stant. At the lower energy the results are compatible with the Faddeev calculation, but also with a mixing parameter $\epsilon = 0$. However, at the higher energy, none of the conventional theories[8,9,16] agrees even qualitatively with the structure of the data. The oscillatory behavior is surprisingly well reproduced by the predictions of Ref. 13 in which effects of dibaryon resonances are explicitly included (the experiment favors $l_\pi = 4$). We consider this as a strong indication for the presence of at least one dibaryon resonance in the π-d channel with a strength compatible with the parameters of Ref. 13. Independent of the details of the model the very observation of strong oscillations is a direct indication of a strong contribution from a higher partial wave interfering with the background.

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Correction to Z_P/Z_T Expansions for Electron Capture

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For highly asymmetric electron-capture collisions, the ratio of the small charge Z_P, here taken to be that of the projectile, to the large charge of the target Z_T forms the natural expansion parameter. Previous treatments with use of Z_P/Z_T expansions obtain the impulse approximation which is shown to err by unknowingly neglecting terms of order $(Z_T/v)^2$; a simple correction factor in the limit of small Z_P/v is derived.

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Electron capture in ion-atom collisions over a broad range of energies has proven to be difficult to treat theoretically. Fragmentary results, mainly at low velocities in nearly symmetric collisions where a molecular representation provides an adequate framework[1] and at asymptotically high velocities where the Thomas double-collision mechanism dominates,[2] exist but no comprehensive picture has emerged. Much recent research[3,4] attempts to define closely the importance of second Born terms, which describe the Thomas mechanism but may have broader significance. For highly asymmetric collisions, such as charge transfer from inner shells of atoms of nuclear charge Z_T to incident bare ions of charge Z_P, for which $Z_T \gg Z_P$, the theoretical description simplifies; only the dominant terms in an expansion in powers of Z_P/Z_T need be retained. This leads to the impulse approximation[5,6] which correctly reproduces the high-energy behavior, but fails at low and not-so-low energies, for example, for 2-MeV protons impinging on argon, even though the usual derivations suggest that the theory is correct to order $(Z_P/v)^2 \sim 0.02$ and hence is applicable in this region. Here v is measured in atomic units.

We have critically examined the impulse approximation to identify the order of the errors. We find that the approximation is fundamentally incorrect in one crucial step and errors of the order of $(Z_T/v)^2$ are introduced. Such errors are large compared to $(Z_P/v)^2$ when $Z_T \gg Z_P$; for example, $(Z_T/v)^2 = 4$ for 2-MeV protons impinging on argon. A simple correction factor, derived in this Letter, corrects the deficiency so that remaining errors are of order $(Z_P/v)^2$. With this factor, we obtain agreement with measurements in argon over a broad range of energies from below the cross-section peak at 4 MeV to the asymptotic region. This represents the first indication of a theoretical framework applicable over the full energy range for highly asymmetric collisions and realizes the previously unattained goal of Z_P/Z_T expansions.

Our starting point is the second Born approximation with the Coulomb, rather than the free-particle, Green's function to incorporate the strong-interaction Z_T/r_T, where r_T is the electron–target-nucleus distance, to all orders. Our model therefore considers a bare charged particle of charge Z_P incident on a one-electron ion with nuclear charge Z_T where $Z_T \gg Z_P$. The nucleus-nucleus potential is neglected in accord with the arguments of Wick.[7] Results of the calculations relate to experiments with multielectron targets via the independent-particle model. Atomic units are used throughout.

Let $\varphi_i(\vec{r}_T)$ represent the initial internal state of the system comprising the electron and the target nucleus $(e+T)$ with binding energy ϵ_i; $\varphi_f(\vec{r}_P)$, the final internal state of the system comprising the electron and the projectile $(e+P)$ with binding energy ϵ_f; and $\vec{K}_i$ and $\vec{K}_f$, the initial and final momentum of P and $(e+P)$, respectively. Then the perturbation expansion for the scattering amplitude to second order in the weak potential V_{Pe} is $A = A_1 + A_2$, where

$$A_1 = \langle \psi_f | V_{Pe} | \psi_i \rangle = \langle \psi_f | V_{Te} | \psi_i \rangle \tag{1}$$

and

$$A_2 \approx A_{2c} = \langle \psi_f | V_{Te} G_c^{+} V_{Pe} | \psi_i \rangle . \tag{2}$$

