Shell-Model Description of the Interacting-Boson–Model Parameters for Two Nondegenerate *j* Shells

Philip D. Duval and Bruce R. Barrett
Department of Physics and Theoretical Physics Division, University of Arizona, Tucson, Arizona 85721
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The interacting-boson–model parameters are examined from a shell-model viewpoint with use of the generalized seniority scheme in two nondegenerate j shells. Predictions for the interacting-boson–model parameters are obtained by constructing the boson image of the corresponding fermion operator and are found to be in good overall agreement with the empirically determined values.

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The interacting-boson model (IBM) of Arima and Iachello has been extremely successful in phenomenologically describing the properties of low-lying energy levels of medium to heavy-mass nuclei.[1] There is currently a great deal of interest in establishing the connection of the IBM to the nuclear shell model, by determining the microscopic structure of the bosons of the IBM.[2,3] When the structure of the boson is known in terms of the underlying fermion space, a relationship can be established between the fermion effective Hamiltonian and the assumed form for the IBM Hamiltonian. This can be done with use of the generalized seniority scheme.[4] So far, all calculations of the IBM parameters have been carried out under the assumption that the valence nucleons occupy a single j shell, with occupancy, $2j+1$, equal to that needed to fill the major shell (e.g., $j=\frac{31}{2}$ for the 50–82 shell). This turns out to be essentially equivalent to the assumption that the single-particle levels are degenerate.

Although the predicted IBM parameter trends with use of the single-j-shell approximation are roughly in agreement with empirically determined values, there are sometimes marked deviations. In this Letter we will show that we can improve the reproduction of the empirical IBM parameters by allowing the valence nucleons to occupy two nondegenerate j shells.

We present a new method of relating the bosons of the IBM to the underlying fermion space for nondegenerate j shells. Other methods exist, such as that of Otsuka and Arima,[5] who used the so-called "number-operator approximation." We do not use this method since it "averages" over the subshell effects in which we are interested and has not been tested in the midshell region. Furthermore, we do not use the technique of Klein and Vallieres,[6] who assign a boson correspondence with each kind of fermion pair.

In the neutron-proton IBM (sometimes called IBM-2), correlated valence neutron pairs and proton pairs are assumed to be neutron and proton bosons of $J=0$ (s bosons) and $J=2$ (d bosons). Particle bosons are counted from the beginning to the middle of a shell; hole bosons, from the middle to the end. For example $^{174}_{74}W_{100}$ has nine neutron-particle bosons and four proton-hole bosons. The properties of nuclei are assumed to arise from interactions among the bosons. The simplest IBM Hamiltonian which incorporates the essential features of the underlying fermionic interactions is of the form[2]

$$H=\epsilon\hat{n}_d+\hat{T}_\pi\cdot\hat{T}_\nu, \tag{1}$$

where ϵ is related to the difference in the s- and d-boson energies and $\hat{n}_d$ is the d-boson number operator. The terms $\hat{T}_\pi$ and $\hat{T}_\nu$ are the proton-boson and neutron-boson quadrupole operators, respectively, which are given by

$$\hat{T}_\rho=\kappa_\rho[(s_\rho^\dagger\tilde{d}_\rho+d_\rho^\dagger s_\rho)^{(2)}+\chi_\rho(d_\rho^\dagger\tilde{d}_\rho)^{(2)}], \quad \rho=\pi,\nu, \tag{2}$$

where χ_ρ is the ratio of the seniority-conserving part of the quadrupole operator to the seniority-nonconserving part, and κ_ρ gives the strength of the quadrupole operator. The strength of the quadrupole *interaction* is given by the product $\kappa=\kappa_\pi\kappa_\nu$, so that Eq. (1) contains only four parameters, ϵ, κ, χ_ν, and χ_π.

To establish the connection with the nuclear shell model, we must first find the correlated-pair states which correspond to the s and d bosons and then equate matrix elements between the fermion space and the boson space, which will establish the structure of the four IBM parameters mentioned above.

If we look at the actual single-particle levels for neutrons in the 50–82 shell and in the 82–126 shell,[7] we see that in the midshell neutron and proton regions the levels seem to naturally form two groups, suggesting a two j-shell analysis. In most of the 50–82 shell the $g_{7/2}$ and the $d_{5/2}$ levels lie close together and are fairly well separated from the $d_{3/2}$, $h_{11/2}$, and $s_{1/2}$ levels. This leads to the choice of $j_1 = \frac{13}{2}$ and $j_2 = \frac{17}{2}$ for the two j shells, found by matching the occupancy of the j shells with the total occupancy of the corresponding groups of single-particle levels. Similarly, in the 82–126 shell, the $h_{9/2}$ and $f_{7/2}$ levels form a $j_1 = \frac{17}{2}$ shell and the $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, and $i_{13/2}$ levels form a $j_2 = \frac{25}{2}$ shell.

