COMMENT

Comment on Riazuddin's Weak-Interaction Model

A recent Letter by Riazuddin[1] claims to have a gauge model of weak interactions based on SU(2) alone, which has the same charged- and neutral-current interactions as the standard $SU(2)\otimes U(1)$ gauge model,[2] but without electroweak unification. There are two flaws in this model as it stands, which make it not a gauge model, and hence not renormalizable:

(A) Mixed left-handed and right-handed fermion states are not form invariant under a gauge transformation. For example, under $\exp(\frac{1}{2}i\vec{\epsilon}\cdot\vec{\tau})$, Riazuddin's doublet $(\nu_L, \cos\alpha\, e_L + \sin\alpha\, e_R)$ acquires a right-handed component in the slot reserved for ν_L. In contrast, a purely-left-handed doublet such as $(\nu, e)_L$ transforms to $(\nu', e')_L$ which can be redefined as $(\nu, e)_L$.

(B) It is meaningless to attach a γ_5 in defining the transformation of the vector gauge bosons, again because the transformed states are not form invariant. The appearance of either flaw will be fatal to the renormalizability of the model, as shown below.

Consider $\nu\bar{\nu}\to W^+W^-$ which proceeds via (a) virtual fermion(s) in the t channel, and/or (b) virtual boson(s) in the s channel. In the standard model,[2] the bad high-energy behavior[3] of this process is canceled[4] between (a) with e and (b) with Z. Specifically, (a) is proportional to $(g/\sqrt{2})^2[\frac{1}{2}(1+\gamma_5)]$ and (b) to $(g/\cos\theta_W)(-g\cos\theta_W)(\frac{1}{2})^2(1+\gamma_5)$. In Riazuddin's model, (a) is proportional to $(g_W\cos\alpha/\sqrt{2})^2[\frac{1}{2}(1+\gamma_5)]$, but (b) is zero because the $W_3W^+W^-$ vertex does not exist. Hence there can be no cancellation, and the model is not renormalizable. Suppose we now remove the γ_5 in the W_μ transformation; then (b) will be proportional to $g_W \times(-g_W)(\frac{1}{2})^2(1+\gamma_5)$, and will still not cancel (a) unless $\cos^2\alpha = 1$.

To reformulate Riazuddin's model into a renormalizable gauge model without changing the standard neutral-current phenomenology, we first remove the γ_5 in the W_μ transformation, and then assign the doublets $(\nu, \cos\alpha\, e + \sin\alpha\, e')_L$ and $(\sin\alpha'\, e + \cos\alpha'\, e', E^{--})_R$, where e' is an unobserved heavy electron, and E^{--} an unobserved doubly charged lepton. To obtain the standard neutral-current structure in the electron sector, we set $\alpha' = \alpha$. Then as far as ν and e interactions are concerned, this model is identical to Riazuddin's. However, ours is a bona fide gauge model and hence renormalizable. For the process $\nu\bar{\nu}\to W^+W^-$, (a) involves both e and e', and (b) W_3. Specifically, (a) is proportional to $(g_W/\sqrt{2})^2(\cos^2\alpha+\sin^2\alpha)[\frac{1}{2}(1+\gamma_5)]$, and (b) to $g_W(-g_W)\times(\frac{1}{2})^2(1+\gamma_5)$.

For $e^+e^-\to W^+W^-$, (a) is proportional to $-(g_W\times\cos\alpha/\sqrt{2})^2[\frac{1}{2}(1+\gamma_5)] + (g_W\sin\alpha/\sqrt{2})^2[\frac{1}{2}(1-\gamma_5)]$, and (b) to $g_W(-g_W)\{-\frac{1}{2}[\frac{1}{2}(1+\gamma_5)]+\frac{1}{2}\sin^2\alpha\}$. There is, again cancellation of the bad high-energy behavior. However, the contribution of the photon to (b) has not been taken into account, and its addition by hand will again destroy the overall renormalizability of the entire theory. In contrast, for the standard model, (a) is proportional to $-(g/\sqrt{2})^2[\frac{1}{2}(1+\gamma_5)]$, and (b) to $(-e)^2$ from the photon and $(g/\cos\theta_W)(-g\cos\theta_W)\{-\frac{1}{2}[\frac{1}{2}(1+\gamma_5)]+\sin^2\theta_W\}$ from the Z boson, so that together they sum to zero with the unification condition $e = g\sin\theta_W$.

Indeed it has been shown by Feynman[5] that starting with a pure SU(2) gauge theory for weak interactions which involves $W^\pm$ bosons, and then imposing local U(1) gauge invariance for the electromagnetic interaction, *requires mixing between* W_3 *and the photon,* and furthermore *the mixing is precisely that of the standard model.*

Finally, the statement of Ref. 1 concerning the relation $m_{W_3}\simeq\sqrt{2}m_W$ is pure speculation and not supported by any theory.

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[1] Riazuddin, Phys. Rev. Lett. 45, 976 (1980).
[2] See Ref. 2 of Riazuddin, Ref. 1.
[3] M. Gell-Mann *et al.*, Phys. Rev. 179, 1518 (1969).
[4] See, for instance, C. H. Llewellyn-Smith, in *Proceedings of the Fifth Hawaii Topical Conference in Particle Physics, Honolulu, Hawaii, 1973,* edited by P. N. Dobson, Jr., V. Z. Peterson, and S. F. Tuan (Univ. of Hawaii Press, Honolulu, 1974), p. 103.
[5] See J. J. Sakurai, in *Proceedings of the Eighth Hawaii Topical Conference in Particle Physics, Honolulu, Hawaii, 1979,* edited by S. Pakvasa and V. Z. Peterson (Univ. of Hawaii Press, Honolulu, 1980), p. 375.

