Nonlinear Saturations of Parasitic Instabilities in High-Efficiency Free-Electron Lasers

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A broad spectrum of unstable parasitic waves can arise from the interaction between the trapped electrons and the radiation generated by a free-electron laser. Imposing a dc electric field with appropriate strength at the onset of trapping can substantially narrow the unstable spectrum and allows considerable enhancement in the radiation intensity. The nonlinear mechanisms which limit the enhancement process are observed to be due to nonlinear frequency shift and detrapping.

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The idea[1] of generating tunable, high-power coherent radiations by passing a relativistic electron beam through a rippled magnetic field, due to the recent advance in the accelerator technology, finally becomes[2,3] a reality. The efficiency of a free-electron laser is intrinsically limited because the growth of the ponderomotive force produced by the interaction of the rippled magnetic field and the signal wave will eventually trap the electrons. There are several schemes which theoretically could substantially enhance the efficiency of a free-electron laser. For instance, if the strength of the rippled magnetic field is increased just before saturation,[4] the depth of the ponderomotive potential well becomes deeper which allows the electrons to give up more energy to photons. Another scheme, suggested by several groups,[5,6] is a variable wiggler, in which the strength and/or the period of the magnet is tapered to maintain the electrons in resonance with photons throughout the length of the device, thereby increasing energy extraction. Still another alternative[7,8] is applying a dc electric field with proper strength at the time of saturation, thereby clamping the trapped electrons in the decelerating phase which in turn transforms the dc energy into high-frequency radiations. Most of the efficiency enhancement calculations use a single-mode approximation which prohibits the parasitic instability[9,10] to occur. In this Letter, we will demonstrate, by using particle simulation (multimodes), that the enhancement process is ultimately terminated by the generation of a parasitic instability due to the interaction of the trapped electrons and the enhanced signal wave. This parasitic instability will play an important role in determining the maximum output power which can be achieved from a free-electron laser. It will be shown later that a considerable amount of improvement in output power can still be achieved by carefully choosing the strength and the turn-on time of the dc electric field.

To study the dynamics of this highly nonlinear process, a $1\frac{2}{2}$-dimensional electromagnetic particle code with periodic boundary condition is used. The parameters are the following: $\gamma_0=(1-\beta^2)^{-1/2}=3$, $ck_0=1.52\,\omega_p$, $P_t/P_0=10^{-3}$, and $\omega_c=qB_r/m_0c=0.53\omega_p$, where $\beta=V_0/c$, V_0 and c are, respectively, the beam velocity and speed of light, k_0 is the wave number of the rippled field, P_t is the beam momentum spread, and B_r is the rippled field strength. This set of parameters corresponds to a relativistic electron beam and a density of 10^{12} cm^{-3}, an energy of 1.5 MeV, and a current of 5 kA/cm^2. The period and the strength of the static magnetic field are, respectively, 2.2 cm and 1.2 kG. The growth rate and the efficiency of this case are $0.07\omega_p$ and 8%, respectively. The trapping of the electrons by the total longitudinal potential wave causes the saturation to occur at $\omega_p t \simeq 150$. This trapping process becomes evident in Fig. 1(a) (dashed lines) which exhibits oscillatory behavior in the time evolution of the signal wave ($ck_s=15.2\omega_p$). The oscillation frequency is at the particle bouncing frequency ω_b, which is given by

$$\omega_b \approx k_l (q\varphi_0/m_0\gamma_0^3)^{1/2}\omega_p, \tag{1}$$

where $ck_l=c(k_0+k_s)=16.7\omega_p$ and φ_0 is the total longitudinal potential. At the onset of trapping, a broad spectrum of parasitic waves with frequencies lower than that of the signal wave becomes unstable. The dashed line in Fig. 1(b) ($ck_9=13.7\times\omega_p$), Fig. 1(c) ($ck_8=12.2\omega_p$), and Fig. 1(d) ($ck_7=10.7\omega_p$) shows the time evolution of the three most unstable parasitic electromagnetic waves. These waves grow with a very large growth rate ($\Gamma_9=0.2\omega_p$, $\Gamma_8=0.17\omega_p$, $\Gamma_7=0.1\omega_p$) which is substantially larger than the original signal growth rate. This instability has been investigated by Kruer, Dawson, and Sudan[9] for large-amplitude electrostatic waves in plasmas and by Kroll, Morton, and Rosenbluth[10] for high-γ_0 free-electron lasers. In this paper the dispersion relation for the parasitic instability will be derived for free-electron lasers following the approach of Ref. 9.

