K-S mode appears first on the outside of the field null after an ion transit time. Since the gradient in density is directed towards the null, any acceleration towards the null has the correct sign for the K-S mode. We have run simulations with these new parameters and find that this picture is qualitatively correct.

While our simulations have focused upon explaining the formation process in FRX experiments, the results have more general significance. If we consider the reconnection rate, defined in the usual manner in the reconnection literature,[8] as the influx rate of fluid toward the X point, we find that this velocity is comparable to the Alfvén velocity. We have already noted in our simulations that self-consistent toroidal field production is necessary for rapid reconnection. This result strongly suggests that it is a rotational discontinuity in the magnetic field, as suggested by Hameiri,[7] and not the slow magnetosonic shock which is responsible for the reconnection. The simple MHD model contains no mechanism that will produce self-consistent B_θ in axisymmetry, if it is not initially present and if there is no initial toroidal flow. In a non-MHD fluid description, it is the gyroviscous stress terms in the pressure tensor which are responsible for finite $\dot{B}_\theta$. This kinetic effect produces B_θ scale lengths on the order of the ion gyroradius and can provide a channel for enhanced diffusion of B_z. The axial field is rotated into small scale B_θ structures by the action of the K-S mode and ion kinetic effects. Because of rapid spatial variation, the field can now be dissipated much more rapidly by classical resistivity.

[1]J. H. Irby, J. F. Drake, and H. R. Griem, Phys. Rev. Lett. 42, 228 (1979).

[2]D. W. Hewett, J. Comp. Physiol. 38, 378 (1980).

[3]Z. A. Pietrzyk, in Proceedings of the Third Symposium on Physics and Technology of Compact Toroids, Los Alamos, New Mexico, 1980 (unpublished).

[4]R. D. Milroy and J. U. Brackbill, in Proceedings of the Third Symposium on Physics and Technology of Compact Toroids, Los Alamos, New Mexico, 1980 (unpublished).

[5]W. T. Armstrong, R. K. Linford, J. Lipson, D. A. Platts, and E. G. Sherwood, to be published.

[6]C. E. Seyler, Phys. Fluids 22, 2324 (1979).

[7]E. Hameiri, J. Plasma Phys. 22, 245 (1979).

[8]V. M. Vasyliunas, Rev. Geophys. Space Phys. 13, 303 (1975).

Nonlinear Theory of the $m = 1$ Mode in Hot Tokamak Plasmas

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The nonlinear behavior of the $m=1$ instability is investigated within two-fluid theory. In contrast to the purely resistive case, the mode is found to saturate at a finite amplitude if diamagnetic effects are sufficiently strong. The main stabilizing process is a nonlinear azimuthal shear flow.

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In toroidal current-driven plasma configurations, the mode with dominant poloidal- and toroidal-mode numbers $(m, n) = (1, 1)$ plays an important role. It is believed to give rise to the so-called sawtooth oscillations or internal disruptions, which periodically mix the central with the surrounding plasma. Since in ideal-magnetohydrodynamics theory the $m=1$ mode is either stable,[1,2] or, if unstable, saturates at a small amplitude,[3] nonideal effects dominate the nonlinear behavior. Of these, resistivity η is the most important for low temperatures. In this regime the nonlinear properties are quite well understood. Computer simulations[4] have confirmed the picture given by Kadomtsev,[5] i.e., a magnetic island growing without saturation until filling the whole volume inside the original $q=1$ surface. It is said that the mode leads to complete reconnection of the helical magnetic flux enclosed. This process gives a very plausible explanation of the internal disruption.

For higher plasma temperatures, however, further effects such as diamagnetic drifts have to be considered. Their main influence on the linear $m=1$ mode is a reduction of the growth rate, if the diamagnetic frequency exceeds the resistive growth rate, $\omega_* > \gamma_r$,[6,7] quite similar to that in the case of $m \geq 2$ tearing modes.[8] The nonlinear properties in this regime, however, have not yet been studied. Instead it had tacitly been assumed that, as for $m \geq 2$, diamagnetic effects do not change the general nonlinear behavior.[9]

