erogeneity than are the extrapolated high-field data.

Thus the amorphous particles produced by spark erosion appear to have a significantly higher degree of chemical disorder than do amorphous ribbons of the same composition because of the higher quench rate from the liquid. The decreased CSRO is reflected in lower μ_{Fe}, T_c, H_{mean}, and other consistent changes in $p(H)$. An anomalously wide transition in the critical region in low applied fields is puzzling in view of a more regular behavior in the critical region in high fields. However, it is clear from the above that amorphous ribbon represents only one class of the amorphous state.

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Spin Waves in a Disordered Medium: A Simple Model with a Mobility Edge

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A computer simulation of an isotopically disordered harmonic crystal with positive and negative masses is presented. This system may be related to a Heisenberg-Mattis random magnet, for which our results give the elementary excitations. Ideas of percolation theory are employed to explain the existence of a mobility edge in two and three dimensions whenever the positive or negative masses, but not both, percolate. For the corresponding random magnet this implies a new kind of magnon spectrum in which localized and extended states coexist.

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Spin waves are *delocalized* boson excitations. They were found[1] as elementary excitations in quantum spin Heisenberg models describing ordered media with translational invariance. However, it has remained unclear whether spin waves can exist in a disordered medium without translational invariance or a well-defined wave vector $\vec{k}$. To examine this question we consider the Heisenberg-Mattis model[2,3] of a random magnet, characterized by the Hamiltonian

$$\mathcal{H} = -\sum_{i,j} \xi_i \xi_j J_{ij} \vec{S}(i) \cdot \vec{S}(j). \qquad (1)$$

The $J_{ij} = J(|i-j|) \geq 0$ represent a finite-range interaction on a d-dimensional cubic lattice, and the ξ_i's are independent, identically distributed random variables taking values $+1$ and -1 with probabilities p and $q = 1 - p$, respectively. Fixing

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p we may take $\langle S_3(i)\rangle = \xi_i S$ as the classical Néel ground state. The model is of some relevance[2] to the spin-glass problem, particularly for $p = q = \frac{1}{2}$, since it may exhibit frozen magnetism without long-range order.

In this Letter, we examine the excitation spectrum by exploiting the equivalence[4] of this model to an isotopically disordered harmonic crystal with positive *and negative* masses. We find no spin waves in one dimension, spin waves when $p < 1 - p_c$ or $p > p_c$ in two dimensions, and spin waves for all p in three dimensions; p_c is the percolation threshold. In many cases there are also localized states in the *same* energy domain. This is best understood in terms of a mobility edge in the equivalent disordered crystal.

The equations of motion for the $S_+(j)$ may be linearized about the ground state, through $S_3(i) := \langle S_3(i)\rangle$, so as to read

$$i\,\partial \vec{S}_+/\partial t = \Phi U \vec{S}_+. \qquad (2)$$

Here we have defined the diagonal unitary matrix U via $(U\vec{x})_i = \xi_i x_i$; since $\xi_i^2 = 1$, $U = U^{-1} = U^\dagger$. For convenience we consider nearest-neighbor interactions only, and put $J_{i,i+1}S = 1$. Then Φ is nothing but the negative of the discretized Laplacian,

$$(\Phi\vec{x})_i = (2d)x_i - \sum_{\langle i,j\rangle} x_j, \qquad (3)$$

where the sum is over all nearest neighbors j of the site i.

Equation (2) leads to the classical eigenvalue problem

$$U\Phi\vec{x} = \omega\vec{x}, \qquad (4)$$

or equivalently, as $U = U^{-1}$,

$$\Phi\vec{x} = \omega U\vec{x}. \qquad (5)$$

The last equation makes clear the relation to an isotopically disordered harmonic crystal; Φ may be interpreted as the interaction matrix and U as the mass matrix, with masses $\xi_i = \pm 1$. In determining the eigenstates of $U\Phi$,[5] one may take advantage of the ideas and intuition which have been developed for the usual isotopic disorder problem.[6]

A complete solution of the elementary excitation problem involves transforming the Hamiltonian (1) into a quadratic boson Hamiltonian by use of a Holstein-Primakoff transformation. This gives the Bose-Einstein statistics which does not follow from (2). In diagonalizing the resulting Hamiltonian one then discovers[4,7] (a) that the relevant symmetry is hyperbolic $[O(n,n)]$ instead of orthogonal $[O(2n)]$; (b) that the eigenvectors of $U\Phi$ determine the diagonalizing transformation completely; (c) that an eigenvalue ω of $U\Phi$ corresponds to an elementary excitation of energy $\hbar|\omega|$—the *absolute value of* ω must be used. Eigenstates of $U\Phi$ may be localized or delocalized, and map into localized or delocalized elementary excitations of $\mathcal{H}$. The delocalized eigenstates correspond to spin waves.