The instability process can be viewed as a Raman scattering of the signal wave off the electrons executing bouncing motion in the ponderomotive potential well. In the small amplitude approximation, the equation describing a driven harmonic oscillator can be used

$$\frac{d^2X_n}{dt^2}+\omega_b^2(X_n-X_{n0}-V_pt)=-\frac{q^2B_r}{m_0c^2k_0\gamma^4}\int\frac{ik_l'A_r(k',\omega')\exp(ik_l'X_n-i\omega't)}{(2\pi)^2}dk'd\omega', \tag{2}$$

where $(X_n-X_{n0}-V_pt)$ is position of the electron relative to the nth trough, $k_l'=k_0+k'$, $V_p=\omega_s/(k_0+k_s)$ is the phase velocity of the ponderomotive potential wave which provides the trapping and A_r is the perturbing vector potential. In writing Eq. (2) the space-charge field is neglected. The density perturbation produced by the perturbed motion of the oscillators coupled with the transverse motion induced by the rippled magnetic field produced a current perturbation which is the source of the parasitic wave. By using some δ-function identities, one obtains

$$A_r(k,\omega)=\frac{\omega_t^2\omega_c^2(k_0+k)\sum_m(k_0+k+mk_p)A_r(k+mk_p,\omega+m\omega_s)}{2k_0^2\gamma_p^5(\Omega^2-\omega_p^2)\epsilon(k,\omega)}, \tag{3}$$

where $\Omega=\omega-(k_0+k)V_p$, $k_p=k_0+k_s$, and ω_t is the plasma frequency for the trapped electrons, $\gamma_p=(1-V_p^2/c^2)^{-1/2}$, and

$$\epsilon(k,\omega)=(\omega^2-c^2k^2-\omega_t^2/\gamma_p). \tag{4}$$

Retaining only the lowest-order coupling in Eq. (3), i.e., $m=0$, we obtain the desired dispersion relation

$$(\Omega^2-\omega_b^2)\left(\omega^2-c^2k^2-\frac{\omega_t^2}{\omega_p}\right)=\frac{\omega_t^2\omega_c^2}{2k_0^2\gamma_p^5}. \tag{5}$$

Substituting the appropriate parameters into Eq. (5) gives $\Gamma_9=0.12\omega_p$, $\Gamma_8=0.1\omega_p$, and $\Gamma_7=0.08\omega_p$, which is smaller than the simulation results ($\Gamma_a=0.22\omega_p$). The discrepancy could be due to the nonlinear frequency shift caused by the difference between the restoring forces for trapped electrons and for untrapped electrons. The instability tends to level off after the unstable wave energy reaches about 10^{-4} times the signal wave energy and eventually saturates because of particle diffusion and detrapping in the phase space (Fig. 2) which destroy the resonance between electrons and parasitic waves.

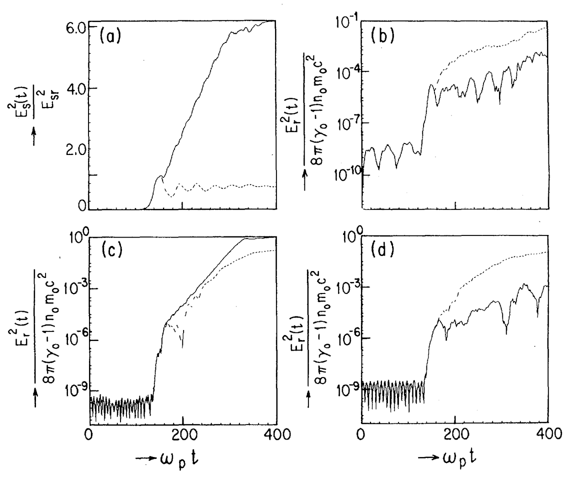

FIG. 1. Time evolution of the electromagnetic wave energies for the cases without imposing a dc electric field (dashed line) and with a dc electric field ($E_0 \simeq E_{sl}$) (a) signal wave, (b) $ck_9=13.7\omega_p$, (c) $ck_8=12.2\omega_p$, and (d) $ck_7=10.7\omega_p$.

