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Z Dependence of Bremsstrahlung Radiation from Free Atoms

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Relative bremsstrahlung cross sections have been measured for free atoms, with atomic numbers in the range $Z=2$ to $Z=92$, at low incident electron energies of 2.5 and 10 keV. The results agree reasonably with a theoretical calculation of Pratt *et al.*, except at the largest Z number, where differences of up to a factor of 2 at low photon energies are observed.

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Electrons being accelerated in the fields of neutral or ionized atoms give rise to the emission of noncharacteristic radiation. This atomic field bremsstrahlung is of fundamental importance as well as of practical relevance for various plasma and fusion projects and in biological applications.[1]

Most modern discussions of bremsstrahlung radiation start with the work of Sommerfeld and co-workers.[2-4] In a nonrelativistic, quantum-mechanical calculation, Sommerfeld and Maue[3] obtained $d\sigma/dk$, the bremsstrahlung cross section differential in photon energy k,

$$\frac{d\sigma}{dk}=\frac{16\pi^2}{3}\alpha^5 a_0^2\frac{Z^2}{k\beta_1^2}\frac{x_0}{[\exp(2\pi n_1)-1][1-\exp(-2\pi n_2)]}\frac{d}{dx_0}|F(x_0)|^2, \tag{1}$$

with $n_{1,2}=\alpha Z/\beta_{1,2}$, $\beta_{1,2}=v_{1,2}/c$, $v_{1,2}$ the ingoing and outgoing electron velocity, a_0 the Bohr radius, $x_0=-4n_1n_2/(n_1-n_2)^2$, and $F=F(in_1, in_2, 1, x_0)$ the hypergeometric function. At low incident electron energies T ($T\lesssim 10$ keV) this result is almost identical to a relativistic calculation of Elwert and Haug.[5] For sufficiently large energies $2\pi n_{1,2}\ll 1$, Eq. (1) reduces to

$$d\sigma/dk=\tfrac{8}{3}\alpha^5 a_0^2(Z^2/k\beta_1^2)\ln(1-x_0). \tag{2}$$

Equation (2) predicts a Z^2 dependence of bremsstrahlung cross section.

Here we report on an investigation of the Z dependence of bremsstrahlung cross section $d\sigma/dk$ for the electron bombardment of free (gaseous) atoms at low incident electron energies ($T=2.5$ to 10 keV). The results are compared with a recent numerical calculation of Pratt *et al.*[6] in which the screening of the nuclear field by the surrounding electrons is taken into account.

The experimental arrangement has been described in detail previously.[7] It consists essentially of an electron gun inside a vacuum chamber, providing beam currents of about 100 μA. Gas is introduced into the chamber through a needle valve producing a pressure typically about 1×10^{-3} mbar for all gases studied with the exception of helium. In the case of helium, pressures up to 4×10^{-3} mbar have been used. A Baratron capacitance manometer was used for the pressure measurements. X rays produced by the collision process were detected, at an observation angle chosen to be 90°, with a Si(Li) detector having an energy resolution of 200 eV at 5.9 keV. The measured signal rates have been normalized to the integrated electron beam current and to the gas pressure. The background contribution (resulting mainly from collisions of electrons with solid material) was small (< 5%), except when helium was used as a gas target and the background contributed about 20% of the total signal. In all cases the background was taken into account. A linear relationship between the measured signal rate and the gas pressure has been confirmed for each gas.

Atomic numbers Z in the range $Z = 2$ (helium) to $Z = 92$ (uranium) were used. In addition to the noble gases helium, neon, argon, krypton, and xenon, two molecular gases (N_2 and UF_6) have been used. In the case of N_2, the measured signal rate has been divided by 2 to account for the higher number of atoms in a molecular gas compared to an atomic gas. In the UF_6 case, the bremsstrahlung contribution from the six fluorine atoms has to be subtracted from the measured signal rate. The uranium contribution has been calculated by multiplying the measured signal rate by

$$(d\sigma/dk)_U/[(d\sigma/dk)_U + 6(d\sigma/dk)_F],$$

where the index refers to uranium and fluorine, respectively, and with the bremsstrahlung cross sections $d\sigma/dk$ taken from the calculation of Pratt *et al.*[6] Because of the comparatively low bremsstrahlung yield of fluorine, this factor was of the order of 0.9.

