tial value when the Gelfand-Levitan equation from the right is used.

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[a]Permanent address: Department of Physics, University of Missouri–Columbia, Columbia, Mo. 65211.

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Evidence for Large-x Corrections in Quantum Chromodynamics

R. Michael Barnett and D. Schlatter
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

Luca Trentadue[a]
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305

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We show there is evidence that a method of summing important logarithmic corrections which are significant in the large-x region leads to a superior description of deep-inelastic scattering data (analyzed with use of the evolution equations). Next-to-leading-order calculations can imitate the impact of this summation method, but at high x it appears that there are higher-order and higher-twist corrections which separate those approaches.

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Considerable effort has been devoted to obtain clean and quantitative predictions in quantum chromodynamics (QCD) for various physical processes.[1] Among these, deep-inelastic processes could, in principle, represent an excellent laboratory for such tests. Although QCD predicts an observable logarithmic violation of scaling for the structure functions, there are difficulties which have prevented a completely satisfactory comparison between theory and experiment.[2] Even when considering only the leading-twist contributions in the operator-product expansion (OPE) there are contributions of higher order in α_s, the running coupling constant. Also, the summation[3-9] of certain logarithmic terms (in moments they are characterized as $\alpha_s \ln^2 n$) leads to large corrections in the $x \to 1$ region ($x = Q^2/2p\cdot q$).

We use the evolution equations[10] to analyze the impact of both second-order and $x \to 1$ corrections to the deep-inelastic structure functions, and we consider the relationship between them. We make direct comparisons between theory and experiment for both electron and neutrino deep-inelastic scattering, using Stanford Linear Accelerator Center–Massachusetts Institute of Technology (SLAC-MIT)[11] and CERN–Dortmund-Heidelberg-Saclay (CDHS)[12] collaboration data. We find that there are dramatic indications in the data to support theoretical expectations.

Brodsky and Lepage[3] have observed that large nonleading contributions to the structure functions, which arise in the $x \to 1$ region because of the gluon radiation corrections of the theory,[13] are related to kinematical constraints. Analyzing deep-inelastic scattering, they observed that the use of the correct kinematic constraints plus the correct argument of α_s in evolution equations introduces an additional term. For the moments of the structure functions, this reproduces the sum of the large $\alpha_s \ln^2 n$ corrections (which come from the $x \to 1$ region). These corrections are the same ones as computed with use of the operator-product expansion[8,9] to order α_s. Similar arguments have been developed by Parisi[4] and by

Curci and Greco.[5] Amati *et al.*,[6] by analysis of the strong-ordering contributions to the invariant charge, have proposed a modified evolution equation which, because of the rescaled argument of the running coupling constant, resums[14] the large corrections. The proof that such a resummation occurs has been given only at the leading-infrared-singularity level. The kinematics-dependent scale in the coupling constant is given by the upper limit on the emitted-gluon invariant mass. The modified leading-order evolution equation for the structure functions (see Refs. 5 and 6) is then, in the nonsinglet (NS) case,

$$Q^2\frac{\partial}{\partial Q^2}F^{\mathrm{NS}}(x,Q^2) = \int_x^1 dz\, F^{\mathrm{NS}}\left(\frac{x}{z},Q^2\right)\left[\frac{\alpha_s(Q^2(1-z)/z)}{2\pi}P(z)\right]_+, \quad (1)$$

where $F^{\mathrm{NS}}=(F_2^{ep}-F_2^{en})$ or $xF_3^{\nu N}$. The + notation indicates the regularization procedure (see Ref. 10). This equation sums the large logarithms which arise when the real emission of gluons cannot compensate for the large, opposite effects of the virtual contributions.

The perturbative approach of Eq. (1) breaks down for $(1-z)\lesssim Q_c^2/Q^2$ [where Q_c^2 is a mass of order 1 GeV^2 such that $\alpha_s(Q_c^2)/2\pi\lesssim 1$]. The structure function becomes of the quark form-factor type and shows a Sudakov-type damping.[1]

Considering the new argument of α_s in Eq. (1), we see that as x becomes large and Q^2 is held fixed, the running coupling constant becomes large. In order to investigate this problem we introduced the following definition[15] of α_s:

$$\alpha_s(Q^2(1-z)/z)=(4\pi/\beta_0)[\log(V^2/\Lambda^2)]^{-1}, \quad (2)$$

where we define[16] $V^2\equiv Q^2(1-z)/z+m^2$, with m a free mass parameter acting as an infrared cutoff. Such a definition, which leaves α_s unchanged at large values of the argument, makes it possible to make a limited estimate of the sensitivity of calculations to this region.[17] Use of this definition (with the parameter m^2) is equivalent to the inclusion of higher-twist terms since

$$\left(\ln\frac{Q^2(1-z)/z+m^2}{\Lambda^2}\right)^{-1}\approx\left(\ln\frac{Q^2(1-z)/z}{\Lambda^2}\right)^{-1}\left(1-\frac{zm^2}{Q^2(1-z)\ln[Q^2(1-z)/z\,\Lambda^2]}\right). \quad (3)$$