When a dc electric field with amplitude $E_0\simeq k_l\varphi_0=E_{sl}$ where φ_0 is the saturated total longitudinal potential ($E_0\approx300$ V/cm) is applied at saturation, the combined action of dc and rf fields causes some of the beam electrons to become runaways while others remain clamped at the decelerating phase of the rf fields [Fig. 3(a)]. The clamped electrons transfer nearly all the dc electric field energy that went into them to the electromagnetic radiation. The escaped beam electrons are lost to the interaction. The gain in output power can in principle be extended indefinitely if the parasitic waves can be prevented from growing. The simulation results show that the unstable parasitic wave spectrum is substantially narrowed upon imposing a dc electric field at $\omega_pt=150$ (Fig. 1 solid line). This is due to the distortion of the potential well and phase shift caused by the dc electric field. In fact only the mode with $ck_r=12.2\omega_p$ remains unstable with a growth rate of $\Gamma_8=0.03\omega_p$ and the signal wave energy is increased to six times the saturation energy (E_{sr}^2) without applying the dc electric field. The en-

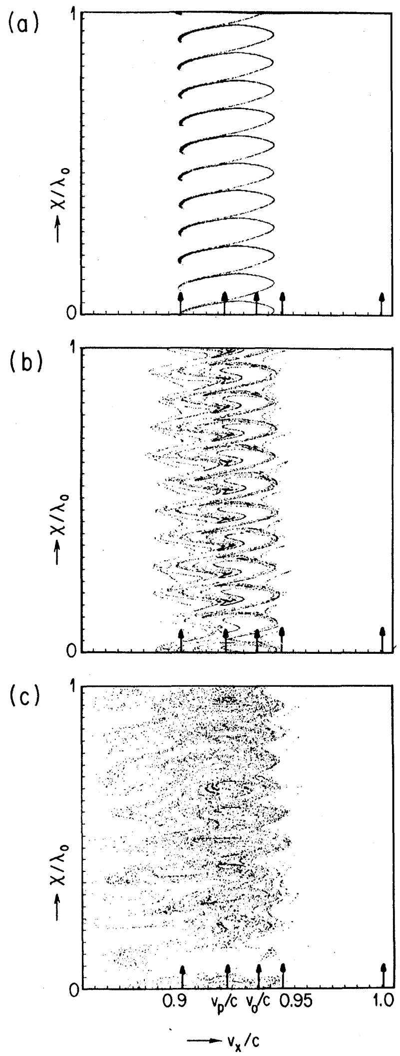

FIG. 2. Time evolution of the phase space for the case without imposing a dc electric field, (a) $\omega_p t$ = 145, (b) $\omega_p t$ = 200, and (c) $\omega_p t$ = 300.

