Solitonlike Propagation of Zero Sound in Superfluid ^{3}He

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We report observations of nonlinear phenomena in the propagation of zero-sound pulses near a collective-mode attenuation peak in ^{3}He-B. These include saturation of the sound absorption, amplitude dependence of the group velocity, and pulse breakup. Our data show effects similar to those observed in nonlinear optical systems. We suggest that above a threshold input level, the sound pulse propagates as a soliton.

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Ultrasonic propagation experiments in superfluid ^{3}He have revealed a variety of attenuation peaks which originate from an excitation, by a sound wave, of collective internal vibrations of the Cooper pairs.[1] These pair vibrations can be somewhat simplistically thought of as being the upper energy level of a two-level quantum system, with the lower level represented by the quiescent superfluid. In a classic paper on self-induced transparency in optical systems, McCall and Hahn[2,3] have shown that it is possible to propagate large resonant pulses through a medium consisting of an aggregate of two-level quantum systems with very little attenuation, even though the medium is almost totally opaque for small resonant pulses. In this work, we experimentally demonstrate the existence of analogous phenomena in the propagation of zero-sound pulses in superfluid ^{3}He. In particular, our studies reveal a number of effects which closely resemble soliton propagation. Magnetic solitons in ^{3}He-A have been studied experimentally with use of NMR techniques by several groups[4-8] and theoretically by Maki and co-workers.[9] These magnetic solitons tend to form static domain walls in the texture of the order parameter and thus do not display the variety of pulse-propagation phenomena seen in the present experiment.

The choice of appropriate coordinates for the experiment on the superfluid phase diagram was based on several criteria: First, thermal relaxation (quasiparticle damping) should be small on the time scale of the pulse (a few microseconds). Second, the attenuation should vary only slowly with temperature to minimize the effect of temperature drifts. Third, the attenuation should not be too high, so that the power level used to saturate the absorption would be compatible with cryogenic capabilities. With all that in mind, we did the measurements at 24 bars and about 1 mK, which is the location of the recently discovered collective-mode peak[10,11] in ^{3}He-B for 100 MHz drive frequency.[12] The sound cell was the same one used in previous work.[10] It contained two 20-MHz X-cut quartz transducers separated by 0.318 cm. The transmitting crystal was excited with rf tone bursts, a few microseconds long. Temperatures were measured with a La-doped cerium magnesium nitrate thermometer calibrated against the Helsinki T_c vs pressure scale.[13] To eliminate spurious effects due to harmonic generation, tuned electronics were used for the input and output. Particular care was paid to exclusion of heating effects. We checked this by applying sequentially two equal pulses and measuring their relative amplitudes as a function of time separation. As seen in the inset of Fig. 1, which shows the attenuation versus temperature in this region, heating caused by the first pulse will alter the amplitude of the second pulse because of the change of attenuation with T. We found no observable heating effects within about 50 μsec, which is three to five times longer than the time of arrival of the signal. The actual power density applied to the liquid is difficult to estimate, but the total observed heat leak corresponds to an upper limit of 1 mW/cm^2 during measurements at the highest power level, or an energy density of about 10 nJ/cm^3 inside the cell.

In Fig. 1, we show a series of received signal traces obtained at a constant temperature for increasing input power. At the lowest level, one broad pulse is observed. This pulse shape is consistent with the considerable dispersion in the vicinity of the collective mode.[11] The group velocity is considerably lower than C_0, the zero-sound velocity. This effect, previously seen by others,[11] led to suggestions that nonlinear effects similar to the ones reported here might be observable.[14] Over some range of low input powers, the superfluid responds linearly. Once the input power is increased above a well-defined thresh-