Motivated by recent experiments such as PLT[10] and ASDEX,[11] where under certain conditions quasisteady $m=1$ oscillations have been observed, we have investigated the $m=1$ instability in the nonlinear regime including diamagnetic drifts, viscosity, and plasma diffusion. Our model equations are based on two-fluid theory, hence neglecting kinetic effects. It is true that recent advances in the linear theory have shown the importance of a kinetic treatment of the electrons.[12,13] But kinetic electron processes, which are confined to a tiny layer around the unperturbed resonant surface, become strongly modified and probably less effective even for very small amplitudes. In addition anomalous processes, which dominate electron transport in all known tokamaks, strongly increase the electron collisionality as compared with the classical value. We therefore believe that the two-fluid approach is a reasonable basis to describe the gross nonlinear behavior of the $m=1$ mode in a hot tokamak plasma.

Since the model equations have already been discussed previously (see, e.g., Ref. 7), we only briefly outline the main approximations implied. Regarding geometry we restrict ourselves to the lowest order in the so-called tokamak expansion with the inverse aspect ratio $a/R \ll 1$ and the ratio of poloidal to toroidal magnetic field components $B_\theta/B_t \sim a/R$. The equilibrium has cylindrical symmetry, while the perturbed state is helically symmetric, depending only on r and $\theta - r/R$. We note that in this approximation the ideal $m=1$ mode is marginally stable. The magnetic field is written in terms of the helical flux ψ and the toroidal field $B_t = B_0 = \text{const}$:

$$\vec{B} = \vec{h} \times \nabla\psi + B_0\vec{h} \cong \hat{z} \times \nabla\psi + B_0\vec{h}, \tag{1}$$

where $\vec{h} \cong \hat{z} + (r/R)\hat{\theta}$ is a vector in the symmetry direction. With neglect of the electron inertia and electron viscosity and under the assumptions that $\nabla_\parallel T_e = 0$ and $\nabla p_i = \gamma_i T_i \nabla n$, the equation for ψ becomes

$$\partial\psi/\partial t + \vec{u}\cdot\nabla\psi = \eta j + \alpha[(T_e + \gamma_i T_i)/n]\hat{z}\cdot(\nabla n \times \nabla\psi), \tag{2}$$

where $j = \nabla^2\psi + 2B_0/R$. The incompressible part $\vec{u} = \hat{z} \times \nabla\varphi$ of the ion velocity is determined by the vorticity $W = \nabla\cdot(n\nabla\varphi)$, which obeys the equation

$$\partial W/\partial t + (\vec{u} - \vec{v}_{i*})\cdot\nabla W - \hat{z}\cdot(\nabla n \times \nabla\tfrac{1}{2}u^2) = \vec{B}\cdot\nabla j + \mu\nabla^2 W, \tag{3}$$

while the equation for the particle density n ($=n_e = n_i$) is

$$\partial n/\partial t + \vec{u}\cdot\nabla n = \alpha\vec{B}\cdot\nabla j + \kappa\nabla^2 n. \tag{4}$$

The terms $\mu\nabla^2 W$ and $\vec{v}_{i*}\cdot\nabla W$ (with $\vec{v}_{i*}$, the ion diamagnetic velocity) represent the effects of collisional viscosity and gyroviscosity, while $\kappa\nabla^2 n = -\langle\nabla\cdot\tilde{\vec{v}}_e\tilde{n}\rangle$ takes into account anomalous particle diffusion due to microfluctuations not contained in the macroscopic description. Equations (2)–(4) are written in dimensionless form, with use as units of a (the wall radius), $n(r_s)$ (with r_s, the radius of the resonant surface), $B_\theta(a)$, and the poloidal Alfvén velocity v_A. The equations contain several small diffusion coefficients, η, μ, and κ; typical values in tokamaks are $\eta \sim 10^{-7}$–10^{-6}, $\mu \sim 0.1\eta$ for classical viscosity, and $\kappa \gtrsim 10\eta$ is the effective particle diffusion. A further small parameter is $\alpha = (c/\omega_{pi})B_\theta(a)/B_0$, which together with $T_{e,i}$ measures the magnitude of the diamagnetic frequencies $\omega_{e*} = -\alpha T_e n'/n$ and $\omega_{i*} = \alpha\gamma_i T_i n'/n$.