The initial and final functions are

$$\psi_i = \exp(i\vec{K}_i \cdot \vec{R}_T)\varphi_i(\vec{r}_T), \quad \psi_f = \exp(i\vec{K}_f \cdot \vec{R}_P)\varphi_f(\vec{r}_P), \tag{3}$$

where $\vec{R}_T$ is the coordinate of the projectile P relative to the target $(e+T)$ and $\vec{R}_P$ is the coordinate of $(e+P)$ relative to T.

The Green's function is approximated by

$$G^{+} \approx G_c^{+} = \frac{1}{E + i\eta - H_0 - V_{Te}}, \tag{4}$$

which includes the strong potential V_{Te} to all orders.

The impulse approximation is obtained by inserting a unit operator $\int\int |\vec{k},\vec{K}\rangle d^3k\, d^3K \langle \vec{k},\vec{K}|$ between ψ_f and V_{Te} in Eq. (2), integrating over d^3K and approximating the off-energy-shell Coulomb wave function

$$\psi_{\vec{k},\epsilon}^{(+)} = |\vec{k}\rangle + \frac{1}{H_T - \epsilon - i\eta} V_{Te} |\vec{k}\rangle \tag{5}$$

by its on-energy-shell version $\psi_{\vec{k}}^{(+)}$ with $\epsilon = \frac{1}{2}k^2$. Since ϵ is of the order of $v^2/2$ and $\epsilon - k^2/2$ is of the order of $Z_P^2/2$, this approximation is implicitly assumed[5] to be of order $(2\epsilon - k^2)/2\epsilon \sim O((Z_P/v)^2)$. We point out that since $\psi_{\vec{k},\epsilon}^{(+)}$ does not uniformly[8] approach $\psi_{\vec{k}}^{(+)}$, a key error of the order of $(Z_T/v)^2$ is introduced. One must correctly evaluate A_{2c}, keeping track of the terms neglected. We do this by evaluating Eq. (2) using the expression given by Macek and Shakeshaft[9] for 1*s*-1*s* charge transfer.

Macek and Shakeshaft[9] give the result, valid to order $(Z_P/v)^2$,

$$A_{2c} = \frac{(2Z_T)^{1/2} Z_T^2 Z_P}{\pi K^2} \int d^3p\, \tilde{\varphi}_f^{*}(\vec{p}) Y(\vec{p}), \tag{6}$$

where

$$Y(\vec{p}) = \frac{-16\pi i v}{(Z_P^2 + p^2)} \frac{\partial}{\partial \mu_1} \int_0^1 d\rho\, \rho^{-iv}(1 - Z\rho)^{-1}(K^2 - v^2 + \mu_1^2 - 2i\mu_1 v)^{-1}, \tag{7}$$

with

$$Z=\frac{4v^2(J^2+\mu_1^2)}{(K^2-v^2+\mu_1^2-2i\mu_1 v)(Z_P^2+p^2)},$$
$$\nu=Z_T/v,\quad \vec{K}=\alpha\vec{K}_i-\vec{K}_f,\quad \vec{J}=\vec{K}_i-\beta\vec{K}_f,\quad \alpha=M_T/(m+M_T),\quad \beta=M_P/(m+M_P), \tag{8}$$

and μ_1 is set equal to Z_T after differentiation. M_T, M_P, and m represent the masses of the target nucleus, the incident nucleus, and the electron, respectively.