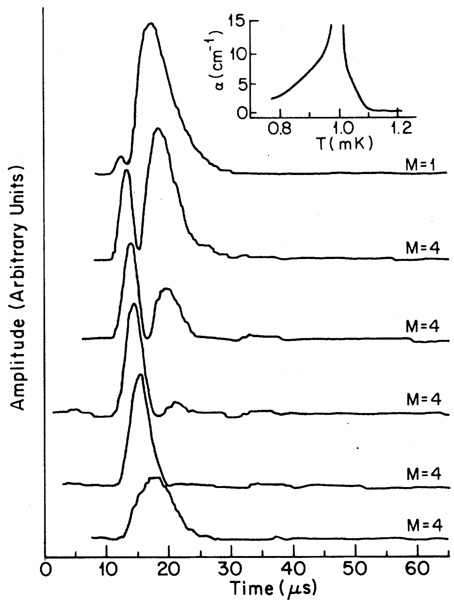

FIG. 1. Typical received signal traces at $T=0.875$ mK obtained with successively increasing input power (bottom to top) showing pulse breakup for large input pulses. Note the magnification of traces by a factor M. The inset shows the small signal attenuation vs T. The asymmetry of the peak indicates that the energy gap is quite insensitive to temperature changes below T/T_c ~ 0.37.

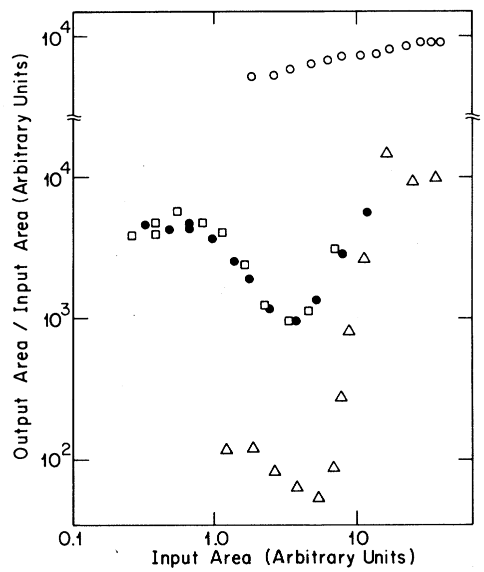

FIG. 2. Total area under the detected signal, divided by the input area, vs the input area. Data points are $T=0.984$ mK, 4.6-μsec pulses (triangles); $T=0.908$ mK, 1.3 μsec (closed circles); $T=0.908$ mK, 7-μsec pulses (squares). Normal liquid phase (open circles). (The ordinate is offset for clarity.)

old, the area under the received pulse becomes power independent. However, the peak amplitude of this pulse continues to increase, with a corresponding decrease in its width, so as to maintain a constant area. Simultaneously, the group velocity starts to increase as well. As the power is further increased, additional, much slower pulses are observed. The number of these pulses is temperature dependent. In the region of the highest attenuation, up to three additional pulses were resolved. The areas and the velocities of these pulses increase rapidly with increasing input power until they merge into one large pulse which then becomes a linear function of the input level. This indicates that the absorption is completely saturated. We repeated the same measurements both in the superfluid (away from attenuation peaks) and in the normal phase, and observed no dependence of the received pulse shape and its velocity on input power. The total area under the received signal (divided by the area of the input pulse) is shown in Fig. 2, as a function of the input pulse area. All sets of data obtained near the attenuation peak show a dip which occurs when the received signal area becomes independent of the input pulse area. The upturn in the data occurs where additional pulses first appear. We plotted the output area against input area, since we found that in this way any dependence on the pulse length or input power was removed for data obtained at a constant temperature. It is important to point out that the dependence of the data in Fig. 2 on the input area instead of on the input power rules out incoherent saturation as an explanation of the results. The measured width (full width at half maximum) of the attenuation peak in frequency was 600 kHz, which is about twice the bandwidth of the receiver crystal. Therefore, spurious effects that can arise when only part of the frequencies contained in the transmitted pulse are absorbed by the liquid can be excluded as well.

In search of an interpretation, we found that the qualitative features of our data are consistent with behavior characteristic of propagating solitons.[2,3,15] The dependence of the data in Fig. 2 on input area is characteristic of soliton solutions of nonlinear wave equations. For example, in the case of self-induced transparency[2,3] pulses with dimensionless areas $\pi < S < 3\pi$ will evolve in the medium into output pulses having an area 2π, where $S(z) = (\mu/\hbar)\int_{-\infty}^{\infty} E(z,t)\,dt$ with E the electric-