We have solved Eqs. (2)–(4) numerically for various values of the parameters using the equilibrium current profile $j_0 = 2(1+s^{-2})/(1+r^2/s^2)^2$ with $s=0.6$ and $q(1)=3.4$, hence $q(0)=0.9$ and $r_s = 0.2$, and choosing a typical bell-shaped density profile. The numerical method uses a finite difference scheme in r and a Fourier decomposition in θ. The principal result is that for large diamagnetic frequencies, such that $\omega_{e*}/\gamma_r \gtrsim 2$, the Kadomtsev picture of complete flux reconnection is no longer valid. Instead the instability saturates at a finite island size. Figure 1(a) shows the evolution of the flux perturbation $\tilde{\psi}(r_s)$ for $\eta = 5\times10^{-7}$, $\mu = 10^{-7}$, $\kappa = 10^{-5}$, and $\omega_{e*}/\gamma_r = 2.7$. The saturated width of the magnetic island is $0.1r_s$.

In order to identify the main saturation process we neglect higher-order harmonics, i.e., consider the quasilinear approximation, which we find to give qualitatively the same nonlinear behavior as the exact equations. By artificially freezing two of the average profiles $j_0(r)$, $n_0(r)$, and $W_0(r)$, we find that the quasilinear change δW_0 provides the dominant stabilizing effect, Hence, in contrast to $m \geq 2$ tearing modes, which saturate by a change of the current profile δj_0, inertia plays

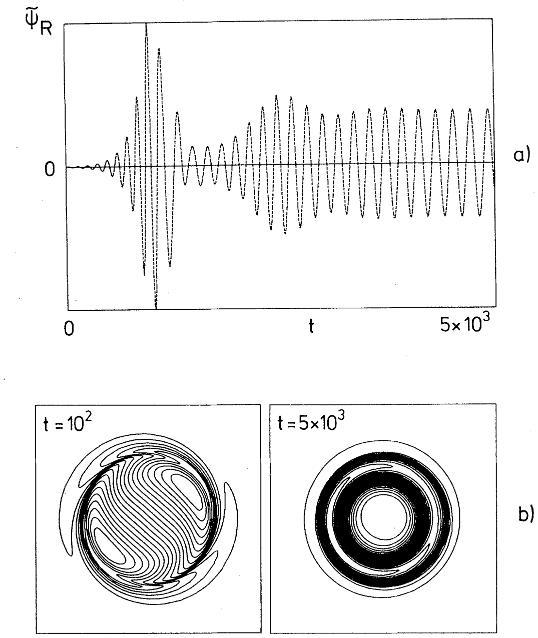

FIG. 1. (a) Time evolution of the real part of the perturbed flux $\tilde{\psi}(r_s)$. (b) Convection pattern $\varphi(r,\theta)$ in the linear phase, $t=10^2$, and at saturation, $t=5\times10^3$.

the most important role in saturating the $m=1$ instability. Figure 1(b) shows the convection pattern $\varphi(r,\theta)$ in the linear phase and at saturation.

Starting from these results we set up a quasilinear description, restricting ourselves to the regime $\omega_* \gg \gamma_r$, where amplitudes are small, as we shall see. From Eq. (4), we obtain

$$\partial W_0/\partial t \simeq \langle \hat{z}\cdot\nabla\tilde{\psi}\times\nabla\tilde{j}\rangle_\theta + \mu W_0'', \qquad (5)$$

where $\tilde{\psi}$ in the nonlinear term is approximated by the linear eigenfunction[6]

$$\tilde{\psi} = \psi_s[\exp(-Ax^2/2) - Ax\int_x^\infty \exp(-Ay^2/2)dy], \quad (6)$$

where $A=[\gamma + i(\omega_* - \omega)/\eta]$, $\omega_* = \omega_{e*} - \omega_{i*}$, $x = r - r_s$. For $\mu \ll \eta$ the viscosity in Eq. (5) is negligible. Integration yields $\delta\varphi_0' \simeq \mathrm{Im}\{\tilde{\psi}\tilde{\psi}''^*\}/\gamma r_s$, from which one computes $\delta W_0' \simeq \delta\varphi_0'''$ at $x=0$, with use of (6),

$$\delta W_0' \cong (2/r_s)[(\omega_* - \omega)/\eta^2]\,|\psi_s|^2. \qquad (7)$$

