Self-Consistent Random-Phase-Approximation Calculations of Muon Capture Rates

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(Received 12 December 1980)

Total muon capture rates on nuclei are calculated for ^{16}O and ^{40}Ca. The distribution of muon capture strength is calculated with use of random-phase–approximation and Tamm-Dancoff–approximation methods in a response-function approach. The random-phase–approximation calculation is in excellent agreement with data.

PACS numbers: 25.40.Hc, 21.60.Jz

This Letter is concerned with the question of muon capture in nuclei. The capture rate proceeds through the elementary process $\mu^- + p \to n + \nu_\mu$ from the $1s$ atomic orbit.[1] The nucleus which captures the muon is excited in the process (to about 25 MeV excitation energy). The states which are predominately reached are the giant multipole vibrations, the analogues of which are known from photoexcitation processes for lighter nuclei.[2] The excited nucleus decays by emitting nucleons, while the difference (about 80 MeV) between the muon mass and the excitation energy of the nucleus is carried away by the neutrino; thus, the mean momentum transfer is $\bar{q} = E_\nu/\hbar c \cong 0.4$ fm^{-1}.

The Primakoff theory[1] gives the following expression for the capture rate Λ_μ of the $1s$ muon:

$$\Lambda_\mu = (m_\mu^2/2\pi\hbar^2 c)|\varphi_\mu|_{\rm av}^2[G_V^2 M_V^2 + 3G_A^2 M_A^2 + (G_P^2 - 2G_P G_A)M_P^2] + \Lambda_\mu', \qquad (1)$$

where $|\varphi_\mu|_{\rm av}$ is the $1s$ muonic atomic wave function averaged over the nucleus. The Λ_μ' is the recoil correction part of Λ_μ.[2] The G_V, G_A, and G_P are the vector, axial-vector, and induced pseudoscalar coupling constants, respectively. The M_V, M_A, and M_P are the matrix elements associated with these vector, axial-vector, and induced pseudoscalar couplings. These matrix elements are given by[3]

$$M_\alpha = \sum_n \left(\frac{\nu_{n0}}{m_\mu}\right)^2 \int \frac{d\hat{\nu}}{4\pi} \left|\langle n|\sum_i O_\alpha(i)|0\rangle\right|^2. \qquad (2)$$

For the vector coupling $\alpha = V$, $\theta_V(i) = \exp(i\vec{q}_{n0}\cdot\vec{r}_i)t_i^+$, for the axial-vector coupling $\alpha = A$, $\theta_A(i) = \exp(i\vec{q}_{n0}\cdot\vec{r}_i)\vec{\sigma}_i t_i^+$, and for the pseudoscalar coupling $\alpha = P$, $\theta_P(i) = \exp(i\vec{q}_{n0}\cdot\vec{r}_i)\vec{\sigma}\cdot\vec{q}_{n0}t_i^+$. The ν_{n0} is the energy of the emitted neutrino, $\nu_{n0} = [m_\mu c^2 - (M_n - M_p)c^2 - E_{n0}]$, where E_{n0} is the excitation energy in the final nucleus $[Z-1, N+1]$ measured from the ground-state energy in the nucleus $[Z, N]$. The $\vec{q}_{n0}$ is the momentum of the neutrino.

As in most of the previous evaluations of the transition rate Λ_μ, we will make the assumption $M_V^2 = M_A^2 = M_P^2$. The equality of the vector, axial-vector, and pseudoscalar matrix elements is valid only if SU(4) is a good nuclear symmetry. However, some detailed calculations[4] give $M_V^2 \cong M_A^2 \cong M_P^2$ and therefore we will assume these equalities. Moreover, since the spin-dependent part of the effective nucleon-nucleon interaction which will be used below is not well adjusted, a reliable calculation of M_A^2 and M_P^2 can be questioned. In a recent random-phase–approximation (RPA) calculation of pion and muon capture in ^{208}Pb, Ebert and Meyer-Ter-Vehn[5] have found $(M_V^2 - M_P^2)/M_V^2 \cong 8.5\%$ and $(M_V^2 - M_A^2)/M_V^2 \cong 19\%$; thus even for heavy nuclei, where SU(4) symmetry is broken more than in light nuclei, $M_V^2 \simeq M_P^2 \simeq M_A^2$. The relation between the total capture rate and the M_V^2 matrix element is then given by[2]