In Fig. 1, the differential bremsstrahlung cross section $d\sigma/dk$ is presented versus the atomic number Z. The incident electron energy was $T = 2.5$ keV; the photon energy was $k = 2.0$ keV. The strong increase of $d\sigma/dk$ with Z is clearly demonstrated. Also given in Fig. 1 is the theoretical calculation of Pratt *et al.*,[6] to which our experi-

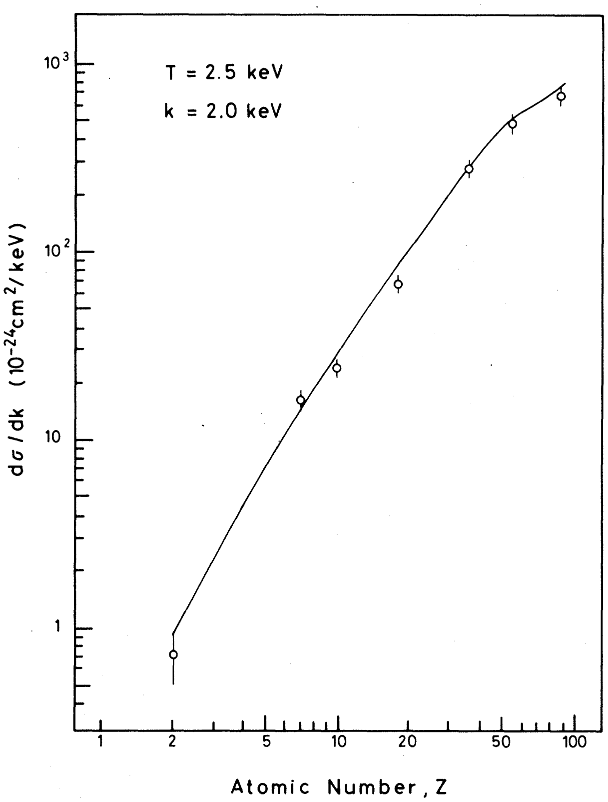

FIG. 1. Bremsstrahlung cross section, $d\sigma/dk$, vs atomic number Z. Circles are the present results; the solid line is the theoretical calculation of Pratt *et al.* (Ref. 6). The incident electron energy was $T = 2.5$ keV; the photon energy was $k = 2$ keV.