It is interesting to compare the magnitude of the nonlinear flows for $m=1$ and $m\geq 2$. In Ref. 14, it was derived that

$$\delta\varphi_0' \cong (m/r_s)[(\omega - \omega_*)/\gamma\eta]\,|\psi_s|^2. \qquad (8)$$

This is small for $m \geq 2$ since the frequency mismatch is small, $\omega - \omega_* = -\gamma^2/\omega_*$, but is large for $m=1$, where $\omega - \omega_* \sim \omega_*$.[7] Writing Eqs. (2)–(4) for the perturbed quantities $\tilde{\psi}$, $\tilde{\varphi}$, and $\tilde{n}$, substituting $\tilde{n}$ from (3), and assuming a time dependence $\exp\{\int^t \Gamma dt'\}$, we obtain

$$(\Gamma + i\omega_*)[\tilde{\psi} - i(xF/\Gamma)\tilde{\varphi}] = \eta\tilde{\psi}'', \qquad (9)$$

$$(\Gamma - i\omega_{i*})\tilde{\varphi}'' - (i/r_s)\delta W_0'\tilde{\varphi} = ixF\tilde{\psi}'', \qquad (10)$$

considering only the quasilinear change δW_0. All coefficients are taken at $r=r_s$, $F=\psi_0''(r_s)/r_s$. To estimate the stabilizing effect of $\delta W_0'$, it is convenient to replace ψ'' in (10) by the left-hand side of (9),

$$\tilde{\varphi}'' - \left[\frac{i}{r_s}\delta W_0' + \frac{x^2F^2}{\eta}\frac{\Gamma + i\omega_*}{\Gamma}\right]\frac{1}{\Gamma - i\omega_{i*}}\tilde{\varphi} = i\frac{xF}{\eta}\frac{\Gamma + i\omega_*}{\Gamma - i\omega_{i*}}\tilde{\psi}. \quad (11)$$

The imaginary part of the expression in brackets in (11) is the only term where $\delta W_0'$ enters. Since $\mathrm{Im}\{(\Gamma + i\omega_*)/\Gamma\} \cong \gamma\omega_*/\omega^2$, one has, for the bracketed factor in Eq. (11),

$$\mathrm{Im}\{[\ ]\} = r_s^{-1}\delta W_0' + (x^2F^2/\eta)\gamma\omega_*/\omega^2. \qquad (12)$$

To obtain the value of $\delta W_0'$ necessary to change the growth rate substantially, we set (12) equal to $x^2F^2\omega_*\gamma_0/\eta\omega^2$, $\gamma_0 = \gamma_r^3/\omega_{e*}\omega_{i*}$ being the growth rate for $\delta W_0' = 0$. For $\gamma = 0$ this relation becomes

$$r_s^{-1}\delta W_0' = (x_0^2F^2/\eta)\gamma_0\omega_*/\omega^2, \qquad (13)$$

which yields an estimate of the saturation amplitude ψ_s, where x_0 is of the order of the current layer width $\delta_s = (\eta/\gamma_0)^{1/2} = \omega_*/F$ or the island half width $\delta_I = 2(\psi_s/Fr_s)^{1/2}$, whichever is larger. It can easily be seen from (13), with use of $\delta W_0'$ from (7), that $\delta_s > \delta_I$ for $\omega_* > \gamma_0$. With $x_0 = \delta_s$ we find the saturation amplitude

$$\psi_s \cong r_s F\eta/\omega_*. \qquad (14)$$

The result is in good agreement with the numerical simulations for $\omega_* \gtrsim 3\gamma_r$. It is interesting to note that Eq. (13) implies a nonlinear flow of the order of the diamagnetic velocity $r_s\omega_*$ confined to a region δ_s around r_s.

For anomalously large viscosity $\mu > \eta$, the left-hand side of (5) can be neglected. A crude estimate of the saturation amplitude in this case gives the somewhat larger value $\psi_s \cong r_sF(\eta\mu)^{1/2}/\omega_*$, which results from the reduction of the nonlinear flows due to viscosity.