Our procedure to exhibit localization and to find a mobility edge is straightforward in principle, although computationally somewhat expensive. We choose a large random U, find all the eigenvalues and eigenvectors of $U\Phi$, and examine the inverse participation ratio (IPR),[8]

$$\sum_i x_i^4 / (\textstyle\sum_i x_i^2)^2, \qquad (6)$$

of each eigenvector $\vec{x}$. Delocalized states are expected to have small IPR, of order N^{-1} for N sites, while localized states show larger IPR val-

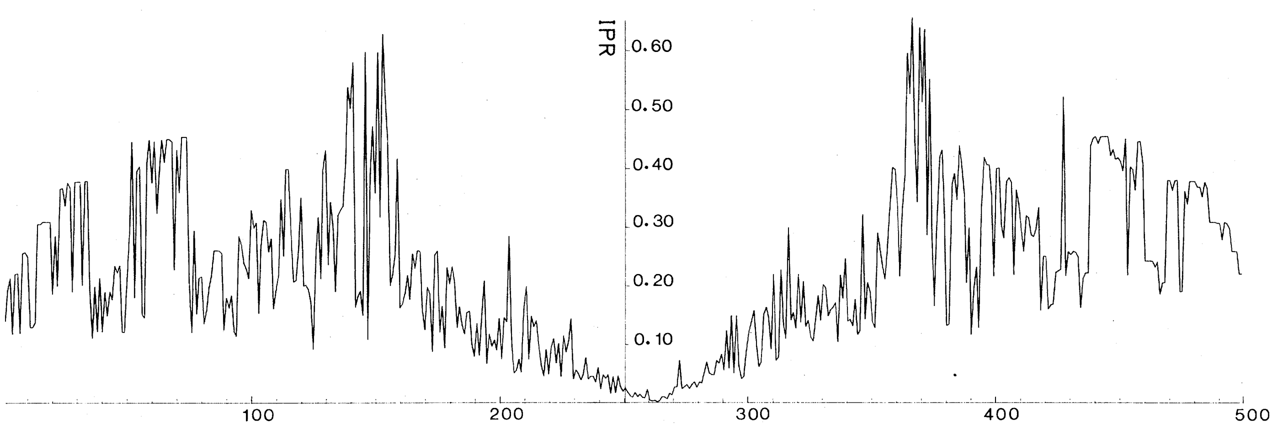

FIG. 1. One dimension: The inverse participation ratio (IPR) for a linear chain of length 500 at $p = 0.50$. The horizontal axis is simply the eigenvalue label k in a sequence $\omega_1 < \omega_2 < \cdots < \omega_k < \cdots < \omega_N$. There is no mobility edge and all states, except $\omega = 0$, are localized in an infinite system.

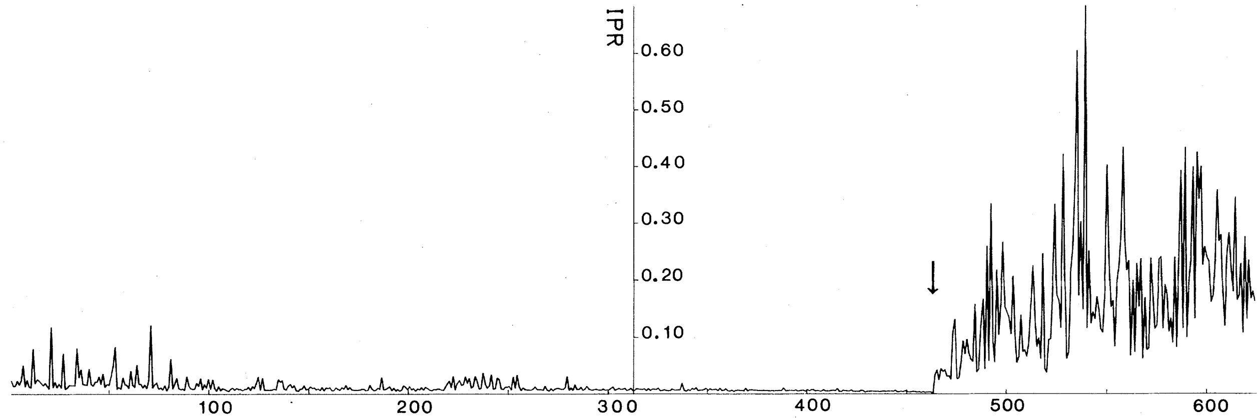

FIG. 2. Three dimensions: IPR for an $8\times 8\times 8$ array at $p = 0.25$. The arrow indicates the mobility edge at $\omega = 0$.

ues.

We have performed the diagonalization in one, two, and three dimensions with lattice sizes up to 500, 25×25, and $8\times 8\times 8$, respectively, employing periodic-boundary conditions. We use the iterative QR algorithm to guarantee numerical stability, knowing that the eigenvalues are not degenerate because of the randomness. For a given p *we need not average* over many U's since the localization/delocalization structure should occur with probability one.