$$\Lambda_\mu = (m_\mu^2 G^2/2\pi\hbar^2 c)\pi^{-1}(Z\alpha m_\mu)^3 R M_V^2 + \Lambda_\mu'. \qquad (3)$$

The first term in Eq. (3) can be written as

$$\Lambda_\mu^{(0)} = 281 R Z^3 |M_V|^2 \ \mathrm{s}^{-1}. \qquad (4)$$

The R in Eqs. (3) and (4) expresses the effect of the muon wave function of Eq. (1).[2]

The evaluation of M_V^2 has proceeded by two different methods. One method involves a closure approximation, while the other involves a term-by-term sum with use of detailed wave functions for the excited states $|n\rangle$ of Eq. (2). We briefly mention the closure approximation here and discuss in detail the other method.

The closure approximation is obtained by defining a mean neutrino energy so that $\sum \nu_{n0}^2 \times |\langle n|\theta_V^+|0\rangle|^2 \equiv \bar{\nu}^2\langle 0|\theta_V^-\theta_V^+|0\rangle$. A direct evaluation of $\langle 0|\theta_V^-\theta_V^+|0\rangle$ with use of a Slater-determinant wave function would give[6]

$$F_V^2 = Z - \sum_{\alpha_i(p),\alpha_j(n)} |\langle \alpha_i(p)|e^{i\vec{q}\cdot\vec{r}}|\alpha_j(n)\rangle|^2, \quad (5)$$

where $\alpha_i(p)$ and $\alpha_j(n)$ are the single-particle proton and neutron orbitals in the ground-state nucleus, respectively. For example, an oscillator description of ^{40}Ca results in

$$F_V^2 = 20 - 20e^{-x}(1 + \tfrac{1}{2}x^2 - \tfrac{1}{10}x^3 + \tfrac{1}{40}x^4), \quad (6)$$

where $x = \bar{q}^2/2\nu$, with $\nu = m\omega/\hbar$. The calculated value of F_V^2 turns out to be twice the value obtained from experiment for ^{40}Ca for $\bar{q} = (80\text{ MeV})/\hbar c$. This factor-of-2 discrepancy between the sum-rule approach which uses a Slater-determinant description of $|0\rangle$ and experiment persists for all nuclei. Thus the closure approximation in its simplest form fails to account for the capture rates. In using the closure approximation, a mean neutrino energy was introduced. This approximation can be made better by introducing energy-weighted sum rules into the description[7] besides the non–energy-weighted strength. Recent attempts in this direction give a capture rate which is 50% larger than experiment.[8] Another improvement is to include ground-state correlations when evaluating $\langle 0|0_V^- 0_V^+|0\rangle$ which also somewhat reduces the discrepancy.

In the present work we calculate Eq. (3) using a development of the response-function method given in Ref. 9. In this approach, we can obtain the distribution of strength of one-body operators using the RPA and Tamm-Dancoff–approximation (TDA) methods in their Green's-function representation. Since μ capture involves a $\Delta T = 1$, $\Delta T_z = +1$ excitation, we employ a generalization of the RPA scheme for such excitations.[10-12] For the lighter nuclei ($A \leq 40$), the capture involves the excitation of $J = 0^+$, 1^-, 2^+, and 3^- isovector excitations, which account for 99% of the vector μ-capture rate.[4] We should emphasize that our calculation contains the complete one-particle, one-hole spectrum including the particle continuum. The calculation employed Skyrme-type forces[13] and we used two types, the SIII and the recently suggested SKM,[14] which gives a more correct value for the compression modulus. Both forces are similar as far as the symmetry energy effects in the nucleus are concerned. They generally give the correct position of the known giant resonances[15] and they fit separation energies,[16] radii, and Coulomb displacement energies.[17]