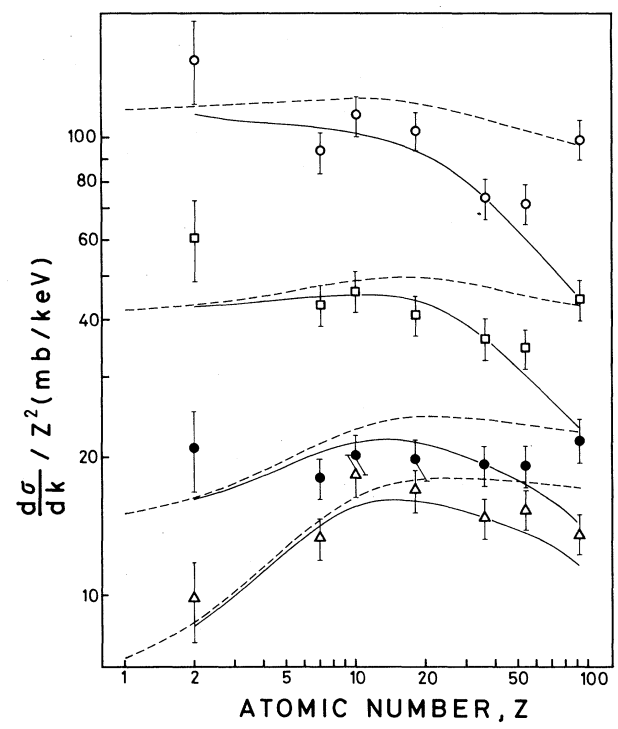

FIG. 2. Scaled bremsstrahlung cross sections $(d\sigma/dk)/Z^2$, vs atomic number Z at an incident electron energy $T = 10$ keV and photon energies of 2 keV (open circles), 4 keV (squares), 7 keV (solid circles), and 9 keV (triangles). The solid lines are the theoretical calculations of Pratt *et al.* (Ref. 6); dashed lines are from the Sommerfeld-Maue formula (Ref. 3) [Eq. (1)].

mental data have been normalized at $Z = 36$.

To have a better comparison between experiment and theory we present, in Fig. 2, data for an incident electron energy $T = 10$ keV and photon energies of $k = 2$, 4, 7, and 9 keV. In Fig. 2 the scaled bremsstrahlung cross section $(d\sigma/dk)/Z^2$ may be seen to be only weakly dependent on Z. This signifies that the measured data roughly support a Z^2 dependence for the bremsstrahlung cross section. Especially at medium Z numbers, there is a good agreement with theory. The helium data are always larger than expected from theory, although the differences are only slightly larger than the experimental error bars. It should be noted here that our data show no indication for electron-electron bremsstrahlung, which, compared to the atomic field bremsstrahlung, should be most important for helium. At the larger Z values, experiment and theory disagree. This disagreement is largest for the small photon energies, amounting to more than a factor of 2 at $k = 2$ keV and $Z = 92$.

The photon energy dependence of the bremsstrahlung cross section is given in Fig. 3. Since no absolute calibration of the x-ray detector has been performed and also the relative sensitivity is not known, the data for different atoms have been normalized at each individual photon energy k to the krypton bremsstrahlung yield. Hence the data presented in Fig. 3 are cross-section ratios, $(d\sigma/dk)_X/(d\sigma/dk)_{\rm Kr}$, where X stands for He, N, Ne, Ar, Xe, and U. The data illustrate once more that for the low Z numbers both the (relative) magnitude and the shape of the bremsstrahlung cross section are well represented by the theoretical data of Pratt *et al.*[6] There is also some indication that at medium Z numbers (Xe) and low photon energies the theoretical data underestimate the experimental results. This becomes obvious for heavy atoms (U), where the theoretical data predict a completely different shape for the cross section, with the above-mentioned discrepancy amounting to a factor of 2 at low photon energies ($k = 2$ keV).

To investigate the origin of this discrepancy one one may compare the calculation of Pratt *et al.* with the bremsstrahlung cross sections $d\sigma/dk$ obtained from the Sommerfeld-Maue formula [Eq. (1)]. Since relativistic effects do not play an important role (apart from retardation of the electromagnetic potentials, which affects the angular distribution of bremsstrahlung photons) at the low incident electron energies considered here, the difference between Pratt's calculations and