To display the results, we arrange the eigenvalues in an ascending sequence $\omega_1 < \omega_2 < \cdots < \omega_k < \cdots < \omega_N$, and plot the IPR against the label k. The eigenvalue $\omega = 0$ occurs at $k \approx pN$, and is always present since it corresponds to the uniform mode $x_i = \text{const}$. Clearly this eigenvector is extended; in fact, it minimizes the IPR of $\vec{x}$.

In *one dimension,* we expect,[4] on the basis of the exponential growth phenomenon, a pure point spectrum with well-localized eigenvectors. This is indeed what we find (Fig. 1), except for a small dip in the IPR around $\omega = 0$ which we interpret as a finite-size effect. Because all states are localized, there are no spin waves in one dimension.

In *three dimensions,* the results depend on whether p and q are above or below the percolation threshold[8-10] $p_c \approx 0.307$. At $p = q = \frac{1}{2}$, we find mainly delocalized states, but localized states appear for $\omega > 0$ at $p < p_c$ (Fig. 2), and for $\omega < 0$ at $q < p_c$. This is to be expected, since in an infinite system the delocalized eigenstates are associated with an infinite cluster[11] either of positive or of negative masses, corresponding to positive or negative eigenvalues, respectively. Whenever $p < p_c$ (as in Fig. 2) or $q < p_c$, so that there is only one infinite cluster and one species gets delocalized while the other has to remain localized, we find a mobility edge at $\omega = 0$ for the disordered crystal. This is *not* observable as such in the random magnet, however, because the positive and negative eigenvalues are combined so as to

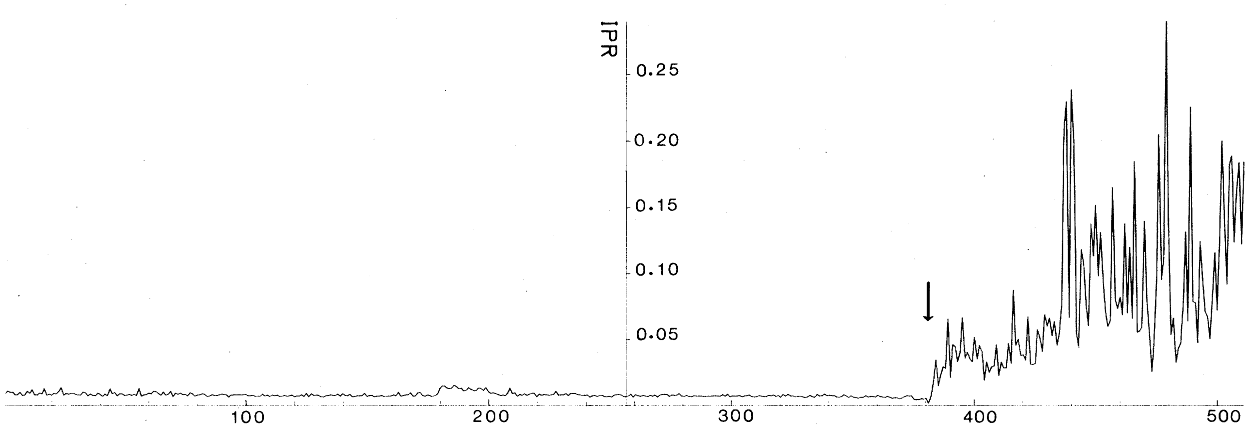

FIG. 3. Two dimensions: IPR for a 25×25 array at $p = 0.30$. The arrow indicates the mobility edge at $\omega = 0$.

form the positive $|\omega|$ excitation spectrum. Now both localized and extended states occur in the same energy range. We thus find spin waves for any p in three dimensions, with localized states densely superimposed in the same energy range for $p < p_c$ or $q < p_c$.

It may be well to point out that, whatever the dimension, a single negative mass in an infinite sea of positive masses always gives rise to a bound state of negative energy.

In *two dimensions,* our results are similar to, though somewhat less clear-cut than those in three dimensions. A mobility edge appears at $\omega = 0$ for $p < p_c < q$ (Fig. 3) or $q < p_c < p$. Now, however, there is a regime where p and q are *both* less than $p_c \approx 0.59$, giving localization of both positive and negative eigenstates. For example, if $p = q = \frac{1}{2}$, we find fairly high IPR values over the whole frequency range, except for $\omega \approx 0$, as in Fig. 1. Thus in two dimensions we have spin waves superimposed on localized states when $p < 1 - p_c$ or $p > p_c$, but only localized states for $1 - p_c < p < p_c$. This may explain the difficulties experienced by Ching and Huber[3] in analyzing their data.

Of course, we have to keep in mind that the states in the delocalized domain *could* be "very weakly localized.[12]" After all, a numerical experiment is not a mathematical proof. Moreover, the mere existence of a mobility edge contradicts, at first sight, some recent results[12,13] for the two-dimensional Anderson model. But one has to realize that the distribution of the masses is *discrete,* whereas the energy distribution for the Anderson model is taken to be continuous (Gaussian[13]). Full details, including the density of states, will be presented elsewhere.

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