One also has

$$K^2+2\epsilon_i=J^2+2\epsilon_f, \tag{9}$$

where

$$\epsilon_f=-\tfrac{1}{2}Z_P^2 \text{ and } \epsilon_i=-\tfrac{1}{2}Z_T^2. \tag{10}$$

Since the Fourier transform of the final-state wave function $\tilde{\varphi}_f^*(\vec{p})$ restricts p to values of the order of Z_P and since $K\approx v$, we have $Z\approx v^2/Z_P^2\gg 1$. Accordingly, we evaluate the integral over ρ by recognizing that it represents a hypergeometric function of large argument, ${}_2F_1(1,\ 1-i\nu;\ 2-i\nu;\ Z)$, and use the appropriate analytic continuation of ${}_2F_1$ for this case:

$${}_2F_1(1,\ 1-i\nu;\ 2-i\nu;\ Z)=(i\nu Z)^{-1}\,{}_2F_1(1,i\nu;\ 1+i\nu;\ 1/Z)+\Gamma(2-i\nu)\Gamma(i\nu)(-Z)^{-1+i\nu}. \tag{11}$$

The ${}_2F_1$ on the right-hand side of Eq. (11) is unity to order $(Z_P/v)^2$ and one can show that the corresponding term $1/(i\nu Z)$, when used in the expression for $A\approx A_1+A_{2c}$, exactly cancels A_1. The remaining term relates to, but does not equal, the impulse-approximation expression.

Upon substituting Eq. (11) into Eqs. (6) and (7) and using the Fourier transform of the final-state wave function,

$$\tilde{\varphi}_f(\vec{p})=\frac{2^{3/2}Z_P^{5/2}}{\pi}\frac{1}{(Z_P^2+p^2)^2}, \tag{12}$$

one has

$$A\approx\frac{2^4Z_T^{3/2}Z_P^{7/2}}{K^2\pi}\frac{\partial}{\partial\mu_1}\frac{1}{\mu_1^2+J^2}\left[\frac{\mu_1^2+K^2-v^2-2iv\mu_1}{4v^2(\mu_1^2+J^2)}\right]^{-i\nu}e^{\pi\nu}\Gamma(1-i\nu)\Gamma(1+i\nu)\int d^3p(Z_P^2+p^2)^{-i\nu-2}. \tag{13}$$

The approximate amplitude Eq. (13) represents the key result of this note. It is identical to the impulse-approximation result[5] except for the factor $e^{\pi\nu/2}\Gamma(1+i\nu)[(Z_P^2+p^2)/4v^2]^{-i\nu}$. Now $\vec{p}$ relates to $\vec{k}$ of Eq. (5) by $\vec{k}=\vec{p}+\vec{v}$, and ϵ is given by[9] $\epsilon=\frac{1}{2}v^2+\vec{v}\cdot\vec{p}-\frac{1}{2}Z_P^2$ so that $(\epsilon-\frac{1}{2}k^2)/4\epsilon\approx-(Z_P^2+p^2)/4v^2$. As shown by Mapleton,[8] we have

$$\psi_{\vec{k},\epsilon}^{(+)}\approx e^{\pi\nu/2}\Gamma(1+i\nu)[|\epsilon-\tfrac{1}{2}k^2|/4\epsilon]^{-i\nu}\psi_{\vec{k}}^{(+)},\quad \epsilon<\tfrac{1}{2}k^2;$$

thus the amplitude in Eq. (13) differs from the impulse-approximation amplitude only by the inclusion of the correct off-energy-shell wave function.

The integral over p is easily evaluated:

$$\int d^3p(Z_P^2+p^2)^{-i\nu-2}=2\pi\Gamma(\tfrac{1}{2}-i\nu)\Gamma(\tfrac{3}{2})Z_P^{-1-2i\nu}/\Gamma(2+i\nu). \tag{14}$$

Substituting Eq. (14) into Eq. (13) and the result into the electron-capture cross section σ,

$$\sigma=(2\pi v^2)^{-1}\int_0^\infty|A|^2K_\perp dK_\perp, \tag{15}$$

with $K_\perp^2=K^2-[\frac{1}{2}v-(Z_P^2-Z_T^2)/v]^2$, defining $x=K_\perp^2/v^2$, and neglecting terms of order $(Z_P/v)^2$, we have for σ the result

$$\sigma=\frac{2^8\pi Z_T^5Z_P^5}{v^{12}}|N(\nu)|^2|M(\nu)|^2\int_{\frac{1}{4}(1+\nu^2)^2}^\infty dx\left(\frac{1+\nu^2}{x^6}+\frac{1+\nu^2}{x^4[(x-x_0)^2+4]^2}+\frac{2(1-\nu^2)(x-x_0)-8\nu^2}{x^5[(x-x_0)^2+4\nu^2]}\right)\times\exp\left[-2\nu\tan^{-1}\frac{2\nu}{x-x_0}\right], \tag{16}$$