When we (later) make use of the second-order-in-$\alpha_s(Q^2)$ evolution equations, we follow the work of Curci, Furmanski, and Petronzio[18] and write (using the $\overline{\mathrm{MS}}$ renormalization scheme[9]):

$$Q^2\frac{\partial}{\partial Q^2}F^{\mathrm{NS}}(x,Q^2)=\int_x^1 dz\, F^{\mathrm{NS}}\left(\frac{x}{z},Q^2\right)\left[\frac{\alpha_s(Q^2)}{2\pi}P^{(1)}(z)+\frac{\alpha_s^2(Q^2)}{(2\pi)^2}P^{(2)}(z)\right]_+, \quad (4)$$

where $P^{(1)}$ and $P^{(2)}$ are defined in Ref. 18.

Now, turning to our results: What is the impact of using the variable V^2 [$\equiv Q^2(1-z)/z+m^2$] instead of Q^2 in Eq. (1)? Clearly one expects to find a lower value[19] of Λ. For the CDHS ν data, Λ decreases from $\Lambda\approx 0.33$ GeV for Q^2 evolution to $\Lambda\approx 0.20$ GeV for V^2 evolution. For the SLAC data, $\Lambda=0.66$ GeV decreases to 0.43 GeV.

More important is the result that the data are better described by V^2 evolution than by Q^2 evolution (first order). We find that the χ^2 for fitting data with V^2 evolution are noticeably better: $\chi^2=90.7$ vs 93.3 for Q^2 evolution (for 76 degrees of freedom) for neutrino data and $\chi^2=62.8$ vs 67.8 (for 72 degrees of freedom) for electron data (χ^2 is very insensitive to the value of m).

This can be understood, in part, by noting that whereas the first-order Q^2-evolution equation implies a fixed, constant value of Λ, the use of this Q^2 evolution to extract Λ from the data results in a Λ which is dependent on x (see Fig. 1). Such a contradiction is not found when V^2 evolution is used to extract Λ from the data; here we

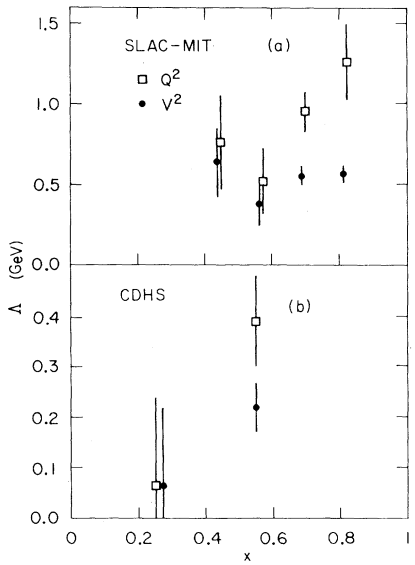

FIG. 1. Λ extracted from SLAC-MIT (Ref. 11) and CDHS (Ref. 12) data for $F_2^p-F_2^n$ and xF_3 with use of first-order Q^2 evolution and with use of V^2 evolution with the data in large-x bins.

find Λ consistent[20] within errors with being independent of x (see Fig. 1). Because V^2 evolution leads to a constant Λ, the resulting global (all x) fit to the data is superior to that from the leading-order Q^2 approach.

How do results that use the evolution equation calculated to second-order in $\alpha_s(Q^2)$ compare with those from leading-order Q^2 evolution? The impact on the overall value of Λ is very small (e.g., $\Lambda^{(1)} \approx 0.33$ GeV and $\Lambda^{(2)} \approx 0.35$ GeV). However, the χ^2 for second-order fits to the data are somewhat better than for first order. The cause of this improved fit is that (as for V^2 evolution) use of second-order evolution, Eq. (4), to extract Λ from data can give a Λ relatively independent of x. That it is *possible* for second-order evolution to imitate the effects of V^2 evolution is easy to see since Eq. (4) can be rewritten as

$$Q^2(\partial/\partial Q^2)F^{\mathrm{NS}}(x,Q^2) = \int_x^1 dz\, F^{\mathrm{NS}}(x/z,Q^2)\{\alpha_s(Q^2 f(z))P^{(1)}(z) + O(\alpha_s{}^3)\}_+. \tag{5}$$

By expanding in powers of $\ln(Q^2/\Lambda^2)$, one finds

$$[P^{(1)}(z)\ln f(z)]_+ \approx [(4\pi/\beta_0)P^{(2)}(z)]_+ + [P^{(1)}(z)(\beta_1/\beta_0{}^2)\ln\ln Q_2/\Lambda^2]_+. \tag{6}$$

Analogous arguments were presented in Refs. 8 and 9 in relating the Λ_n scheme to other second-order renormalization schemes. To see whether this approximate equivalence does in fact occur, we treated the output of the V^2-evolution equation (1) as "data," and then checked the x dependence of Λ extracted with use of the second-order evolution equation (4). The results are shown in Fig. 2, where we also compare with the use of the first-order Q^2-evolution equation. Clearly the second-order equation (unlike the first-order equation) is consistent with giving a constant Λ. The fact that both methods simultaneously provide x-independent values for Λ confirms that the Q^2-evolution equation corrected to second order can to some extent handle the essential kinematic constraints as the V^2 evolution does.

