence in the parts $\langle B_R(E)| \equiv \langle \chi_R{}^\dagger|Q_\beta V_e{}^\beta(E)P_\beta$ of the numerators of the pole terms in (9). We infer from (16) and identity (i) that

$$\langle\chi_R{}^\dagger|Q_\beta(1+\mathfrak{N})^{-1}=\langle\chi_R{}^\dagger|Q_\beta+\langle p_R|. \qquad (17)$$

Equation (17) when combined with $\langle B_R(E)|$, (3), and (7b) yields

$$\langle B_R(E)| =\langle\chi_R{}^\dagger|Q_\beta V^\beta P_\beta-\langle p_R|G^{-1}(E)P_\beta, \qquad (18)$$

$$\langle\chi_R{}^\dagger|Q_\beta G_\beta{}^{-1}(E)\mathfrak{A}(\hat{\beta}) =\langle\chi_R{}^\dagger|Q_\beta V^\beta\mathfrak{A}(\hat{\beta})-\langle p_R|\mathfrak{A}(\hat{\beta})G^{-1}(E), \qquad (19)$$

$$\langle\chi_R{}^\dagger|Q_\beta G_\beta{}^{-1}(E) =\langle\chi_R{}^\dagger|Q_\beta[V^\beta Q_\beta+(E-E_R)]+\langle p_R|V^\beta Q_\beta, \qquad (20)$$

respectively. The combination of (19), (20), and identity (i) yields

$$\langle\chi_R{}^\dagger|Q_\beta V^\beta P_\beta =\langle p_R|G^{-1}(E)P_\beta-\langle p_R|(E-E_R). \qquad (21)$$

Equations (18) and (21) imply our basic consequence of (16):

$$\langle B_R(E)| =-\langle p_R|(E-E_R). \qquad (22)$$

This demonstrates that *the residues of the poles of (9) vanish when (12) does not hold, so that (13) is generally valid and its resonance interpretation is unambiguous.*

The case when $|p_R\rangle=0$ is of special interest because (16) then implies that $Q_\beta|\chi_R{}^\dagger\rangle$ is an isolated Pauli-forbidden state, and we see from (22) that *there is no contribution from this state to* $\mathfrak{V}_{\rm opt}$ *at any E.* Our analysis of (16) did not depend upon the normalizability of $Q_\beta|\chi_R{}^\dagger\rangle$ so that our argument applies equally well to continuum solutions of (7b). We then conclude from (6) and (22) that *there is no contribution to the spectral representation of the Green's function for* $\mathfrak{V}_{\rm opt}$ *arising solely from a Pauli-forbidden resonance or continuum state.*

The results of this Letter extend previous work[6,18] on the discrete structure in elastic scattering in providing an *exact* characterization of the resonances for the scattering of two *arbitrary* complex fragments composed of identical fermions. This is formulated with incorporation of the scattering boundary conditions within a practical definition of the optical potential which involves relatively simple projection operators. As a consequence, we anticipate new applications in the study of compound and intermediate resonance structure in nuclear scattering[18] as well as in atomic scattering[19] problems.

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