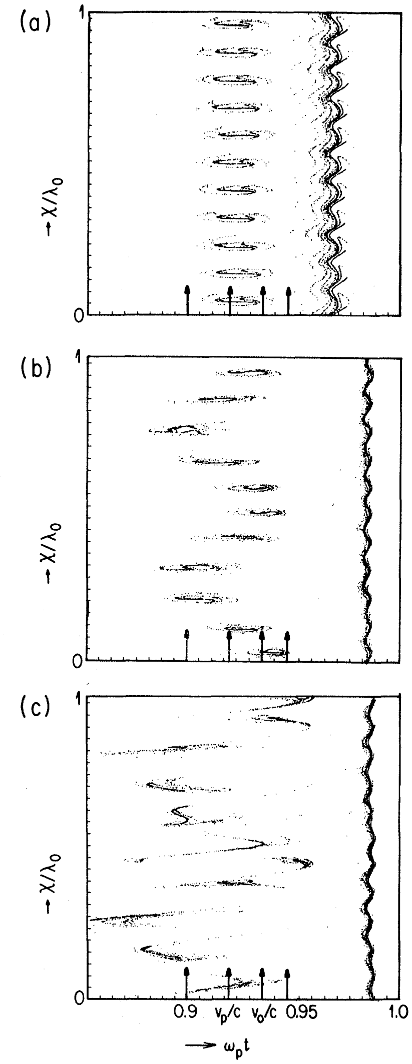

FIG. 3. Time evolution of the phase space for the case with a dc electric field of $E_0 \simeq E_{sl}$, (a) $\omega_p t$ = 210, (b) $\omega_p t$ = 300, and (c) $\omega_p t$ = 325.

hancement process is eventually terminated due to the detrapping of resonant electrons by the ponderomotive wave with $ck_l = c(k_0 + k_r) = 13.7\omega_p$, which is close to the original ponderomotive wave number $c(k_0 + k_s) = 16.7\omega_p$ [Fig. 3(c)]. At the same time the instability also generates a long-wavelength ponderomotive wave with wave number $c(k_s - k_r) = 3\omega_p$ [Fig. 3(b)] which does not cause any detrapping.

The results with $E_0 = 2E_{sl}$ indicate that the parasitic waves are completely suppressed for a while upon imposing the dc electric field and the mode with $ck_r = 12.2\omega_p$ begins to grow at $\omega_p t = 200$ but this mode was saturated without causing any significant detrapping. (See Fig. 4.) The saturation is due to the frequency change [Eq. (1)] introduced by the increase of φ_0. This frequency change also renders the mode with $ck_r = 10.7\omega_p$ to become unstable at $\omega_p t = 450$ and eventually detraps the resonant electrons. In this case the signal wave energy was enhanced to 25 times E_{sr}^2. However the increase rate in this case (only 25% of electrons are trapped) is lower thus requiring a longer system. The overall efficiency is small since the dc electric field spent a large amount of its energy to accelerate the runaway electrons. The efficiency enhancement factor can be substantially increased if the runaway elec-

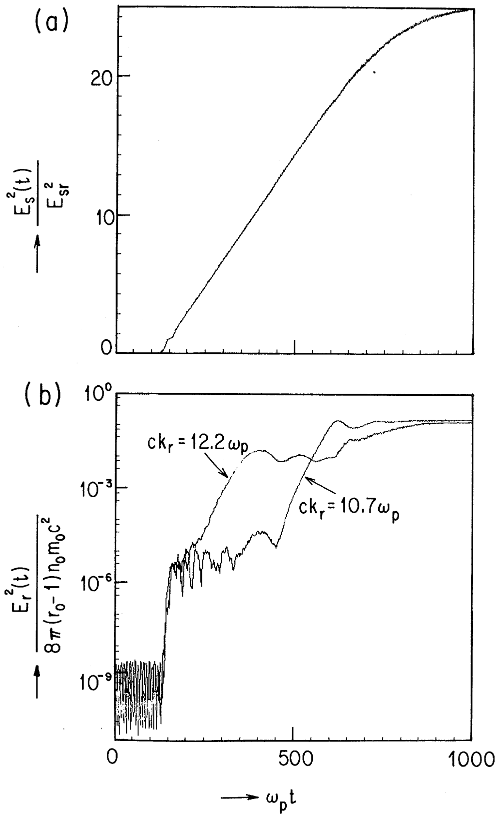

FIG. 4. Time evolution of the electromagnetic wave energies for the case with $E_0 \simeq 2E_{sl}$, (a) signal wave and (b) $ck_8 = 12.2\omega_p$ and $ck_7 = 10.7\omega_p$.

trons are scraped off the system.

The simulation results indicate that the output power and the efficiency of a free-electron laser can be substantially improved by applying an appropriate strength of a dc electric field at saturation. The enhancement process is ultimately terminated by the detrapping of resonant electrons caused by the parasitic instability.

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