How do curves from these different approaches compare? In Fig. 3(a) we show $(xF_3^{(2)} - xF_3^{(1)})/xF_3^{(1)}$ [which we found looks very similar to $(xF_3^{(V^2)} - xF_3^{(1)})/xF_3^{(1)}$]. Large differences for xF_3 in these approaches only occur for large x (as expected).[21] The fact that the differences in Fig. 3(b) are significant only at very large x may indicate that the ability of the second-order equation to reproduce the impact of the V^2-evolution approach fails at these x values, so that the higher-order terms (third order and higher) are increasingly important (and also higher-twist corrections).[22] This is not unexpected since the second-order equation cannot account for terms such as $\alpha_s{}^2(Q^2)\ln^4[1/(1-x)]$ and $xm^2/Q^2(1-x)$.

If one adjusts the V^2-evolution parameters to fit the second-order evolution curves, the best fit has the parameter $m^2 \approx 0.9$ GeV^2 (close to $m_p{}^2 = 0.88$). It is not possible to determine m^2 from the present data, since they are not precise

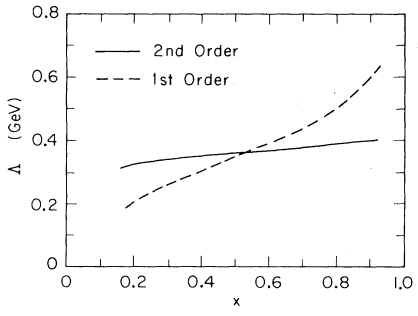

FIG. 2. Λ extracted with use of first-order and second-order Q^2 evolution from "theoretical data" created from the output of the V^2-evolution equation. The input value of Λ was 0.2 GeV. Below $x = 0.2$, there is little sensitivity to the value of Λ.

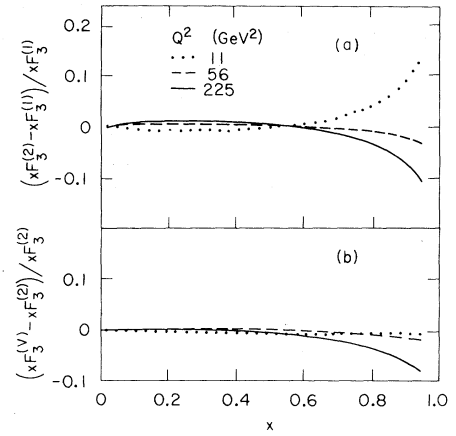

FIG. 3. The fractional differences between (a) second-order and first-order-in-$\alpha_s(Q^2)$ structure functions and (b) the V^2-evolved and the second-order structure functions obtained by use of the corresponding evolution equations. The same x parametrizations were used in all cases. In (b) the particular Q^2 value at which the high-x difference is (approximately) zero is not significant; it varies with the choice of $Q_0{}^2$ (the starting point of evolution).

enough at large x.

We believe that the analysis of the structure functions via the evolution equations has clear advantages over moment analyses. The integration over x needed to obtain moments requires the extrapolation of data into unmeasured regions. In doing moment analyses we found that our results were critically sensitive to the nature of the extrapolation.

In conclusion, the use of the modified evolution equation (in terms of V^2) leads to clearly superior descriptions of the neutrino and electron data (compared to that from leading-order Q^2 evolution). The second-order evolution equation (in Q^2) can imitate the impact of the V^2-evolution equation (by making Λ roughly x independent). However, at large x the two approaches diverge, and one can hypothesize that it is the corrections higher than second order in α_s and of higher-twist type, inherent in the V^2 approach, which distinguish them.

We feel that much could be learned from having more data in the large-x region. Even for moderate-x regions, use of the V^2-evolution equations for the analysis of data is indicated by our results.

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[a]On leave from Istituto di Fisica dell'Università, Parma, Italy, and Istituto Nazionale di Física Nucleare, Sezione di Milano, Italy.

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[15]Compare with J. Richardson, Phys. Lett. 82B, 272 (1979).

[16]This definition has the same form as that of W^2, but it is not W^2: $W^2 = Q^2(1-x)/x + m_p^2$.

[17]The value of Λ from fits can vary by ± 20 MeV (for $\Lambda \approx 200$ MeV) depending on the value of m^2.

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[20]The statistics of the data are poor when single-x bins are used, because low-x data are mostly at low Q^2 and high-x data are at high Q^2.

[21]It is possible to adjust parameters to minimize the differences at large x, but then the differences for $x < 0.8$ are significantly increased.

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