The fermion $J=0$ pair creation operator for two nondegenerate levels can be written as

$$A^{\dagger(0)} = b_1 A_1{}^{\dagger(0)} + b_2 A_2{}^{\dagger(0)}, \tag{3}$$

where b_1 and b_2 are the probability amplitudes for levels 1 and 2, respectively (i.e., $b_1^2 + b_2^2 = 1$), and

$$A_i{}^{\dagger(0)} = [a_i{}^\dagger a_i{}^\dagger]^{(0)} = \sum_m a_{im}{}^\dagger a_{i-m}{}^\dagger.$$

If the levels are degenerate, it is clear that $b_1 = (\Omega_1/\Omega)^{1/2}$ and $b_2 = (\Omega_2/\Omega)^{1/2}$, where $\Omega = \Omega_1 + \Omega_2$, and $\Omega_i = \frac{1}{2}(2j_i + 1)$ is half the level degeneracy. This is apparent, since neither level is favored if they are degenerate, and therefore the probability of finding a pair in level 1 or 2 is proportional to its occupancy. We now define

$$\alpha_i \equiv b_i (\Omega_i/\Omega)^{-1/2}. \tag{4}$$

Substitution into Eq. (3) gives

$$\Omega^{1/2} A^{\dagger(0)} = \alpha_1 \Omega_1{}^{1/2} A_1{}^{\dagger(0)} + \alpha_2 \Omega_2{}^{1/2} A_2{}^{\dagger(0)}$$

or

$$S_+ = \alpha_1 S_{1+} + \alpha_2 S_{2+}, \tag{5}$$

where S_1 and S_2 are the usual quasispin operators[8] for levels 1 and 2. Equation (4) and the normalization condition on the b_i give

$$\alpha_1{}^2 \Omega_1 + \alpha_2{}^2 \Omega_2 = \Omega. \tag{6}$$

The fermion $J=2$ pair creation operator is given by

$$D_\mu{}^\dagger = P A_\mu{}^{\dagger(2)}, \tag{7}$$

where

$$A_\mu{}^{\dagger(2)} = \sum_{i,j=1}^{2} \beta_{ij} A_{ij\mu}{}^{\dagger(2)} = \sum_{i,j=1}^{2} \beta_{ij} \sum_{mm'} (im\, jm' | 2\mu) a_i{}^\dagger a_j{}^\dagger,$$

and P is a projection operator which ensures that the seniority is increased by two units under the action of $D_\mu{}^\dagger$. In our present work we will take

$$D_\mu{}^\dagger = b_1 D_{11\mu}{}^\dagger + b_2 D_{22\mu}{}^\dagger, \tag{8}$$

where

$$D_{ij\mu}{}^\dagger = P A_{ij\mu}{}^{\dagger(2)},$$

i.e., we do not include a mixed term $b_{12} D_{12\mu}{}^\dagger$ and take the probability amplitudes the same as for the $J=0$ pair creation operator. We realize that these are both major assumptions but feel that they are a reasonable starting point to investigate the effects produced in a two–nondegenerate–j-shell calculation. The first approximation greatly simplifies the angular momentum algebra, and the second reduces the number of parameters in our calculation. In a forthcoming paper[9] we will discuss in more detail the validity of these approximations and how they can be improved.

We are now in a position to determine the trends of the IBM parameters ϵ, κ, χ_ν, and χ_π as a function of neutron and proton number. As was done in the single–j-shell approximation,[2] we can find the form of κ_ρ and χ_ρ ($\rho = \pi, \nu$) by equating matrix elements of the quadrupole operator in the fermion space with the corresponding matrix elements in the boson space. That is, we set

$$\langle s^n \| \hat{T}_\rho \| s^{n-1} d \rangle_B = \langle S^N \| \hat{q}_\rho \| S^{N-1} D \rangle_F$$

and

$$\langle s^{n-1} d \| \hat{T}_\rho \| s^{n-1} d \rangle_B = \langle S^{N-1} D \| \hat{q}_\rho \| S^{N-1} D \rangle_F,$$

where n is the number of bosons and N is the number of fermion pairs. The subscripts B and F denote a boson matrix element and a fermion matrix element, respectively. The fermion quadrupole operator is given by

$$\hat{q}_\rho = \sum_{i,j} \langle i | q_\rho{}^{(2)} | j \rangle a_i{}^\dagger a_j.$$

The fermion states $|S^N\rangle_F$ and $|S^{N-1}, D\rangle_F$ are formed by setting

$$|S^N\rangle_F = \mathfrak{N}_N{}^{-1} S_+{}^N |0\rangle_F$$

and

$$|S^{N-1}, D\rangle_F = \mathfrak{N}_{N-1}{}^{-1} S_+{}^{N-1} (D^\dagger/\sqrt{2}) |0\rangle_F,$$

where $\mathfrak{N}_N$ and $\mathfrak{N}_{N-1}$ are normalization constants.