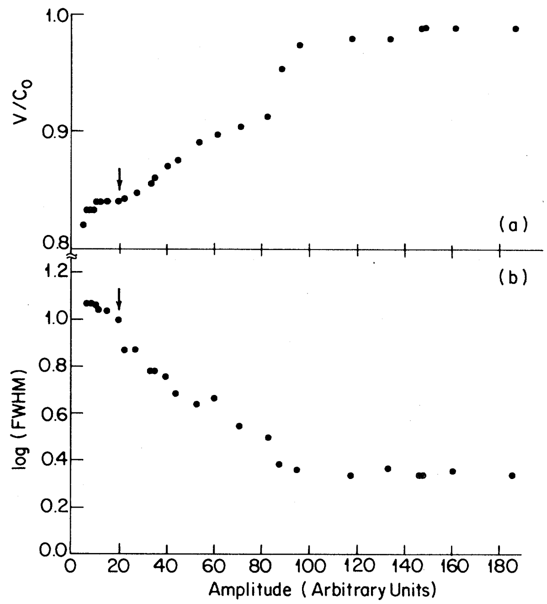

FIG. 3. (a) Group velocity vs amplitude for the first received pulse. (b) Logarithm of the full width at half maximum of the first received pulse vs amplitude, illustrating pulse compression. Arrows indicate the onset of nonlinear behavior. (Data was taken at 1.05 mK.)

field envelope and μ the dipole matrix element. [The analogous expression for area in the case of our experiment is $\int_{-\infty}^{\infty}|V(t)|\,dt$, where $V(t)$ is the detected signal envelope in volts.] As mentioned above, we indeed observed a region in Fig. 2 where the area of the output pulse is independent of the input area. For $S > 3\pi$, this particular theory[2,3] predicts a breakup of the output pulse, which we also observed, as seen in the traces in Fig. 1. (The value of input area above which pulse breakup will occur depends on the differential equation that describes the system under consideration.) The velocity of solitons increases with their amplitude, whereas their width decreases (pulse compression). These two features in our data are shown in Figs. 3(a) and 3(b) for the first received pulse, as well as in Fig. 1. It is also known that solitons can propagate through each other without any change of shape. We performed such "scattering" experiments by splitting the input pulse to excite both transducers simultaneously, sending large sound pulses through each other in the liquid. We found no difference between pulse shapes detected this way and those obtained without scattering. In contrast, this particular experiment, when carried out at a temperature away from the attenuation peak, showed distorted pulse shapes. Despite the overall qualitative agreement, a conclusive interpretation of our data can only be made with use of the solutions of the nonlinear wave equation describing zero sound in superfluid ^{3}He. Such an equation is yet to be derived. Therefore other interpretations of our results are not ruled out. For comparison purposes only, we analyzed the data using the self-induced–transparency models for nondegenerate[2,3] and degenerate[15] optical systems. The degenerate case[15] also corresponds to spin waves in ^{3}He-B. (The collective mode investigated is fivefold degenerate with respect to magnetic field,[16,17] and the measurements were done in zero field.) We did not find quantitative agreement with these models.

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[1]For a review, see P. Wölfle, in *Progress in Low Temperature Physics*, edited by D. F. Brewer (North-Holland, Amsterdam, 1978), Vol. 7A, p. 191.

[2]S. L. McCall and E. L. Hahn, Phys. Rev. 183, 457 (1969).

[3]For a review, see L. Allen and J. H. Eberly, *Optical Resonance and Two Level Atoms* (Wiley, New York, 1975).

[4]O. Avenel, M. E. Bernier, E. J. Varoquaux, and C. Vibet, in *Low-Temperature Physics-LT 14*, edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975), Vol. 5, p. 429.

[5]C. M. Gould and D. M. Lee, Phys. Rev. Lett. 37, 1223 (1975).

[6]R. W. Giannetta, E. N. Smith, and D. M. Lee, Phys. Lett. 62A, 335 (1977).

[7]J. Kokko, M. A. Paalanen, R. C. Richardson, and Y. Takano, J. Phys. C 11, L125 (1978).

[8]H. M. Bozler and T. J. Bartolac, Phys. Lett. 71A, 75 (1979).

[9]K. Maki and P. Kumar, Phys. Rev. B 17, 1088 (1978); R. Bruinsma and K. Maki, to be published.