In the transition region $\omega_* \sim \gamma_r$, where ψ_s rapidly increases with decreasing ω_*, plasma diffusion is important in preventing quasilinear flattening of the density profile, which would otherwise quench the diamagnetic frequency as discussed in Ref. 14, and thus lead into the regime $\omega_* < \gamma_r$, where complete reconnection occurs. Hence for large amplitude the saturation value depends strongly on κ. It also appears from the simulations that in the transition regime the saturation process as described here does not lead to a completely stationary island configuration, but that growth continues, though on a much slower time scale.

In conclusion, we have shown by numerical simulations as well as analytical considerations that for sufficiently large diamagnetic drifts the $m=1$ resistive kink mode in a tokamaklike configuration saturates at small island size. In contrast to $m \geq 2$ tearing modes, the saturated configuration does not represent an equilibrium with $j = j(\psi)$, but contains a strong azimuthal flow of the order and in the direction of the electron diamagnetic velocity. Applying the results to tokamak experiments, we see that internal disruptions only occur when the current peaking and the quasilinear density flattening is large enough, so that $\gamma_r \gtrsim \omega_{e*}$. It may, however, happen that the presence of a finite $m=1$ island increases the energy loss from the central part in such a way that γ_r will not grow any further. This would lead to (quasi-) stationary $m=1$ oscillations as observed recently.[10,11]

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[1] M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. 35, 1638 (1975).

[2] W. Kerner, R. Gruber, and F. Troyon, Phys. Rev. Lett. 44, 536 (1980).

[3] M. N. Rosenbluth, P. Y. Dagazian, and P. H. Rutherford, Phys. Fluids 16, 1894 (1973).

[4] A. Sykes and J. A. Wesson, Phys. Rev. Lett. 37, 140 (1976); B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nucl. Fusion 16, 528 (1976).

[5] B. B. Kadomtsev, Fiz. Plazmy 1, 710 (1975) [Sov. J. Plasma Phys. 1, 389 (1975)].

[6] B. Basu and B. Coppi, Massachusetts Institute of Technology Report No. PRR-76/38, 1976 (unpublished).

[7] B. V. Waddell, G. Laval, and M. N. Rosenbluth, Oak Ridge National Laboratory Report No. ORNL/TM-5968, 1977 (unpublished).

[8] H. P. Furth and P. H. Rutherford, Princeton University Report No. MATT-872, 1971 (unpublished).

[9] G. L. Jahns, M. Soler, B. V. Waddell, J. D. Callen, and H. R. Hicks, Nucl. Fusion 18, 609 (1978).

[10] N. R. Sauthoff, S. von Goeler, D. R. Eames, and W. Stodiek, in Proceedings of the IAEA Symposium on Current Disruptions in Toroidal Devices (to be published), paper C5, Max-Planck-Institute für Plasmaphysik, Garching, Report No. IPP-3/51, 1979. (PLT, Princeton Large Torus.)

[11] W. Engelhardt, private communication. (ASDEX, axisymmetric divertor experiment.)

[12] J. F. Drake, Y. C. Lee, L. Chen, P. H. Rutherford, P. K. Kaw, J. Y. Hsu, and C. S. Liu, Nucl. Fusion 18, 1583 (1978).

[13] B. Basu and B. Coppi, Massachusetts Institute of Technology Report No. PRR-79/15, 1979 (unpublished).

[14] D. Biskamp, Nucl. Fusion 19, 777 (1979).

Magnetic Trapped-Particle Modes

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It is shown that very-low-frequency magnetic modes may be unstable for systems such as tandem mirrors which contain plasma trapped in regions of unfavorable curvature. Onset of the instability occurs when the diamagnetic plasma pressure is sufficient to reverse particle drift velocities.

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At very low frequencies the motion of plasma is no longer governed by the familiar $\vec{E} \times \vec{B}/B^2$ drift, but rather by the magnetic (∇B and curvature) drifts. If the magnetic field is perturbed, the drift orbits of course change, and we may give the following crude picture of an instability if $|\vec{B}|$ is perturbed by an amount $B_1(x)$. Trapped particles (with small $v_\parallel$) tend to move on constant-B surfaces. Thus the pressure of these particles is nearly a function of $|B|$ and the perturbed pressure due to the fraction, f, of these trapped particles is $p_1 = +fB_1|\nabla p_0|/|\nabla B_0|$. The