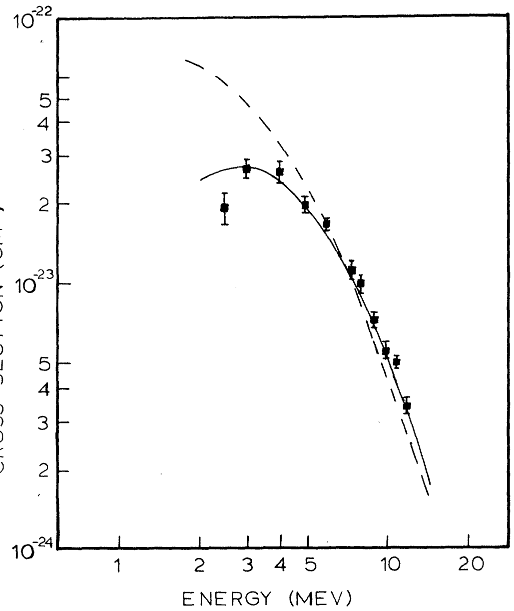

FIG. 1. The cross sections for capture of argon K-shell electrons by protons. Experimental data are from MacDonald, Cocke, and Eidson (Ref. 10), the dashed curve is the impulse-approximation result of Kocbach (Ref. 12), and the solid curve incorporates the correction factor of Eq. (18).

where

$$x_0 = (1-\nu)^2,$$

$$|N(\nu)|^2 = 2\pi\nu/(1-e^{-2\pi\nu}), \quad (17)$$

and

$$|M(\nu)|^2 = 2/[(1+\nu^2)(1+e^{-2\pi\nu})]. \quad (18)$$

Except for the correction factor $|M(\nu)|^2$, Eq. (16) is identical to the impulse-approximation expression of Briggs.[5] This factor stems from the nonuniform approach of $\psi_{\vec{k},\epsilon}^{(+)}$ to $\psi_{\vec{k}}^{(+)}$ as $\epsilon \to \frac{1}{2}k^2$. For $\nu < 1$, $|M(\nu)|^2$ never differs from unity by a factor greater than 2, but for $\nu \gg 1$ it can be much smaller than unity; for example, when $v = Z_P$, $Z_P = 1$, and $Z_T = 18$, corresponding to 25-keV protons impinging on argon, the factor is of the order of 0.005. Low-energy cross sections are thus greatly reduced by this factor, but high-energy ones are altered by less than a factor of 2.

Figure 1 shows the K-shell charge-transfer cross section for protons in argon. The dashed curve is without $|M(\nu)|^2$, the solid curve is with the correction, and the points are the experimental data of Ref. 10. The comparison strongly supports the correction factor $|M(\nu)|^2$, although the lowest-energy point deviates from theory by 10%, an amount slightly greater than $O((Z_P/v)^2)$. Since we have carefully kept track of the order of the errors in the evaluation and have verified that they are indeed negligible, the overall good agreement is expected, although a remeasurement at 2.5 MeV is indicated. The main remaining uncertainty is the use of unscreened Coulomb wave functions to treat inner shells. Experience with ionization calculations indicates[11] that this could be the source of errors of the order of 30%.

These results agree with the equivalent numerical calculations of Macek and Shakeshaft[9] at 5 and 10 MeV, but their 2.5-MeV point is a factor of 2 below that given by Eq. (16). We have no explanation for this disagreement but are currently investigating the discrepancy. Cancellations between first and second Born terms, noted here in Eq. (11), could make the numerical integrations of Ref. 9 somewhat uncertain.