[10]R. W. Giannetta, A. Ahonen, E. Polturak, J. Saunders,

E. K. Zeise, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 45, 262 (1980).
[11]P. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder, J. B. Ketterson, and W. P. Halperin, Phys. Rev. Lett. 45, 266 (1980).
[12]We have also observed similar phenomena to those reported here near collective-mode peaks in the A phase.
[13]T. A. Alvesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, Phys. Rev. Lett. 44, 1976 (1980).
[14]D. B. Mast, B. K. Sarma, J. R. Owers-Bradley, I. D. Calder, J. B. Ketterson, and W. P. Halperin, Bull. Am. Phys. Soc. 25, 497 (1980).
[15]R. K. Bullough, P. J. Caudrey, and H. M. Gibbs, in *Solitons and Condensed Matter Physics*, edited by A. R. Bishop (Springer-Verlag, Berlin, 1978), p. 107.
[16]L. Tewordt and N. Schopohl, J. Low Temp. Phys. 37, 421 (1979).
[17]O. Avenel, E. Varoquaux, and H. Ebisawa, Phys. Rev. Lett. 45, 1952 (1980).

Quenched Randomness in the Two-Dimensional Ferromagnetic Planar Model

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A model of a disordered planar model is introduced and solved with use of the replica method. The model is designed to mimic the inhomogeneities known to exist in helium and granular superconducting films. The results indicate that at onset, the universal results, predicted on the basis of renormalization-group analysis, hold to lowest order in the parameter measuring the disorder. The disorder has the important effect of increasing the vortex pairs couplings, resulting in an initial *increase* of the critical temperature. Possible consequences of the results to related experiments are also presented.

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The understanding of superfluidity in two-dimensional (2D) systems has been rapidly expanding in the last few years, from the seminal papers by Kosterlitz and Thouless (KT)[1] and Berezinskii,[2] to the specific[3,4] predictions by Nelson and Kosterlitz (NK).[5] There have been several experiments on helium films by Bishop and Reppy,[6] by Rudnick,[6] by Telschow and Hallock,[6] and by Webster, Webster, and Chester[6] that have tested the NK prediction. Also recent experiments in superconducting films have been interpreted in terms of the KT theory.[7] In spite of this success, there is still some controversy on the interpretation of the experimental results. In particular, Dash[8] has argued strongly that the ^{4}He experiments were done on substrates that are far from homogeneous, such that what has actually been measured is not genuine 2D superfluidity. Dash further says that even if the substrate was flat, the films themselves would not be uniform. The same criticism applies to the experiments done in superconducting films: The materials used in these experiments are either granular or amorphous.

In this communication a model of a disordered planar model is introduced and solved with use of the replica method.[9] The model is thought to mimic a granular superconductor or an inhomogeneous He film. The language of the superconductors will be used throughout while having in mind that the same model applies to the helium case.

Granular superconductors consist of metallic grains embedded in an insulating matrix. At high temperatures the grains are essentially decoupled from each other. When lowering the temperature the grains interact via a Josephson coupling that can lead to long-range correlations. This has led several authors to write the Hamiltonian of a granular superconductor at low temperatures as a ferromagnetic planar model[10]

$$H/k_B T = \sum_{\vec{r},\vec{r}'} J_{\vec{r},\vec{r}'} \cos(\theta_{\vec{r}} - \theta_{\vec{r}'}), \tag{1}$$

where k_B is the Boltzmann constant (set to 1 from now on) and T is the temperature. The variable $J_{\vec{r},\vec{r}'}$ is related to the Josephson coupling between grains. Early approximations[10] took $J_{\vec{r},\vec{r}'}$ as a nearest-neighbor (nn) constant. However, in a more realistic model $J_{\vec{r},\vec{r}'}$ is a random variable arising from the nonuniform size and shape distribution of the grains. Here $J_{\vec{r},\vec{r}'}$ will be taken as a nn independent random variable (irv). The justification for taking $J_{\vec{r},\vec{r}'}$ as a nn variable is similar in spirit to that given by Edwards and Anderson to justify their model for a spin-glass.[9] Start by defining $J_{\vec{r},\vec{r}'}$ as

$$J_{\vec{r},\vec{r}'} = J_0 \exp(-z_{\vec{r},\vec{r}'}). \tag{2}$$