After calculating the RPA Green's function, the distribution of strength is evaluated for each multipole and for each neutrino momentum transfer q_{n0} corresponding to the nuclear excitation energy. The energy range included in the integration was such as to exhaust about 99% of the strength for each of the considered multipoles. For the 0^+ and 2^+ multipoles, this required us to take into account rather high-energy tails of the respective distributions.

Tables I and II give our results for the capture rate for ^{16}O and ^{40}Ca. The values of R of Eq. (4) were taken from Ref. 2 and they are 0.79 and 0.44 for ^{16}O and ^{40}Ca, respectively. The contribution of each multipolarity to the total μ-capture rate $\Lambda_\mu^{(0)}$ of Eq. (4) is listed in these tables. For

TABLE I. Muon capture rate in ^{16}O (all units are 10^5 s^{-1}). Experimentally the capture rate $\Lambda_\mu = 0.97$.

Force		$L=0$	$L=1$	$L=2$	$\Lambda_\mu^{(0)}$	Λ_μ
SIII	single-particle	0.04	1.44	0.12	1.60	1.76
	TDA	0.02	0.93	0.07	1.02	1.18
	RPA	0.02	0.71	0.07	0.80	0.96
SKM	single-particle	0.05	1.60	0.14	1.79	1.95
	TDA	0.03	1.07	0.09	1.19	1.35
	RPA	0.02	0.80	0.08	0.90	1.06

TABLE II. Muon capture rate in ^{40}Ca (all units are 10^5 s^{-1}). Experimentally the capture rate $\Lambda_\mu = 25.5$.

Force		$L=0$	$L=1$	$L=2$	$L=3$	$\Lambda_\mu^{(0)}$	Λ_μ
SIII	single-particle	2.10	41.0	4.78	1.58	49.46	53.06
	TDA	0.96	23.13	4.0	0.50	28.59	32.19
	RPA	0.82	15.50	3.58	0.46	20.36	23.96
SKM	single-particle	2.40	43.0	4.9	1.65	51.95	55.55
	TDA	1.12	23.50	4.20	0.55	29.37	32.97
	RPA	0.89	15.82	3.85	0.50	21.06	24.66

each interaction, the first row gives the results for a single-particle model in which the residual particle-hole interaction is neglected and the calculated strength is determined by the Hartree-Fock (HF) states only. The total capture rate for the single-particle model agrees with the closure result obtained from Eq. (6) for ^{40}Ca. The next row gives the TDA results while the last row lists the RPA results. A comparison of the results of rows one and three shows that in ^{16}O the RPA evaluation is a factor of 2 smaller than the pure single-particle results whereas in ^{40}Ca this factor is even larger. The total TDA results fall in between these two. Note that the non–energy-weighted sum-rule calculation would not be affected by a TDA calculation since the ground state remains unchanged and therefore so does $\langle 0 | \theta_V^- \theta_V^+ \times | 0\rangle$. Thus the difference between rows one and two arises from the correlations in the excited states.

It was noted in the past that the non–energy-weighted transition strength to the giant dipole depends on the particle-hole correlations and this strength is strongly reduced when these are included.[18] Indeed, in a recent work, the photonuclear bremstrahlung sum rule $\sigma_{-1} \equiv \int \sigma(E) E^{-1} dE$ was evaluated with use of the same method as in this work and the results (in the case of the two forces used here) agree well with experiment.[19] Of course, as discussed by Foldy and Walecka,[2] the photonuclear cross section to the $J=1^-$ states is closely connected to the μ-capture rate because of isospin symmetry and because, in lighter nuclei, the μ-capture rate is dominated by the dipole (as can be seen in Tables I and II). We should note, however, that although the μ-capture rate is largely determined by the non–energy-weighted strength, in order to achieve an accurate theoretical result one must take into account linearly and higher-order energy-weighted sum rules. These higher moments of the strength distribution are subject to exchange-current corrections.