The factor $|M(\nu)|^2$ has been derived only in the limit that $Z_P/v \ll 1$ and $Z_P/Z_T \ll 1$ but the nonuniform convergence of off-energy-shell Coulomb functions to on-energy-shell ones has broad implications for all second-Born-type calculations in atomic collisions. Virtually all high-energy approximations which employ Coulomb Green's functions must be reexamined in light of the demonstrated physical consequences of the nonuniform convergence of $\psi_{\vec{k},\epsilon}^{(+)}$ to $\psi_{\vec{k}}^{(+)}$.

Experimental data relevant to this factor are desirable. The low-energy region, in particular, is important. In this connection, we note that since $|M(\nu)|^2$ is independent of proton scattering angle, the impact-parameter dependence is correctly given by the impulse approximation up to an energy-dependent multiplicative constant. Also, since $|M(\nu)|^2$ is independent of Z_P for $Z_P \ll Z_T$, the impulse approximation and Eq. (16) both predict a Z_P^5 dependence for the cross section.

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Resonance-Enhanced Transient Reflectivity via Exciton Polaritons

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The reflection of a light pulse from a spatially dispersive medium is investigated theoretically. For laser frequency at exciton resonance, spatial dispersion enhances the reflected transients associated with the light pulse. For the case of CdS and GaAs crystals, the transient intensities are about 10% of the incident intensity at a time 0.1 ps after the trailing edge of the reflected signal. The theory predicts a crossover from exponential to slow power-law decay rate of transient reflectivity; this occurs at a characteristic time $\zeta_c \sim 1$ ps after the trailing edge of the pulse for CdS and GaAs.

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This Letter reports results of a theoretical investigation of transient optical reflectivity from spatially dispersive media such as CdS and GaAs.[1] The laser frequency of an incident pulse is taken near one-photon resonance with the transverse exciton-polariton frequency ω_t. We show that enhanced (transient) reflectivity persists for several picoseconds after a laser pulse is cut off. In general both the leading and the trailing pulse edges give rise to transients. Experimentally it may be more convenient to look for trailing-edge transient reflectivity since steady-state reflectivity will then not interfere with measurements. For sufficiently long pulses ($T >$ a few picoseconds), transients from the two edges will be essentially decoupled and can be considered independently. Our results show that transient reflectivity consists of a "local" part and a "nonlocal" part. The former, although the only one present in a local media, is at least an order of magnitude smaller than the latter. The measured transient reflectivity will thus almost completely arise from spatial dispersion: To the leading order, it varies as $M^{-3/4}$ with the exciton mass. Initially the time decay of reflectivity is exponential, crossing over to inverse-power decay, at about 1 ps in CdS or GaAs. Its maximum magnitude is about 10% of the incident intensity. These effects should be measurable.

The origin of spatial dispersion or nonlocality is due to coupling of an exciton state (with center-of-mass motion included) to a photon, producing an exciton polariton. This results in a wave-vector–dependent optical dielectric response. Consequently there is an "additional" transverse-propagating mode compared to the case of a local medium ($M=\infty$). In particular there is a (transverse) mode in the "pseudogap" frequency region, $\omega_t > \omega > \omega_l$, where ω_l is the longitudinal frequency. Steady-state reflectivity, transmittivity, and inelastic scattering in such media have been well studied[1]: Reflectivity is generally maximum in the pseudogap region although smaller than that of a local medium. Transient optical transmission was investigated theoretically by Birman and Frankel,[2] who showed that a new exciton precursor occurs; Johnson[3] discussed transient oscillations in transmittivity of a plate.

We consider transient reflectivity from a semi-infinite nonlocal medium occupying half-space $z > L$, with $z < L$ being vacuum. A detector is placed at $z=0$. The nonlocal medium is taken to have a scalar isotropic dielectric susceptibility[4]

$$\chi(\vec{r},\vec{r}\,') = \chi(\vec{r}-\vec{r}\,') - \chi(\vec{\zeta}-\vec{\zeta}\,', z+z'), \quad (1)$$

where $\vec{\zeta}=(x,y)$ is the transverse part of $\vec{r}$. At time $t=0$ and at $z=0$, a square optical pulse, with laser frequency ω_0 and duration T, is incident normally on the crystal surface. The elec-