In a straightforward manner we find

$$\kappa_\rho \sqrt{n} = \frac{1}{\mathfrak{N}_N \mathfrak{N}_{N-1}} \frac{\sqrt{10}}{N} \sum_{n,m}^{N} T_{nm} \left(C_1 \frac{m(\Omega_1 - m)}{\Omega_1(\Omega_1 - 1)} + C_2 \frac{n(\Omega_2 - n)}{\Omega_2(\Omega_2 - 1)} \right), \tag{9}$$

where

$$C_i = \frac{\langle r^2\rangle_i}{\sqrt{\pi}}\Omega_i(-1)^{\Omega_i}\begin{pmatrix} j_i & 2 & j_i \\ -\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}\left(\frac{\Omega_i}{\Omega}\right)^{1/2}, \quad T_{nm} = \frac{N!\alpha_1^{2m}\alpha_2^{2n}\Omega_1!\Omega_2!}{n!m!(\Omega_1-m)!(\Omega_2-n)!},$$

and $\langle r^2\rangle_i$ is the mean square radius in level i. The restrictions on the above summation are that $n+m=N$, $m\leq\Omega_1$, and $n\leq\Omega_2$. The expression for χ_ρ follows in a similar manner but is more complex in its structure. The prediction for the parameter ϵ is the same as that for the single j shell, namely that ϵ is independent of neutron number. This is due to the fact that ϵ gives the difference in energy of a D pair and an S pair, and we assume the single-particle levels to be constant in energy (constant α_1 and α_2) throughout the shell.

As α_1 and α_2 are varied from the degenerate case ($\alpha_1=\alpha_2=1$) to the case where level 1 is much more favored over level 2 ($\alpha_1\gg\alpha_2$), it turns out that most of the effect is manifest in the parameter χ_ρ. In Fig. 1, we show different curves of χ_ν versus neutron number, where each curve represents different values of α_1 and α_2 for the case of $j_1=\frac{13}{2}$ and $j_2=\frac{17}{2}$ (i.e., the 50–82 shell).

We have diagonalized a simple δ-function pairing interaction in both the 50–82 and 82–126 neutron-shell–model spaces in order to get an estimate of reasonable values for α_1 and α_2 in these shells. In Fig. 2, we show the predicted values of χ_ν versus neutron number for α_1 and α_2 determined in this manner. In the 50–82 shell, the curve of χ_ν is in reasonably good agreement with the empirically determined curve.[10]

In earlier work[11] we performed IBM calculations for the tungsten isotopes and noted difficulties in matching the experimentally observed small splitting among the energy levels in the β band for the midshell isotopes. We have redone our calculations[9] for these midshell W isotopes, using our two-j-shell predictions of χ_π, χ_ν, and κ and have obtained a remarkable improvement in our agreement with the data. The calculated β band is now much more compressed. This is due primarily to the predicted, low, negative value of χ_ν. Previously we had used positive values for χ_ν for several midshell isotopes.

It turns out that fairly negative values for both χ_ν and χ_π are needed to give a good SU(3) or rotational dynamical symmetry in the midshell region.[1] Thus the midshell region where χ_ν is roughly constant and negative, as seen in Fig. 2, is an important result, since it helps account for the rotational character of nuclides in the midshell regions.

In closing, we note that our method is being extended to the general case of many, nondegenerate, single-particle levels, and hence the previous restrictions of the D pair creation operator can be eliminated. We believe, however, that, in light of the natural grouping of the single-particle levels in the rare-earth region, our two-j-shell predictions give a good account of the gen-

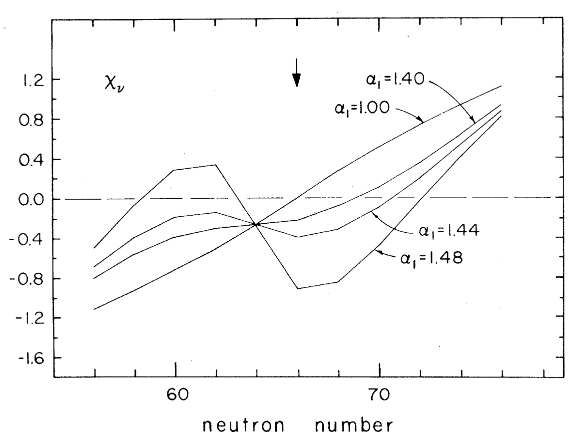

FIG. 1. Two-j-shell predictions for χ_ν vs neutron number in the 50–82 shell for various values of weighting factor, α_1. The arrow indicates midshell.