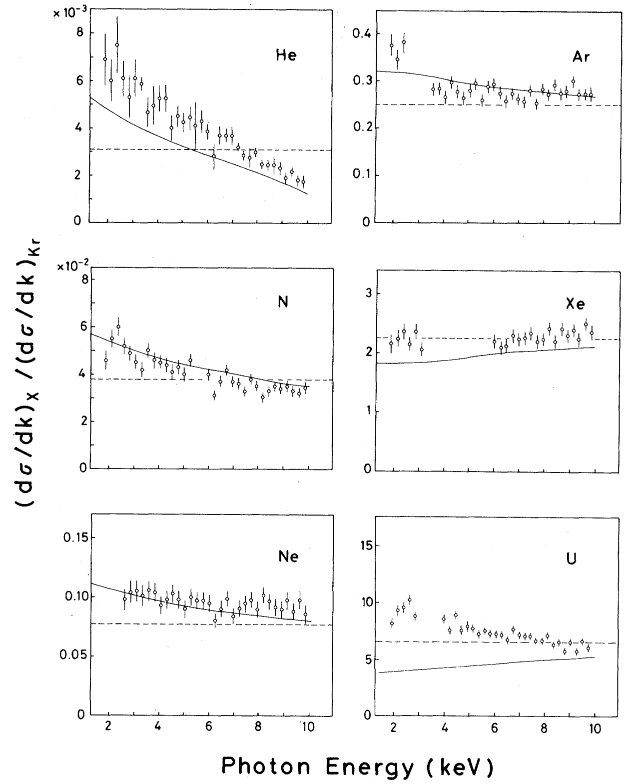

FIG. 3. Relative bremsstrahlung cross section $(d\sigma/dk)_X/(d\sigma/dk)_{\rm Kr}$ vs photon energy k. X stands for He, N, Ne, Ar, Xe, and U. Circles are the present results, solid lines are the theoretical calculations of Pratt *et al.* (Ref. 6), and dashed lines are the Z^2 dependence (see text).

the Sommerfeld-Maue formula [Eq. (1)] should be due to the screening of the atomic electrons. At the low Z numbers, the bremsstrahlung cross sections obtained from Eq. (1) and from the calculation of Pratt *et al.*[6] are almost identical to each other (Fig. 2). The difference at the larger Z numbers is attributed to the screening of the atomic electrons, which has been accounted for in the calculation of Pratt *et al.*, but not in Eq. (1). From this comparison between theories, it follows that screening is most important at low photon energies. We do not think, however, that the discrepancy between our data and and the calculations of Pratt *et al.* is due to an overestimation of the screening effect. A more likely cause, to us, seems to be that second-order effects (e.g., two-photon bremsstrahlung) become important for large Z numbers. Such effects have, to our knowledge, so far not been considered in detail in any calculation.

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X-Ray Raman and Compton Scattering in the Vicinity of a Deep Atomic Level

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In this Letter we report the first observation of inelastic x-ray scattering from an atom in which both Raman and Compton processes have been observed simultaneously. It has been observed that x-ray Raman and Compton scattering in the vicinity of a deep atomic level exhibit different behavior as a function of incident photon energy.

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Inelastic scattering of x rays at energies in the vicinity of a deep-core–electron binding energy has attracted recent experimental and theoretical attention. The scattering of a characteristic x-ray line from targets of various elements whose K binding energies lie near the x-ray photon energy has been observed by Sparks[1] and investigated more recently by Suortti.[2] Eisenberger, Platzman, and Winick,[3,4] using the powerful technique of tunable synchrotron radiation excitation, have studied resonant x-ray inelastic scattering as the incident photon energy was varied in the vicinity of the K edge of copper in a metallic target.

The observed inelastic scattering is attributed to both the $\vec{p}\cdot\vec{A}$ term in second order and the A^2 term in first order of perturbation theory in the Hamiltonian describing the interaction of electrons and the radiation field. In the vicinity of an absorption discontinuity, the $\vec{p}\cdot\vec{A}$ contribution in second order exhibits a resonant character which dominates the usual Compton scattering by free electrons which is described by the A^2 term.

Inelastic scattering of x rays by bound electrons is accompanied by either an excitation of a core electron into the continuum, or, by excitation of the core electron into a bound excited state of the system. According to Sommerfeld,[5] the former possibility, which results in a continuous photon band with a definite limit on the high-energy side, is ascribed to Compton scattering of bound electrons, while the latter results in discrete inelastically scattered photon lines which are classed as Raman lines. Thus whether or not inelastic x-ray scattering of bound electrons in the vicinity of an absorption discontinuity is called Compton or Raman scattering depends on the final state of the core electron.

In this paper, we present the first observation of resonant x-ray Raman scattering in the vicinity of a discrete inner level of a quasiatomic system. Furthermore, the high-resolution inelastically scattered photon spectra obtained allow us to identify explicitly the contributions due to Compton and Raman lines and to study their be-