The last columns in Tables I and II give the total muon-capture rates including recoil corrections as obtained from Foldy and Walecka. From this column we see that the RPA results with recoil included are in excellent agreement with experiment.

To summarize, this Letter has reported a calculation of muon capture rates for ^{16}O and ^{40}Ca. These calculations, we feel, are the most detailed and least approximate of the various methods for calculating muon capture. Our calculations show that the RPA correlations are important in obtaining agreement with data. We intend to extend the calculation to heavier nuclei, especially to those with a considerable neutron excess. Another aim will be to study the Primakoff parametrization of the capture rate as a function of A and Z.

We thank N. C. Mukhopadhyay for several interesting discussions. This research was supported in part by a grant from the U. S.-Israel Binational Science Foundation, Jerusalem, and in part by the National Science Foundation.

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Identification of $\Delta S = 1$ Transitions in ^{13}C by Measurement of Pion Inelastic Excitation Functions

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(Received 17 February 1981)

Differential cross sections for $^{13}\mathrm{C}(\pi,\pi')$ were measured between 100 and 300 MeV for momentum transfers of $1.1\hbar\ \mathrm{fm}^{-1}$ and $1.4\hbar\ \mathrm{fm}^{-1}$. In this energy range the different energy dependences of the spin-dependent and spin-independent parts of the pion-nucleon interaction provide a very sensitive method of discriminating between transitions that proceed with a spin transfer ($\Delta S = 1$) or without a spin transfer ($\Delta S = 0$). Five transitions in ^{13}C were found to be dominated by the $\Delta S = 1$ transition density amplitude.

PACS numbers: 25.80.+f, 27.20.+n

The work of Moore *et al.*[1] indicated that new information can be obtained from the measurement of inelastic pion excitation functions. In this Letter we report on the use of excitation functions of inelastic pion scattering to distinguish between $\Delta S = 0$ and $\Delta S = 1$ transitions, where ΔS is the spin transfer to the target nucleus. Differential cross sections for two transitions in ^{13}C, known to proceed predominantly by $\Delta S = 0$, were found to have energy dependences very different from that of a recently determined[2,3] pure $\Delta S = 1$, pure neutron particle-hole excitation of a stretched state. Four other transitions were found to be dominated by $\Delta S = 1$. Such an effect was also seen in the work of Cottingame *et al.*[4] in which natural and unnatural parity transitions in $^{12}\mathrm{C}(\pi,\pi')$ had dramatically different energy dependences. The explanation[5] of these different energy dependences is based on two facts. Firstly, at energies near the [3, 3] resonance the spin-dependent and spin-independent parts of the pion-nucleus scattering amplitude have quite different energy dependences for a given momentum transfer. Secondly, transitions that involve a spin transfer, $\Delta S = 1$, to the target can be caused only by the spin-dependent part of the force and transitions without a spin transfer, $\Delta S = 0$, are predominantly due to the spin-independent part of the force.

The pion-*nucleon* scattering amplitude can be written in the following form if the interaction is dominated by the [3, 3] resonance,[6]

$$f(k,k') = \alpha(k)(2\cos\theta + i\vec{\sigma}\cdot\hat{n}\sin\theta),$$

where k (k') is the pion momentum before (after) the collision, θ is the scattering angle in the pion-nucleon center-of-mass frame, $\vec{\sigma}$ is the spin operator for the nucleon, and $\hat{n}$ is the normal to the scattering plane. The coefficient $\alpha(k)$ is given in terms of the pion-nucleon phase shift δ by $\alpha(k) = k^{-1}\exp(i\delta)\sin\delta$. Only the operator $\vec{\sigma}\cdot\hat{n}$ can