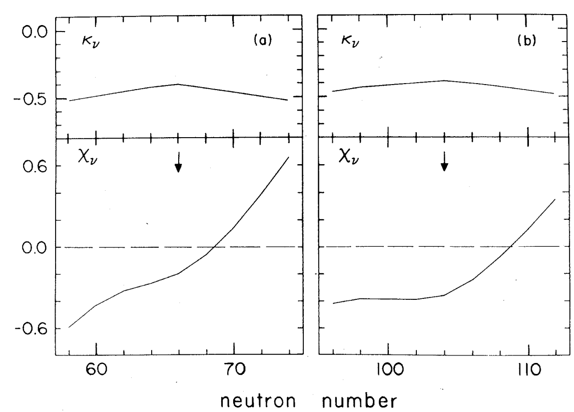

FIG. 2. Two-j-shell predictions for κ_ν and χ_ν in (a), the 50–82 shell ($\alpha_1=1.39$, $\alpha_2=0.524$) and (b), the 82–126 shell ($\alpha_1=1.46$, $\alpha_2=0.465$). The units for κ_ν are arbitrary. The arrows indicate midshell.

eral parameter trends.

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Measurement of the Average and Longitudinal Recoil Polarizations in the Reaction $^{12}C(\mu^-, \nu)^{12}B$(g.s.): Magnitude of the Induced Pseudoscalar Coupling

L. Ph. Roesch, V. L. Telegdi, P. Truttmann, and A. Zehnder

Laboratory for Nuclear Physics, Swiss Federal Institute of Technology, CH-8093 Zurich, Switzerland

and

L. Grenacs and L. Palffy

Institut de Physique Corpusculaire, Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium

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The polarizations, $P_{av} \equiv \langle \vec{J} \cdot \vec{\sigma}_\mu \rangle / J$ and $P_L \equiv \langle \vec{J} \cdot \hat{v} \rangle / J$, of the recoils $^{12}C(\mu^-, \nu)^{12}B$(g.s.) were measured *simultaneously* by selective recoil implantation. Their ratio R' is largely immune to the systematics of P_{av} and P_L and more dependent on the dynamics than either. Our result (normalized to unit $\vec{P}_\mu$), R(g.s.) = − 0.506(41), yields g_P/g_A = 9.0(1.7) (impulse approximation) and $F_P/F_A\,(q_m^{\,2}) = -1.03(14)$ (elementary-particle treatment), to be compared to partial conservation of axial-vector current (PCAC) predictions of 7 and − 0.99, respectively. Thus PCAC is quantitatively verified.

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The reaction $^{12}C(\mu^-, \nu)^{12}B$(g.s.) is for several reasons particularly useful for elucidating the nature of the hadronic axial current. On the one hand, all the observables of this transition are accessible to actual measurements; on the other hand, a sufficient number of well-determined observables (e.g., β-decay asymmetry coefficients) in related transitions in the $A = 12$ triad is available to extract the relevant form factors rather uniquely. This is particularly true for the "induced" pseudoscalar piece of the current which, as is well known, does not intervene appreciably in β decay and which constitutes the last quantitatively open question in this field. In fact, other μ-capture experiments were so far prevented either by statistical limitations and/or by theoretical difficulties in interpretation [e.g., μ capture by protons, $^{16}O(\mu^-, \nu)^{16}N(0^-)$] from providing a reliable answer of sufficient accuracy.

In a $0 \rightarrow 1$ capture there are three independent observables,[1] viz., the capture rate Γ^{cap}, the polarization P_{av} of the recoil nucleus (here ^{12}B) along the muon spin, $P_{av} \equiv \langle \vec{J} \cdot \vec{\sigma}_\mu \rangle / J$, and the longitudinal polarization of the recoil nucleus, $P_L \equiv \langle \vec{J} \cdot \hat{v} \rangle / J$ (where $\langle \vec{\sigma}_\mu \rangle = \vec{P}_\mu$ = muon polarization, $\vec{J}$ = nuclear spin, $\hat{v}$ = recoil direction).[2] These ob-